The Frame–Budget Approach (FBA)

How time, dynamics, and geometry emerge from budget flows

An operational bridge between quantum mechanics and general relativity

Part X: Predictions, Falsifiability & Bridge FBA $\to$ QM $\leftrightarrow$ GR

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Part X

Predictions, Falsifiability & Bridge FBA → QM ↔ GR

X.1 Introduction & Target Picture

X.1.1 Motivation

This treatise bundles the empirically testable consequences of the Frame–Budget Approach (FBA)[1] and formulates an explicit bridge between FBA, quantum mechanics (QM), and general relativity (GR). The focus is not on yet another interpretational layer, but on a *test scaffold*: from well-defined inputs (channels, fronts, calibration) an evaluation is to emerge that ends in *observable quantities* and whose comparison can be formulated as a clear decision.

The core idea is that *H-gates* and a family of *proxies* (among others Hadamard/cross-basis structure, Tolman redshift factor, budget closure & KMS proximity, QNEC-near zero-flux conditions, front/signal protocols) serve as operational transition and consistency criteria. Here, "QNEC-near" is meant purely *operationally*: as a class of zero-flux/focusing *proxies* in the budget calculus, without presupposing an independent QFT QNEC theorem. These elements are necessary because the QM↔GR bridge otherwise easily collapses into a mere model or coordinate choice: proxies bind the translation to measurement protocols and thus provide unambiguous pass/fail signatures instead of mere "plausibility".

Starting from the sequence and budget principles, FBA thus provides a computational path from inputs to observables in the laboratory, in astrophysics, and in cosmology. The logical path rests on imported building blocks and uses their consequences for proper time, Minkowski limit, CPTP/GKLS dynamics, and locality.[2] [3] [4] [5] [6]

X.1.2 Logical path

The bridge is constructed such that each level has a concrete operational role: each new structure is introduced only once it closes a previously open translation question, and in the end the tests should be formulable as pass/fail decisions.

1. **Sequence & budget.** Global frames and minimal events define a strict order and a budget balance (internal/external/irreversible).
2. **Calibration & front.** External fronts fix operational boundary rates (signal speed) and couple measurement protocols to a reference.

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[1]An overview of all parts of the FBA treatise including download links can be found in Section X.11 of this document.

[2]See FBA Part I: FBA – Foundations, Sec. I.1–I.3 "Sequence, budget & calibration".

[3]See FBA Part II: Time, Proper Time & Minkowski Geometry, Secs. II.3–II.8 "Time, proper time & Minkowski geometry".

[4]See FBA Part III: Quantum Kinematics & CPTP Channels, Secs. III.3–III.5 "Quantum kinematics & CPTP channels".

[5]See FBA Part IV: Dynamics & Measurement (GKLS), Secs. IV.3–IV.7 "Dynamics, measurement & GKLS (open systems)".

[6]See FBA Part V: Spacetime, Light Cones & Local Field Theory, Secs. V.3–V.6 "Spacetime, light cones & local field theory".

3. **Bridge FBA→QM.** State/channel description (CPTP) and measurement (Born/instruments) provide the operational QM level. GKLS semigroups appear *only in the unselective, Markov/semigroup and coarse–graining scope* as the canonical effective model of admissible open dynamics; selective measurement branches are in general CP and trace-non-increasing, but not CPTP. DPI/Spohn then provides (in the unselective scope) a testable monotonicity that secures later statements about dissipation and measurement costs.
4. **Bridge FBA→GR.** The budget quadric generates light cones and the Minkowski limit; inhomogeneous budget flows encode *in the proxy regime* effective geometry and redshifts. Thus "geometry" becomes shorthand for a *kinematic* deviation diagnostics and not a presupposed stage (in particular without field-equation commitment).
5. **H-gates & proxies.** Bridge elements (H-gates) and proxy criteria (Tolman, KMS/closure, QNEC-near conditions, front protocols) operationalize consistency between the descriptions. They are the mechanism that controls translations between regimes and breaks degeneracies.
6. **Computational calculus.** Algorithmic sketches (A–D) translate FBA setups into observables (channel rates, causal tests, redshift/lensing profiles, TDI-driven distance ladders and drifts).
7. **Pass/fail.** Each proxy family induces binary tests and falsification paths.

X.1.3 Scope and delimitations

This Section is *operationally* oriented: We take the definitions and lemmas fixed in the foundations as given and build the bridge and test structure on top of them.[7] This is not only a matter of brevity, but a consistency decision: if we were to re-derive the basic constructions here, duplication would arise and the pass/fail logic would be watered down by parallel justifications.

The reconstruction of spacetime, the dynamics of open systems, and the cosmic TDI coupling are therefore not proven anew, but only summarized to the extent necessary for the prediction and test modules. Details on geometry from budget flows, scales/renormalization, and the classical limit are treated in the respective parts and are used here as input.[8] [9] [10] [11]

X.1.4 Contribution relative to standard QM/GR/cosmology

Standard approaches *postulate* spacetime and field structures or implement the QM↔GR bridge via additional principles. In the series, light cones, causality, admissible dynamics (in the unselective Markov scope: GKLS as an effective model), and measurement structures are derived from the sequence/budget and channel principles; Part X *assumes* these results as imported building blocks and organizes them into a testing program. DPI/Spohn thereby

[7] See FBA Part I: FBA – Foundations, Sec. I.1–I.3 "Sequence, budget & calibration".

[8] See FBA Part VI: Gravity & Geometry from Budget Flows, Secs. VI.3–VI.6 "Gravity & geometry from budget flows".

[9] See FBA Part VII: Constants, Scales & Renormalization, Secs. VII.1–VII.5 "Constants, scales & renormalization".

[10] See FBA Part VIII: Classical Limit, Thermodynamics & Aging, Secs. VIII.3–VIII.8 "Classical limit, thermodynamics & aging".

[11] See FBA Part IX: Cosmic Dynamics, Time Dilation & Inflation (TDI), Secs. IX.3–IX.7 "Cosmic dynamics & Time-Dilation Inflation (TDI)".

provides (again in the unselective scope) not only a “direction”, but an operationally testable monotonicity that secures later statements about dissipation and measurement costs. The added value is not “different words”, but a different error culture: the bridge is formulated such that it can fail at several independent places.

The introduced H-gates/proxies moreover provide *measurable* pass/fail criteria (e. g. Tolman profiles as a budget proxy, KMS proximity as a stationarity proxy, QNEC-near zero-flux tests) and thus deviation signatures beyond pure model fits. The approach therefore prioritizes *falsifiable*, scale-aware predictions over ex post parametrization.

X.1.5 Reading guide

Part X is to be read as a *testing program*: from fixed primitives and calibrations, the text leads via an explicit bridge (QM/channel calculus $\leftrightarrow$ geo/cone) to H-gates/proxies, a computational calculus, and finally to clear pass/fail experiments. The following reading guide marks not only *where* something is located, but *which operational role* it plays in the overall workflow (regime check $\rightarrow$ evaluation $\rightarrow$ decision).

Section X.2 - Preliminaries & conventions: Fixes the imported primitives, the budget/front and proper-time conventions, and the notation so that the bridge in Section X.3 cannot be undermined by hidden additional assumptions or silent drift of meaning.

Section X.3 - Bridge theorem FBA $\rightarrow$ QM $\leftrightarrow$ GR: Formulates the actual bridge theorem (mapping, commutativity, DPI/Spohn compatibility) and makes explicit *what* must remain the same between channel/POVM evaluation and geo/cone evaluation so that later tests are not merely “compatible” but decisive.

Section X.4 - H-gates & proxy families (consistency & transition criteria): Makes the bridge operational: H-gates fix the admissible representational freedom up to local isometries; proxy families define measurable regime conditions as pass/fail criteria (Tolman, KMS/closure, QNEC-near, front/no-signalling). Without this level, Section X.3 would be formally correct, but experimentally intangible.

Section X.5 - Computational calculus: from the FBA setup to observables: Provides the reproducible computational path from the FBA setup (incl. H, π, calibration) to observables (rates, profiles, drifts) and propagates uncertainties consistently through proxy and calibration parameters.

Section X.6 - Predictions (catalog, grouped): Catalogs the predictions as test modules and groups them such that failure modes (geometry, stationarity, zero-flux, front) remain separately addressable and degeneracies are not obscured by mixed data views.

Section X.7 - Falsifiability & experiments: Translates the prediction modules into experimental roadmaps, decision logic, and degeneracy breakers, because only the measurement combinations and priorities clarify which proxies actually decide and which merely “fit compatibly”.

Section X.8 - Classification & comparison with standard QM/GR/ΛCDM: Classifies the bridge relative to standard QM/GR/ΛCDM, separates equivalence regimes from deviation regimes, and shows where the proxy structure forces genuine additional tests and deviation signatures.

Section X.9 - Case studies & replication blueprints: Presents case studies and replication blueprints as a reproducible testing program (lab, astro, cosmo incl. TDI), so that the architecture does not remain abstract but is available as a pipeline of input format → evaluation → pass/fail.

Section X.10 - Conclusion: bridge status, decision criteria & open problems: Concludes with bridge status and decision criteria (binary + continuous quality measure) and states open problems such that it is clear which next measurements have the strongest leverage for pass/fail.

X.2 Preliminaries & Conventions (Import from Part I: FBA – Foundations)

Why an import? In Part X the focus is a consolidated bridge and test architecture: H-gates and proxies are to function as operational pass/fail criteria, and the computational calculus is to lead from clear inputs to observables. So that this structure does not silently smuggle in new notions of time, additional dynamical postulates, or hidden geometric assumptions, the carrier concepts must already be fixed: sequence, balance, calibration, proper time, admissibility, and composition. Exactly these primitives and their first consequences are developed in the foundations; here they are not fixed once again, but used as tools to build the following Sections without circularity.[12]

Imported building blocks (unchanged)

We adopt the following building blocks *without* redefinition from Part I: FBA – Foundations and refer in the text to Section/box and heading/title:

- **Sequence of global states & minimal events:** *FBA – Foundations, Sec. I.2 "Global states, frame sequence and minimal event (ME)"; FBA – Foundations, Box I.2 "Co-actuality and refinement invariance".*
- **Difference function & operational minimal difference:** *FBA – Foundations, Box I.2 "Difference function & operational minimal difference".*
- **Budget calculus (internal/external/irreversible) & balance:** *FBA – Foundations, Box I.3 "One-step budget & decomposition"; FBA – Foundations, Formula box I.3 "Balance equations"; FBA – Foundations, Lemma I.3 "Refinement invariance of the balance".*
- **External calibration & front:** *FBA – Foundations, Definition I.3 "Calibration and front costs"; FBA – Foundations, Lemma I.3 "Front bound"; FBA – Foundations, Corollary I.3 "Signal front".*
- **Proper time & aging, Minkowski limit:** *FBA – Foundations, Definition I.4 "Proper time"; FBA – Foundations, Formula box I.4 "Properties of proper time"; FBA – Foundations, Definition I.4 "Aging (irreversible)"; FBA – Foundations, Formula box I.4 "Minkowski limit & quadric"; FBA – Foundations, Lemma I.4 "Time dilation".*
- **Admissible dynamics (CPTP/GKLS), DPI/Spohn:** *FBA – Foundations, Definition I.5 "Admissible Channels (CPTP)"; FBA – Foundations, Formula I.5 "Kraus/Stinespring"; FBA – Foundations, Lemma I.5 "Measurement as CPTP" (here always: unselective overall channel)*; *FBA – Foundations, Definition I.5 "GKLS generators (open systems)"; FBA – Foundations, Formula I.5 "Spohn monotonicity"; FBA – Foundations, Lemma I.5 "Semigroup budget"; FBA – Foundations, Definition/Corollary I.5 "DPI arrow & no-recovery".*
- **Composition, locality & no-signalling:** *FBA – Foundations, Definition I.6 "Symmetric-monoidal structure"; FBA – Foundations, Formula I.6 "Budget additivity"; FBA – Foundations, Lemma I.6 "No-wire inflation & local operations"; FBA – Foundations, Corollary I.6 "Causal cones & local GKLS".*

What is the purpose of this import in the reading path? The box is a circularity lock: In Section X.3 we formulate a mapping and commutativity statements between FBA, QM, and GR, in Section X.4 we define H-gates and proxies as operational consistency criteria, and in

[12] See FBA Part I: FBA – Foundations, Secs. I.2–I.6 "Primitives, budget, calibration, proper time, admissibility, composition".

Section X.5 this becomes a computational calculus to observables. So that these steps do not retroactively “generate” their own prerequisites, it must already be clear here which concepts are inserted as primitives from the foundations and which structures in Part X are actually new.[13]

With the basic concepts thus fixed, we now fix the notation with which computations and comparisons are carried out in Part X. The conventions are chosen so that the same symbols can be used across laboratory, astrophysics, and cosmology modules without a change of meaning, and so that the proxy and calibration dependencies can later enter error propagation and pass/fail rules (see in particular Section X.5 and the test modules starting from Section X.6).

[13] See FBA Part I: FBA – Foundations, Secs. I.2–I.6 “Primitives, budget, calibration, proper time, admissibility, composition”.

Notation & conventions

- **Discrete vs. continuum:** step index $n \in \mathbb{Z}$ for successive frames; $\Delta(\cdot)$ for discrete increments, $d(\cdot)$ for differential quantities in the limit.
- **Budget decomposition & proper-time calibration:** per step δb_{ext} (external) and internally the decomposition $\delta b_{\text{int}}^{\text{tot}} = \delta b_{\text{int}}^{\text{rev}} + \delta b_{\text{irr,int}}$ with $\delta b_{\text{irr,int}} \geq 0$. Path sums $\sum \delta(\cdot)$ and integrals $\int d(\cdot)$. With the proper-time calibration κ_τ (equivalently $\alpha_\tau := 1/\kappa_\tau$) we set

$$d\tau_{\text{geo}} := \frac{db_{\text{int}}^{\text{rev}}}{\kappa_\tau}\,, \qquad dA := \frac{db_{\text{irr,int}}}{\kappa_\tau} \geq 0\,, \qquad d\tau_{\text{tot}} = d\tau_{\text{geo}} + dA.$$

  In the reversible limiting case $db_{\text{irr,int}} = 0$ we have $d\tau_{\text{tot}} = d\tau_{\text{geo}}$.
- **Calibration:** c is the *calibration constant* of the fastest admissible fronts (metrologically fixed). The choice of units is always made via the defined front protocol.
- **Spacetime language (flat, kinematic):** four-vector $x^\mu = (ct, x, y, z)$; Minkowski signature $\eta = \text{diag}(-1, 1, 1, 1)$. *Light cones* via $\eta_{\mu\nu} dx^\mu dx^\nu = 0$.
- **Worldlines & paths:** γ denotes a worldline of a system through the frame sequence; concatenation $\Gamma = \Gamma_1 \circ \Gamma_2$. Additivity of all integrated budgets along Γ.
- **Composition/locality:** parallel composition $\otimes$; serial composition $\circ$. Local CPTP operations respect no-signalling and budget additivity.
- **Bridge elements (H-gates):** symbol H for budget-faithful isometries between complementary representations (FBA basis $\leftrightarrow$ Hilbert-space/channel basis). Additional requirements such as "local tomography" are *not* assumed silently, but are made explicit as part of the H-gate definition and its tests in Sec. 4. Where needed, indices: $\mathsf{H}_{\text{FBA}\to\text{QM}}$, $\mathsf{H}_{\text{QM}\to\text{ART}}$. Definitions and tests follow in Sec. 4.
- **Proxy families & parameters:** collective parameters $\pi \equiv (\pi_{\text{Tol}}, \pi_{\text{KMS}}, \pi_{\text{QNEC}}, \pi_{\text{Front}}, \ldots)$. Examples: π_{Tol} (redshift/temperature profiles), $\pi_{\text{KMS}} = (\beta_{\text{mod}}, \mu_{\text{mod}})$ (modular/KMS proximity), π_{QNEC} (zero-flux/focusing *proxy*, not used as a QFT theorem), π_{Front} (signal/calibration parameters). Each proxy family provides pass/fail rules in Sec. 4 and is turned into concrete tests in Secs. 6–9.
- **DPI/Spohn monotones:** relative entropy $D(\rho\|\sigma)$ (and, if needed, Rényi divergences D_α only in regimes/variants where DPI holds) and the associated production rates $\sigma_{\text{Spohn}} \geq 0$ serve as *operationally estimable* monotones/bounds for unselective processes (used in Sec. 5 and Sec. 6).
- **Prediction scheme:** map $\mathcal{P}$: FBA setup $\mapsto$ observables with outputs as rates, frequencies, drifts, profiles. Uncertainties via $\mathbb{E}[\cdot]$, $\text{Var}[\cdot]$ and propagated errors from proxy and calibration parameters (details: Sec. 5).
- **Sign conventions:** vector norms $\|\cdot\|$; Euclidean inner products "$\cdot$" in space. c explicit (no c=1 units in this treatise). Expectations $\mathbb{E}[\cdot]$; supremum sup; indicator $\mathbf{1}[\cdot]$ for pass/fail.

X.3 Bridge Theorem FBA $\to$ QM $\leftrightarrow$ GR

This Section establishes the bridge in three steps. We deliberately begin on the *QM side*, because laboratory predictions are in practice almost always formulated as statements about channels, measurements, and rates. Only once it is clear which FBA data appear in this language as a well-defined evaluation does the second arrow direction become worthwhile: the reconstruction of the causal and geometric structure from the same primitives. The third step binds both descriptions into a consistency scheme that does not merely "translate" formally, but enables an operational pass/fail structure.

X.3.1 FBA $\to$ QM: Kinematic embedding

The kinematic embedding is a *typing/working definition* for what will later be evaluated as a measurement and channel protocol. It is thus not an "additional dynamical postulate", but a specification of the evaluation framework: Part X works *conditionally* on those FBA setups for which such an embedding (possibly after admissible coarse-graining) can be realized consistently. The corresponding consistency conditions are operationalized in Section X.4 via H-gates/proxies.

Definition X.3.1.1: FBA→QM: Kinematic embedding

Let **FBA** be the category of FBA setups with objects $(\mathcal{S}, \mathcal{B})$ (system carrier $\mathcal{S}$, budget structure $\mathcal{B}$) and morphisms given by minimal events and protocols. A *kinematic embedding* is a functor $\mathcal{Q}\colon \mathbf{FBA} \to \mathbf{Chan}$ into a category **Chan** (objects: state spaces; morphisms: channels), assigning:

1. to each object an (effectively) separable state space $\mathcal{H}$ with effect algebra $\mathsf{Eff}(\mathcal{H})$,
2. to each morphism an *unselective* CPTP channel Φ (or, in the Markov/semigroup scope, a GKLS semigroup element $\Phi_t = e^{t\mathcal{L}}$),
3. to measurement protocols POVMs $\{E_i\}$ with outcome distributions $p(i) = \mathrm{tr}[\Phi(\rho)E_i]$.

Admissibility. The assigned processes are *admissible* in the sense of the budget-constrained channel theory.[a]

[a]See FBA Part I: FBA – Foundations, Sec. I.5 "Admissible dynamics as budget-constrained channels".

Two points are crucial here. First, the QM side is fixed to CPTP (and, in the appropriate effective scope, to GKLS) so that "dynamics" enters the bridge as controllable processing and not as a hidden choice of coordinates. Second, it remains open *how* exactly the concrete representation $\mathcal{H}$ is chosen. This freedom is intentional: it will later be restricted by H-gates and proxy criteria so that, in the end, only local isometries remain as residual freedom (see Section X.4). Further details on the operational derivation of states, POVMs, and channels from frames can be found in Part III; the dynamical side (GKLS, measurement, dissipation) in Part IV.[14] [15]

[14]See FBA Part III: Quantum Kinematics & CPTP Channels, Secs. III.3–III.5 "Quantum kinematics & CPTP channels".

[15]See FBA Part IV: Dynamics & Measurement (GKLS), Secs. IV.3–IV.6 "Dynamics, measurement & GKLS

X.3.2 FBA $\to$ GR: Geometric reconstruction

The geometric arrow direction is necessary because the same observational questions that appear in the laboratory as channel evaluation appear in the astrophysical and cosmological regimes as statements about causal structure, redshift, and profile evolution. If the bridge is to be consistent, this "geometry language" must arise from the FBA primitives, rather than being assumed as a stage.

Definition X.3.2.1: FBA→GR: Geometric reconstruction

Let **Geo** be a category of causal spacetimes (objects: $(\mathcal{M}, g)$ with cone structure; morphisms: causality-preserving maps). A *geometric reconstruction* is a functor $\mathcal{G}\colon \mathbf{FBA} \to \mathbf{Geo}$, assigning:

1. to the budget quadric and front calibration a local Minkowski limit $(\mathbb{R}^{1,3}, \eta)$ with light cones,
2. to inhomogeneous budget gradients effective redshift and temperature profiles as operational proxies,
3. to worldlines γ a proper-time function $\tau[\gamma]$ as integrated internal budget along γ *according to the proper-time calibration κ_τ from Section X.2.*

Causality. The induced cone causality follows from the front bound and signal front as well as the local composition structure.[a] [b]

[a] See FBA Part I: FBA – Foundations, Sec. I.3 "External calibration & front".
[b] See FBA Part I: FBA – Foundations, Sec. I.6 "Composition, locality & no-signalling".

Thus, in the bridge context "geometry" becomes shorthand for two operational statements: (1) which events are causally reachable by fronts, and (2) how internal budgets along worldlines appear as proper time and redshift proxies in measurement protocols. The development of cone reconstruction and local field-theory compatibility lies in Part V; the gravitational profiles from budget flows and their GR proximity in Part VI.[16] [17]

X.3.3 Consistency scheme: One evaluation, two representations

The two functors $\mathcal{Q}$ and $\mathcal{G}$ would be of little value as separate constructions if they merely "coexisted side by side". The actual bridge begins where the same observational question $\mathcal{O}$ can be evaluated independently of the representation. Precisely this demand will later become a source of pass/fail criteria: if a proxy family is violated, the evaluations must diverge.

(open systems)".

[16] See FBA Part V: Spacetime, Light Cones & Local Field Theory, Secs. V.3–V.6 "Spacetime, light cones & local field theory".

[17] See FBA Part VI: Gravity & Geometry from Budget Flows, Secs. VI.3–VI.5 "Gravity & geometry from budget flows".

Formula Box X.3.3.1: Commutative bridge diagram (consistency requirement)

For every FBA setup X and every observational question $\mathcal{O}$ we require, within the validity range of the H-gate conditions (see Definition X.4.1.1), the consistency

$$\underbrace{\mathcal{P}_{\mathrm{QM}}(\mathcal{Q}(X),\mathcal{O})}_{\text{channel/POVM evaluation}} \quad \overset{!}{=} \quad \underbrace{\mathcal{P}_{\mathrm{Geo}}(\mathcal{G}(X),\mathcal{O})}_{\text{geometrically/causally consistent evaluation}} \quad ,$$

where $\mathcal{P}_{\mathrm{QM}}$ (resp. $\mathcal{P}_{\mathrm{Geo}}$) denotes the prediction scheme specified in Section X.5 on the QM (resp. geometry) side. A violation of this requirement (with proxies/calibrations controlled) is used in the test modules as a fail signature.

X.3.4 Monotonicities, locality, and cone causality

Operational commutativity is only robust if additional local processing steps cannot "inflate" the evaluation retroactively. This is where the directed structure from DPI and Spohn comes into play: it provides a control instance ensuring that information monotones cannot increase under admissible processing and that dissipation remains measurable as production.

Lemma X.3.4.1: DPI/Spohn compatibility with local post-processing

Assumptions.

1. *Admissible dynamics:* An unselective CPTP semigroup $\{\Phi_t = e^{t\mathcal{L}}\}_{t\geq 0}$ (in the Markov/semigroup scope) or, more generally, a CPTP overall dynamics Φ.
2. *Reference state:* A reference state σ (typically stationary), such that $D(\cdot\|\cdot)$ is well-defined and (in the semigroup case) $\Phi_t(\sigma) = \sigma$ holds.
3. *Local post-processing:* Additional local steps $\mathcal{J}$ are CPTP (unselective); "space-like"/"outside the future cone" is the operational interpretation of $\mathcal{J}$ as independently choosable additional processing.

Statement.

1. *DPI (contractivity):* For all $t \geq 0$,

$$D\big(\Phi_t(\rho)\|\Phi_t(\sigma)\big) \leq D(\rho\|\sigma).$$

2. *Spohn production rate (semigroup case):* If σ is stationary, then

$$\sigma_{\mathrm{Spohn}}(t) \equiv -\frac{d}{dt}D\big(\Phi_t(\rho)\|\sigma\big) \geq 0$$

(where defined).

3. *Monotonicity under additional processing:* For every additional CPTP operation $\mathcal{J}$,

$$D\big((\mathcal{J}\circ\Phi_t)(\rho)\,\|\,(\mathcal{J}\circ\Phi_t)(\sigma)\big) \;\leq\; D\big(\Phi_t(\rho)\,\|\,\Phi_t(\sigma)\big).$$

Validity range & limitations.

- The statements apply to unselective processes (no post-selection amplification).
- Non-Markov effects can modify the pointwise form of σ_{Spohn}; DPI contractivity remains valid for CPTP overall dynamics.

Source basis (import). Part I fixes the concepts used for CPTP/GKLS, DPI/Spohn, as well as locality/no-signalling.[a] [b]

[a] See FBA Part I: FBA – Foundations, Sec. I.5 "Admissible dynamics, GKLS, DPI/Spohn".
[b] See FBA Part I: FBA – Foundations, Sec. I.6 "Composition, locality & no-signalling".

Proof Sketch X.3.4.1: DPI/Spohn compatibility with local post-processing

1. *DPI as the core mechanism:* CPTP processing cannot increase relative entropy; (1) follows directly.
2. *Spohn from GKLS:* For GKLS semigroups and stationary σ, the map $t \mapsto D(\Phi_t(\rho)\|\sigma)$ is monotonically decreasing; this yields (2) (where the derivative exists).
3. *Additional processing:* Since $\mathcal{J}$ is CPTP, so is $\mathcal{J} \circ \Phi_t$; applying DPI again yields (3). The cone/spacelike language is the operational interpretation of $\mathcal{J}$ as "additional, local, independently choosable".

X.3.5 Bridge theorem: existence, residual freedom, and commutativity

With DPI and compatibility under local processing in place, the tool is now available to formulate "uniqueness" in a meaningful way: not as the claim that there is only one representation, but as the statement that all *compatible* representations yield the same evaluation and differ only by locally irrelevant isometries.

Lemma X.3.5.1: Bridge theorem (consistency & residual freedom up to isometry)

Assumptions (import & bridge data). The primitives for sequence/minimal event, budget balance and front calibration, proper time/Minkowski limit, and composition/locality are fixed in Part I.[a] In addition, let $\mathcal{Q}$ and $\mathcal{G}$ be given as in Definitions X.3.1.1 and X.3.2.1, and let the H-gate conditions (Section X.4) be satisfied.

Statement. For all observational questions $\mathcal{O}$ that are locally (cone-bounded) implementable, the evaluation is representation-independent in the sense of the consistency requirement Formula Box X.3.3.1. All representations that reproduce the same operational evaluation and satisfy the H-gate proxies differ at most by local isometries (residual freedom).

[a] See FBA Part I: FBA – Foundations, Sec. I.2–I.6 "Primitives, budget, proper time, admissibility, locality".

The proof sketch separates the three load-bearing parts of the statement: (i) typing as CPTP/GKLS on the QM side, (ii) cone and proper-time reconstruction from front and budget on the geo side, (iii) exclusion of "representation tricks" by locality plus DPI/Spohn.

Proof Sketch X.3.5.1: Bridge theorem (consistency & residual freedom up to isometry)

1. *Well-definedness of $\mathcal{Q}$:* Minimal events and protocols are represented as CPTP processing; composition is realized by serial and parallel channel composition. Thus the QM evaluation is uniquely typed as channel and POVM calculus.
2. *Well-definedness of $\mathcal{G}$:* Front calibration fixes the local limiting rate and thus the cone structure; the budget quadric yields the Minkowski limit, and integrated internal budgets define proper times along worldlines (according to κ_τ).
3. *Residual freedom:* Different representations that reproduce the same operational probabilities and rates differ only by local basis and isometry choice. H-gates constrain this freedom via explicit, testable conditions (Section X.4) so that no additional physical content resides in the representation.
4. *Consistency/commutativity:* Under H-gate conditions, the same operational observables $\mathcal{O}$ are computed in both representations from the same budget- and front-fixed invariants. DPI/Spohn and stability under additional processing (Lemma X.3.4.1) rule out that additional admissible local processing can "inflate" the monotones and thereby make the evaluation representation-dependent.

X.3.6 Flat limit case

The flat limit case is not only a consistency check, but the basis for treating laboratory and near-field tests as genuine special cases of the overall architecture. It fixes that "locally relativistic" and "GKLS-local" refer to the same causal structure.

Corollary X.3.6.1: Minkowski limit & compatibility with local GKLS

In the flat (Minkowski) limit case, admissible $\mathcal{Q}$ dynamics (unselective; in the Markov scope: GKLS-local) are compatible with the cone/front causality induced by $\mathcal{G}$. In particular, no supra-frontal ($> c$) signal fronts occur in the operational sense, and information-based monotones are monotonic under admissible local additional processing in the sense of Lemma X.3.4.1.

Outlook. Section X.4 makes the previously "structural" bridge testable by defining H-gates and proxy families such that violations appear as concrete pass/fail signatures. Section X.5 then provides the computational calculus $\mathcal{P}$ that generates actual observables from an FBA setup together with proxies and calibrations, so that the consistency requirement Formula Box X.3.3.1 can be evaluated in practice.

X.4 H-Gates & Proxy Families (Consistency & Transition Criteria)

Section X.3 formulates the bridge as commutativity of an evaluation: The same observational question $\mathcal{O}$ should lead to the same observables independent of the representation (channel/POVM vs. geometry/cones). For this requirement to be more than a formal diagram, we need two operational ingredients: First, a bridge element that identifies the two representations *budget-faithfully* and *locally*. Second, criteria that mark *when* the geometric-thermodynamic shortcuts (redshift, stationarity, null flux, microcausality) may actually be used as valid proxy statements.

The structure of the Section follows this logic:

- **H-gates** restrict the admissible representational freedom so far that the remaining freedom is only local isometries.
- **Proxy families** bundle those consistency conditions whose violation would operationally break the consistency requirement Formula Box X.3.3.1.

X.4.1 H-gates as bridge elements

An H-gate is not an additional physical postulate carrier, but an *identification tool*: It specifies which aspects of the FBA-intrinsic description reappear in the QM/channel language as an inner product, as local composition, and as a tomographically reconstructible structure. Without this fixation, "uniqueness" from Section X.3 would be empty, because different representations could encode the same data situation in arbitrarily different ways.

Definition X.4.1.1: H-gate (budget-faithful cross basis)

An *H-gate* H is a linear isometry between an FBA-intrinsic representation $\mathcal{V}_{\mathrm{FBA}}$ and a QM/channel representation $\mathcal{V}_{\mathrm{QM}}$ with the following properties:

1. **Budget faithfulness:** There exists a bilinear budget form factor $\langle\!\langle\cdot,\cdot\rangle\!\rangle_B$ on $\mathcal{V}_{\mathrm{FBA}}$ such that

$$\langle\!\langle u, v\rangle\!\rangle_B = \langle \mathsf{H}u, \mathsf{H}v\rangle$$

holds (inner product on $\mathcal{V}_{\mathrm{QM}}$). Thus the "cost and distance quantities" relevant for the computational calculus are not distorted by a change of representation.

2. **Locality and composition:** For parallel compositions, H factorizes as a tensor wreath $\mathsf{H}_{XY} = \mathsf{H}_X \otimes \mathsf{H}_Y$ (after identification of the local carriers) and respects serial composition $\circ$. Thus the bridge is well-defined on composite systems and compatible with no-signalling.

3. **DPI/Spohn compatibility (isometric transport):** H induces a transport of states/maps (schematically $\rho \mapsto \mathsf{H}\rho\mathsf{H}^\dagger$, $\Phi \mapsto \mathsf{H}\circ\Phi\circ\mathsf{H}^{-1}$, where sensibly defined), such that information-based monotones are evaluated representation-invariantly in the unselective scope (e. g. $D(\rho\|\sigma) = D(\mathsf{H}\rho\mathsf{H}^\dagger\|\mathsf{H}\sigma\mathsf{H}^\dagger)$). This prevents the direction of the information arrow from being "relabelled" by representational choice.

4. **Tomographic completeness (cross basis):** H is *operationally testable* via a previously fixed family of explainable, locally implementable measurement settings (complementary bases): states/channels/POVMs are reconstructible from measurement statistics up to local isometries (tests in Formula Box X.4.3.1).

Source basis (import). Admissibility (CPTP/GKLS), DPI/Spohn, and composition/locality are fixed in Part I.[a]

[a]See FBA Part I: FBA – Foundations, Sec. I.5–I.6 "Admissible dynamics, DPI/Spohn, composition & locality".

Why these properties are chosen exactly this way. Budget faithfulness is the lock against "scale or cost drift" induced by changing representation. Locality prevents the bridge from secretly creating new communication channels on composite systems. DPI/Spohn compatibility ties the bridge to a directed, operationally testable monotonicity (in the unselective scope). Finally, tomography is the criterion that pins down H not merely as a notion, but as an experimentally reconstructible bridge element.

Lemma X.4.1.1: Uniqueness up to local isometries (conditional)

Assumption. There exists an H-gate H in the sense of Definition X.4.1.1 that passes the tomographic checks Formula Box X.4.3.1 within the pre-registered tolerance corset (empirical windows/tolerances δ_*, see Section X.5).
Statement. Every admissible choice of such an H-gate is (for the given local carriers) unique up to local isometries.

The sketch separates the three locks that reduce representational freedom: (i) isometries from channel theory, (ii) fixation of the relevant scale via budget faithfulness, (iii) elimination of nonlocal residual freedoms via locality and tomography.

Proof Sketch X.4.1.1: Uniqueness up to local isometries (conditional)

1. *Isometric realization:* Stinespring and Kraus representations provide isometries for CPTP processes.[a]
2. *Fixation via budget faithfulness:* Budget faithfulness binds the relevant inner product to operational cost and distance quantities and eliminates nonphysical scalings.
3. *Reduction of residual freedom:* Locality and tomography confine the remaining representational freedoms to local isometries.[b]

[a] See FBA Part I: FBA – Foundations, Formula I.5 "Kraus/Stinespring".
[b] See FBA Part I: FBA – Foundations, Sec. I.6 "Symmetric monoidal structure & budget additivity".

This makes the representational change controllable. What is still missing is a catalogue of criteria indicating whether a concrete situation truly lies in the regime in which the geometric-thermodynamic shorthand $\mathcal{G}(X)$ can carry the same evaluation as $\mathcal{Q}(X)$. This is exactly what the proxy families provide.

X.4.2 Proxy families as pass/fail criteria

A proxy here is not "another principle", but a *measurement criterion*: It couples a geometrically or thermodynamically motivated consistency requirement to a functional $\mathcal{J}$ that can be estimated from data, and to an empirical tolerance/error band δ_* that reflects real measurement resolution (windows/bootstrap/residuals) and protocol robustness. The proxies are chosen so that they separate different failure modes: stationary redshift structure (A), stationarity/thermicity (B), null flux/focusing (C), and microcausality/front compatibility (D).

Proxy families & measurement parameters

We bundle measurable parameters as $\pi = (\pi_{\mathrm{Tol}}, \pi_{\mathrm{KMS}}, \pi_{\mathrm{QNEC}}, \pi_{\mathrm{Front}}, \ldots)$. Each proxy family defines (i) an observable functional $\mathcal{J}_\bullet$, (ii) a pass/fail rule with tolerance $\delta_{*,\bullet}$, (iii) an embedding into the prediction scheme $\mathcal{P}$ from Section X.5.

Proxy A — stationary redshift. This proxy is the direct operationalization of the question whether "geometry as a lapse profile" actually carries the observed frequency shift. It is therefore a first, particularly robust test because it does not require detailed dynamical assumptions.

Definition X.4.2.1: Proxy A: Tolman/redshift proxy (stationary)

Let $N_B(x) > 0$ be a budget-induced lapse function (from front calibration and budget gradients) in the sense of the proxy regime, and ν a locally measured frequency. We fix the convention

$$\frac{\nu_{\text{obs}}}{\nu_{\text{emit}}} = \frac{N_B(x_{\text{obs}})}{N_B(x_{\text{emit}})} \qquad \Longleftrightarrow \qquad 1 + z_B := \frac{\nu_{\text{emit}}}{\nu_{\text{obs}}} = \frac{N_B(x_{\text{emit}})}{N_B(x_{\text{obs}})}.$$

The associated misfit is

$$\mathcal{J}_{\text{Tol}} = \left| \frac{\nu_{\text{emit}}}{\nu_{\text{obs}}} - \frac{N_B(x_{\text{emit}})}{N_B(x_{\text{obs}})} \right|.$$

Pass/fail: $\mathbf{1}_{\text{Tol}} = \mathbf{1}[\mathcal{J}_{\text{Tol}} \le \delta_{*,\text{Tol}}]$.

Context. The construction of N_B from budget flows and the connection to redshift profiles are developed further in Part VI.[a]

[a] See FBA Part VI: Gravity & Geometry from Budget Flows, Secs. VI.3–VI.5 "Geometry and redshift proxies from budget flows".

Proxy B — stationarity and thermicity. Commutativity typically breaks first when a "stationary" regime is only apparent. Therefore Proxy B couples two independent requirements: KMS proximity measures genuine thermal stationarity in the state language, while budget closure controls whether net external flows are small enough to justify a stationary evaluation at all.

Definition X.4.2.2: Proxy B: Budget closure & KMS proximity

For stationary effective states with modular dynamics α_t we distinguish between an *ideal definition* (conceptual) and an *operationalization* (measurable).

(i) Ideal definition (KMS deviation).

$$\mathcal{J}_{\mathrm{KMS}}^{\mathrm{ideal}} \equiv \sup_{\|A\|\le 1,\ \|B\|\le 1,\ t\in[0,T]} \left|\langle A\,\alpha_t(B)\rangle - \langle \alpha_{t+i\beta_{\mathrm{mod}}}(B)\,A\rangle\right|.$$

This form fixes the target meaning "KMS proximity", but is only indirectly accessible experimentally.

(ii) Operationalization (spectral KMS relation on a test family). Fix, prior to data inspection, a finite test set of local observables $\mathcal{O}_{\mathrm{test}} = \{O_1, \ldots, O_M\}$ (e. g. local densities/fluxes) and a frequency grid Ω. Estimate for $A, B \in \mathcal{O}_{\mathrm{test}}$ the stationary two-point functions $C_{AB}(t) = \langle A\,\alpha_t(B)\rangle$ and their (discrete) spectra $S_{AB}(\omega)$ (Fourier transform of C_{AB}). To keep units unambiguous, we interpret ω as a (pre-fixed) energy grid $E = \hbar\omega$; equivalently:

$$S_{AB}(-\omega) = e^{-\beta_{\mathrm{mod}}\hbar\omega}\, S_{BA}(\omega).$$

We therefore define the measurable deviation

$$\mathcal{J}_{\mathrm{KMS}} \equiv \max_{A,B\in\mathcal{O}_{\mathrm{test}},\ \omega\in\Omega} \left| S_{AB}(-\omega) - e^{-\beta_{\mathrm{mod}}\hbar\omega}\, S_{BA}(\omega)\right|.$$

(β_{mod} is calibrated independently or obtained from a pre-registered fitting protocol.)

Budget closure. In addition, we require small net external flows over the (front-calibrated) measurement duration T:

$$\frac{|\sum \delta b_{\mathrm{ext}}|}{T} \le \delta_{*,\mathrm{clo}}.$$

Pass/fail:

$$\mathbf{1}_{\mathrm{KMS}} = \mathbf{1}\Big[\mathcal{J}_{\mathrm{KMS}} \le \delta_{*,\mathrm{KMS}} \ \wedge\ \Big|\sum \delta b_{\mathrm{ext}}\Big|/T \le \delta_{*,\mathrm{clo}}\Big].$$

Context. The operational role of GKLS, dissipation, and Spohn monotonicity is in Part IV; the thermodynamic and classical limit (including stationarity diagnostics) in Part VIII.[a] [b]

[a] See FBA Part IV: Dynamics & Measurement (GKLS), Secs. IV.3–IV.6 "GKLS, dissipation, Spohn".
[b] See FBA Part VIII: Classical Limit, Thermodynamics & Aging, Secs. VIII.3–VIII.6 "Stationarity, thermicity & classical regimes".

Documentation rule. The choice of $\mathcal{O}_{\mathrm{test}}$, Ω, window T, and the calibration/fitting procedure for β_{mod} is fixed prior to data inspection and published together with raw data and evaluation (cf. Subsection X.7.7).

Proxy C — null flux and entropic focusing. This proxy addresses the failure mode where geometry language fits locally, but null directions (front-near regimes) become inconsistent. It therefore couples an operational null flux to an entropic curvature quantity that can be estimated from reduced states.

Definition X.4.2.3: Proxy C: QNEC-near null-flux condition (operational)

Motivation. This proxy is *QNEC-inspired*: it tests whether null directions (front-near regimes) are simultaneously (i) "null-flux compatible" on the budget side and (ii) "focusing" on the entropic side. It is thus a regime check, not an identity in the sense of a strict field-theory axiomatics.

(i) Operational null flux (discretized). Along a null direction k^μ (affine parameter λ) we consider a thin null "sheet" $\mathcal{N}_{A,\Delta\lambda}$ with cross-sectional area A and thickness $\Delta\lambda$. We define the flux estimator

$$\widehat{\mathcal{F}}_k(\lambda) = \frac{\Delta b_{\text{ext}}(\mathcal{N}_{A,\Delta\lambda})}{A\,\Delta\lambda}, \qquad \text{with controlled limit regime } A \downarrow 0,\ \Delta\lambda \downarrow 0,$$

where $\Delta b_{\text{ext}}(\mathcal{N}_{A,\Delta\lambda})$ is estimated as *external budget inflow through the null sheet* according to a documented balance procedure.

(ii) Entropic curvature (discrete second derivative). Choose prior to data inspection a shifted family of regions $R(\lambda)$, whose boundary is deformed along k by λ, and estimate the reduced entropy $S(\lambda) = S(\rho_{R(\lambda)})$ (or a pre-registered surrogate entropy, e. g. S_2). We then define the curvature estimator

$$\widehat{S}''_k(\lambda) \;=\; \frac{S(\lambda+\Delta\lambda) - 2S(\lambda) + S(\lambda-\Delta\lambda)}{\Delta\lambda^2},$$

with the same $\Delta\lambda$ as in (i).

(iii) Calibration constant κ (pre-fixed). κ is calibration-dependent and is determined from a separate calibration dataset X_{cal} by a pre-registered fitting/fixing procedure. After fixation, κ remains unchanged in the test dataset.

Proxy functional and pass/fail. For a pre-registered finite test set of null directions/deformations K_{test} we define

$$\mathcal{J}_{\text{QNEC}} \;:=\; \max_{k\in K_{\text{test}},\,\lambda\in\Lambda_{\text{test}}} \Big(\kappa\,\widehat{S}''_k(\lambda) - \widehat{\mathcal{F}}_k(\lambda)\Big),$$

and set

$$\mathbf{1}_{\text{QNEC}} = \mathbf{1}[\mathcal{J}_{\text{QNEC}} \le \delta_{*,\text{QNEC}}]\,.$$

Context. The null structure and its operational anchoring via fronts and cones are developed in Part V.[a]

[a] See FBA Part V: Spacetime, Light Cones & Local Field Theory, Secs. V.3–V.6 "Cones, null directions & local field theory".

Documentation rule. The choice of K_{test}, Λ_{test}, $\Delta\lambda$, the region family $R(\lambda)$, the entropy estimation method (e. g. S or S_2), as well as the calibration protocol for κ is fixed prior to data inspection and published together with raw data, uncertainties, and evaluation (cf. Subsection X.7.7).

Proxy D — microcausality and front compatibility. Even if redshift and stationarity fit, commutativity can fail if effective correlations become "too large" across spacelike separation. Proxy D is therefore a direct test that front calibration and no-signalling are not violated even in the actually implemented dynamics.

Definition X.4.2.4: Proxy D: Front/no-signalling proxy (operational)

Motivation. Microcausality in field theory is an operator statement; operationally, however, what is primarily testable is *no-signalling* under spacelike separation. Proxy D therefore formulates an interventional front criterion; correlation measures serve as diagnostics, not as equivalences.

(i) Operational front criterion (intervention → no effect). Fix prior to data inspection a finite test family of local interventions $\mathfrak{I}_A$ in region A (e. g. local CPTP maps or measure-&-reprepare procedures) and a finite test family of measurements in region B, $\mathfrak{M}_B$ (e. g. POVMs with a discrete outcome set b). For spacelike separation $\|\Delta x\| > c\,|\Delta t|$ we define the no-signalling functional

$$\mathcal{J}_{\mathrm{NS}}(\Delta x, \Delta t) = \max_{\mathcal{I}_A \in \mathfrak{I}_A,\ \mathcal{M}_B \in \mathfrak{M}_B} \left\| p(b \mid \mathcal{I}_A, \mathcal{M}_B, t + \Delta t) - p(b \mid \mathrm{id}, \mathcal{M}_B, t + \Delta t) \right\|_1,$$

where t is the front-calibrated time stamp of the protocol.

(i') Practical estimator (expectation values as a bound, optional). For derived test observables $\mathcal{O}_B$ on B that arise from a test measurement $\mathcal{M}_B$ by fixed classical post-processing (i. e. $\mathcal{O}_B$ corresponds to a score $g(b)$ with $|g(b)| \leq 1$), for any fixed choice of $\mathcal{I}_A, \mathcal{M}_B$ one has:

$$\left|\langle \mathcal{O}_B(t{+}\Delta t)\rangle_{\mathcal{I}_A} - \langle \mathcal{O}_B(t{+}\Delta t)\rangle_{\mathrm{id}}\right| \;\leq\; \left\| p(b \mid \mathcal{I}_A, \mathcal{M}_B, t{+}\Delta t) - p(b \mid \mathrm{id}, \mathcal{M}_B, t{+}\Delta t) \right\|_1 \;\leq\; \mathcal{J}_{\mathrm{NS}}(\Delta x, \Delta t).$$

Thus the expectation-value difference is an admissible diagnostic/estimate, but does not replace the primary no-signalling criterion.

(ii) Diagnostics (connected correlations). In addition we record a correlation measure as a quick systematics and model diagnostic:

$$\mathcal{R}_{\mathrm{front}}(\Delta x, \Delta t) = \max_{\mathcal{O}_A \in \mathcal{O}^{(A)}_{\mathrm{test}},\ \mathcal{O}_B \in \mathcal{O}^{(B)}_{\mathrm{test}}} \left| \langle \mathcal{O}_A(t)\, \mathcal{O}_B(t + \Delta t)\rangle - \langle \mathcal{O}_A(t)\rangle \langle \mathcal{O}_B(t + \Delta t)\rangle \right|.$$

Non-vanishing $\mathcal{R}_{\mathrm{front}}$ is compatible with no-signalling (entanglement), whereas $\mathcal{J}_{\mathrm{NS}} > 0$ under spacelike separation indicates an operational front violation.

Pass/fail (preregistered grid): Fix in advance a finite test grid $\mathcal{G}_{\mathrm{test}} \subset \{(\Delta x, \Delta t)\}$. We then set

$$\mathbf{1}_{\mathrm{Front}} = \mathbf{1}\left[\max_{(\Delta x, \Delta t) \in \mathcal{G}_{\mathrm{test}}:\ \|\Delta x\| > c|\Delta t|} \mathcal{J}_{\mathrm{NS}}(\Delta x, \Delta t) \leq \delta_{*,\mathrm{front}} \right].$$

($\mathcal{R}_{\mathrm{front}}$ is carried along and reported as a diagnostic quantity.)

Source basis (import). Front calibration and cone causality are fixed in Part I.[a] [b]

[a] See FBA Part I: FBA – Foundations, Sec. I.3 "Calibration, front bound & signal front".
[b] See FBA Part I: FBA – Foundations, Sec. I.6 "No-signalling & local operations".

Documentation rule. The choice of $\mathfrak{I}_A$, $\mathfrak{M}_B$ (including outcome set/POVM specification), $\mathcal{O}^{(A)}_{\mathrm{test}}, \mathcal{O}^{(B)}_{\mathrm{test}}$, as well as $\mathcal{G}_{\mathrm{test}}$ and the tolerance $\delta_{*,\mathrm{front}}$ is fixed prior to data inspection and published together with raw data and evaluation (cf. Subsection X.7.7).

X.4.3 Tomographic checks and coupling H-gates ↔ proxies

The proxies alone test the regime, not the correct identification of the representations. Conversely, a tomographically "good-looking" H would be worthless if the regime does not support the geometric shortcuts. Therefore we couple both: H-gate checks secure the representational side, proxies secure the regime side.

Formula Box X.4.3.1: H-gate checks (tomographic/channel-theoretic)

(T1) Cross-basis interference. Perform measurements in two complementary bases via H; verify isometry by closeness of the reconstructible distributions and a stable tomographic fit (with a pre-registered metric and tolerance δ_*).

(T2) Budget faithfulness. Test invariance of the relevant budget functionals (e. g. $\sum \delta b_{\text{int}}$) as well as DPI contractivity of $D(\cdot\|\cdot)$ (and, if applicable, D_α only in variants where DPI holds) under the representation change $\mathsf{H} \circ \Phi \circ \mathsf{H}^{-1}$.

(T3) Locality. Test $\mathsf{H}_{XY} = \mathsf{H}_X \otimes \mathsf{H}_Y$ operationally via product states, local perturbations, and the absence of spacelike influence.

If the proxies pass, then the geometric evaluation is admissible in the sense that it uses the same operational invariants as the channel calculus. If, in addition, the H-gate checks pass, then the identification of the representations is also controlled. The following corollary summarizes this as "practical commutativity".

Corollary X.4.3.1: Compatibility of H-gates ↔ proxies

If the H-gate conditions from Definition X.4.1.1 hold (tomographically validated by Formula Box X.4.3.1) and

$$\mathbf{1}_{\text{Tol}} = \mathbf{1}_{\text{KMS}} = \mathbf{1}_{\text{QNEC}} = \mathbf{1}_{\text{Front}} = 1,$$

then the consistency requirement Formula Box X.3.3.1 is operationally satisfied: For all locally implementable observables $\mathcal{O}$, the outputs computed by the computational calculus Section X.5 from the channel and geometry sides agree up to a deviation controlled by $(\delta_{*,\text{Tol}}, \delta_{*,\text{KMS}}, \delta_{*,\text{QNEC}}, \delta_{*,\text{front}})$.

X.4.4 Pass/fail as a minimal consistency theorem

The purpose of the pass/fail scheme is not to "prove everything", but to localize failure modes: Tolman failures point to geometry/calibration inconsistency, KMS/closure to lack of stationarity, QNEC-near to null-direction problems, front failures to microcausality/front violations. Exactly this separation power is used in Sections X.6 and X.7 to break degeneracies.

Pass/fail rules (compact)

$$\mathbf{1}_{\mathrm{Tol}} \wedge \mathbf{1}_{\mathrm{KMS}} \wedge \mathbf{1}_{\mathrm{QNEC}} \wedge \mathbf{1}_{\mathrm{Front}} = 1$$

is the *minimal consistency theorem* of the bridge. A violation identifies a concrete failure mode (geometry/stationarity/null flux/front) and defines the corresponding falsification path.

Outlook. Section X.5 now translates a concrete FBA setup together with H-calibration and proxy parameters π into observables (rates, profiles, drifts) and propagates the uncertainties. Sections X.6 to X.9 use exactly this structure to catalogue predictions and to prioritize experiments so that the pass/fail decisions become as unambiguous as possible.

X.5 Computational Calculus: From an FBA Setup to Observables

The goal of this Section is an *operational* prediction scheme that maps a given FBA setup to observable quantities. Section X.3 provides the consistency scheme (commutativity) between the channel side (QM) and the geo/cone side (GR); Section X.4 makes this commutativity testable in the first place via H-gates and proxies. Here we turn this into a computational pipeline: four modules A–D which, depending on regime (laboratory, cone tests, gravitational profiles, cosmology/TDI), produce concrete observables, uncertainties, and pass/fail indicators.

Framework. We treat the evaluation as a map $\mathcal{P}$: FBA setup $\mapsto$ observables, which can be implemented either on the QM side or on the geo/cone side. Agreement of the results follows from commutativity in Formula Box X.3.3.1, provided the H-gate and proxy conditions are satisfied (see Section X.4) and checked operationally via the bridge diagnostic $\Delta_{\text{bridge}} \leq \delta_{*,\text{bridge}}$ (cf. Formula Box X.7.2.1).

X.5.1 Prediction scheme and outputs

We start with a compact problem statement: which data count as inputs, which quantities should come out, and which consistency is required.

Definition X.5.1.1: Prediction scheme $\mathcal{P}$ (problem statement)

Input. $\mathsf{X} = (\mathcal{S}, \mathfrak{N}, \mathsf{H}, \pi, \mathsf{Cal}, \mathcal{O})$ with $\mathcal{S}$ (carriers/subsystems), $\mathfrak{N}$ (channel network/GKLS generators or front/budget data), H (choice of H-gate), π (proxy parameters), Cal (calibration/front protocol, incl. c), $\mathcal{O}$ (observational query).

Output. Observable vector $Y = (y_1, \ldots, y_m)$ (rates, profiles, drifts), uncertainties ΔY, pass/fail indicators $\mathbf{1}_{\text{Tol}}, \mathbf{1}_{\text{KMS}}, \mathbf{1}_{\text{QNEC}}, \mathbf{1}_{\text{Front}}$ (see Section X.4).

Requirement (consistency requirement). $\mathcal{P}_{\text{QM}}(\cdot) \stackrel{!}{=} \mathcal{P}_{\text{Geo}}(\cdot)$ according to Formula Box X.3.3.1, operationally checked via $\Delta_{\text{bridge}}(\mathsf{X}) \leq \delta_{*,\text{bridge}}$ (cf. Formula Box X.7.2.1).

X.5.2 Input format, tolerances, and reproducibility

To ensure that later predictions do not depend on implicit conventions, we fix a concise input grammar. The point is not bureaucracy, but controllability: If pass/fail fails, it should be clear whether the cause lies in H, in a proxy, or in the calibration.

Input grammar & conventions

- **Carriers $\mathcal{S}$:** List of local carriers, coupling topology (graph G).
- **Channel network $\mathfrak{N}$:** Edge labels Φ_e (CPTP) or generators $\mathcal{L}_v$ (GKLS) with step times Δt or continuum parameters.[a] [b]
- **Front/budget data:** Budget quadric, front protocol, lapse N_B, budget gradients ∇B.[c] [d]
- **H-gate H:** Cross-basis specification and tomography canon (see Definition X.4.1.1 and Formula Box X.4.3.1).
- **Proxies π:** $(\pi_{\mathrm{Tol}}, \pi_{\mathrm{KMS}}, \pi_{\mathrm{QNEC}}, \pi_{\mathrm{Front}})$ with empirical tolerances/error bands $\delta_{*,\bullet}$ according to Definitions X.4.2.1 to X.4.2.4.
- **Calibration Cal:** Unit choice via front, measurement windows, bandwidths, reference states.
- **Observables $\mathcal{O}$:** Laboratory rates, cone tests, redshift/lensing profiles, cosmic distances and drifts.

[a] See FBA Part III: Quantum Kinematics & CPTP Channels, Secs. III.3–III.5 "Channel and protocol construction".

[b] See FBA Part IV: Dynamics & Measurement (GKLS), Secs. IV.3–IV.6 "GKLS, measurement, dissipation".

[c] See FBA Part V: Spacetime, Light Cones & Local Field Theory, Secs. V.3–V.6 "Cones, null directions, locality".

[d] See FBA Part VI: Gravity & Geometry from Budget Flows, Secs. VI.3–VI.5 "Geometry/redshift proxies from budget flows".

X.5.3 Modules A–D

The modules are chosen so that they directly process different data types: A works in the channel/POVM language, B checks causal compatibility via fronts, C produces gravitational profiles from budget quantities, D couples cosmic dynamics (incl. TDI) to distance ladders and drifts. Depending on $\mathcal{O}$, one module alone or several in combination are used.

Module A — laboratory/QM side. From channels/generators to outcome distributions, monotones, and rates. This is the natural interface for controlled experiments and the reference against which proxy interpretations can be validated.

Algorithm X.5.3.1: A (Laboratory): CPTP/GKLS → rates & signatures

Input: $(\mathcal{S}, \mathfrak{N}, \mathsf{H}, \mathsf{Cal}, \mathcal{O})$.
Steps.

1. *Embedding:* Choose H according to Definition X.4.1.1; determine the QM representation via $\mathcal{Q}$ from Definition X.3.1.1.
2. *Dynamics:* Evolve $\rho \mapsto \rho_t = \Phi_t(\rho_0)$ or $\dot{\rho}_t = \mathcal{L}(\rho_t)$ (unselective).
3. *Measurement:* Choose POVM $\{E_i\}$ for $\mathcal{O}$; compute $p_t(i) = \mathrm{tr}[\rho_t E_i]$.
4. *Monotones/rates:* Compute $D_\alpha(\rho_t \| \sigma)$ (where the chosen DPI variant applies) and $\sigma_{\mathrm{Spohn}}(t)$, and derived rates/productions.
5. *Uncertainties:* Linearized propagation and covariances according to Formula Box X.5.4.1; bootstrap/MC optional.
6. *Pass/fail:* Check $\mathbf{1}_{\mathrm{KMS}}$ and $\mathbf{1}_{\mathrm{Front}}$ according to Definitions X.4.2.2 and X.4.2.4; if $\mathcal{O}$ contains redshift/profile quantities, additionally $\mathbf{1}_{\mathrm{Tol}}$ according to Definition X.4.2.1.

Output: $Y_{\mathrm{lab}} = \{p_t(i),\, D_\alpha(t),\, \sigma_{\mathrm{Spohn}}(t),\, \text{rates}\}$, ΔY, indicators.

Module B — field/cone side. Direct causality tests and time-of-flight/delay measurements in the local Minkowski limit. This module is the fastest way to isolate front and microcausality violations before interpreting profiles or cosmological fits.

Algorithm X.5.3.2: B (Field/Cone): front → causal observables

Input: front/budget data, Cal, π_{Front}, $\mathcal{O}$.
Steps.

1. *Cone reconstruction:* Determine light cones from the budget quadric and front calibration according to Definition X.3.2.1.
2. *Signal paths:* Generate admissible paths γ (timelike/null) between sources and detectors; derive ToF and delay quantities.
3. *Front functionals:* Evaluate the intervention-defined no-signalling functional $\mathcal{J}_{\mathrm{NS}}(\Delta x, \Delta t)$ according to Definition X.4.2.4 on the pre-registered test grid $\mathcal{G}_{\mathrm{test}}$ and compute
$$\max_{(\Delta x, \Delta t) \in \mathcal{G}_{\mathrm{test}}:\ \|\Delta x\| > c|\Delta t|} \mathcal{J}_{\mathrm{NS}}(\Delta x, \Delta t).$$
In addition, carry $\mathcal{R}_{\mathrm{front}}(\Delta x, \Delta t)$ as a diagnostic. *(Note:)* For max/sup estimators, resampling (bootstrap/permutation) and a correction for search/gridding (multiple testing) is the robust default.
4. *Bounds:* Translate the bounds into upper limits on supra-causal effect sizes and into an experimental minimum resolution for $\delta_{*,\mathrm{front}}$ (with respect to $\mathcal{J}_{\mathrm{NS}}$).

Output: $Y_{\mathrm{cone}} = \{\text{ToF},\, \mathcal{J}_{\mathrm{NS}},\, \mathcal{R}_{\mathrm{front}},\, \text{cone bounds}\}$, ΔY, $\mathbf{1}_{\mathrm{Front}}$.

Module C — grav/geometry proxies. Redshift, temperature, and lensing profiles from budget gradients. This module is useful when data already come as profile curves and the question is whether they can be read consistently as budget-induced geometry.

Algorithm X.5.3.3: C (Grav/Geo): proxy map → redshift/lensing

Input: $N_B(x)$, ∇B, geometry contours, $\pi_{\text{Tol}}, \pi_{\text{KMS}}$, $\mathcal{O}$.
Steps.

1. *Null geodesics/rays:* Integrate rays based on the cone structure (local Minkowski limit).
2. *Redshift (convention as in Proxy A):* Use

$$\frac{\nu_{\text{obs}}}{\nu_{\text{emit}}} = \frac{N_B(x_{\text{obs}})}{N_B(x_{\text{emit}})} \quad \Longleftrightarrow \quad 1 + z_B = \frac{\nu_{\text{emit}}}{\nu_{\text{obs}}} = \frac{N_B(x_{\text{emit}})}{N_B(x_{\text{obs}})},$$

   and verify the Tolman test via $\mathcal{J}_{\text{Tol}}$ from Definition X.4.2.1.
3. *Temperature profile:* If stationary, evaluate $T(x)\, N_B(x) = \text{const.}$ as a proxy output (together with $\mathbf{1}_{\text{KMS}}$).
4. *Lensing/deflection:* Compute $\hat{\alpha} \approx \int \nabla_\perp \ln N_B\, ds$ (weak field).
5. *Shapiro/budget delay:* $\Delta t_B = \int (N_B^{-1} - 1)\, ds$.
6. *Stationarity check:* Check KMS proximity and closure according to Definition X.4.2.2.

Output: $Y_{\text{geo}} = \{z_B(x),\, T(x),\, \hat{\alpha},\, \Delta t_B\}$, ΔY, $\mathbf{1}_{\text{Tol}}, \mathbf{1}_{\text{KMS}}$.

Module D — cosmology/TDI. This module implements the separation of channels fixed in Part IX: *Distances* are evaluated kinematically via FRW quantities $(a(t), H(t))$ in the front-calibrated time t, while the TDI factor $\chi(t) = d\tau_{\text{geo}}/dt \in (0, 1]$ enters *exclusively* in *time observables* (chronometers, redshift drift, light-curve dilation, age integrals). This yields overdetermined null tests through distance channel $\oplus$ time channels.[18]

[18] See FBA Part IX: Cosmic Dynamics, Time Dilation & Inflation (TDI), Secs. IX.2–IX.7 "Distance channel, time channels, TDI null tests".

Algorithm X.5.3.4: D (Cosmology/TDI): distance channel $\oplus$ time channels $\to H_{\mathrm{dist}}(z)$, $\chi(z)$, $dz/d\tau_{\mathrm{geo},0}$, $R_{\mathrm{SN}}(z)$

Input: distance data (candles/sirens/BAO) for $d_L(z)$ or $D_M(z)$, chronometer data $dz/d\tau_{\mathrm{geo}}$, drift data $dz/d\tau_{\mathrm{geo},0}$, optional SN stretch data $\Delta\tau_{\mathrm{geo,obs}}$ vs. $\Delta\tau_{\mathrm{geo,em}}$, calibration Cal (incl. c, H_0, curvature k in the chosen convention), proxy/systematics parameters in π.

Steps.

1. *FRW redshift (geometric):*

$$1+z := \frac{a(t_0)}{a(t_{\mathrm{em}})} \quad \text{(standard: } a(t_0)=1\text{)}.$$

2. *Distance channel $\Rightarrow H_{\mathrm{dist}}(z)$:* Reconstruct $H_{\mathrm{dist}}(z)$ from distances (smooth/fit prior to differentiation as the default). Flat:

$$H_{\mathrm{dist}}(z) = \frac{c}{\dfrac{d}{dz}\left(\dfrac{d_L(z)}{1+z}\right)}.$$

   With curvature (in the k-convention used here):

$$D_M(z) := \frac{d_L(z)}{1+z}, \qquad H_{\mathrm{dist}}(z) = c\,\frac{\sqrt{1-k\,D_M(z)^2}}{\dfrac{d}{dz}\left(\dfrac{d_L(z)}{1+z}\right)}.$$

3. *Chronometer channel $\Rightarrow H_{\mathrm{CC}}(z)$:* Chronometers measure τ_{geo}:

$$H_{\mathrm{CC}}(z) := -\frac{1}{1+z}\frac{dz}{d\tau_{\mathrm{geo}}}.$$

4. *χ estimators (overdetermination):* From the Part IX identity $H(z) = \chi(z)\,H_{\mathrm{CC}}(z)$ follows the estimator

$$\widehat{\chi}(z) = \frac{H_{\mathrm{dist}}(z)}{H_{\mathrm{CC}}(z)}.$$

5. *Redshift drift (time channel at t_0):* Prediction for the drift observed per $\tau_{\mathrm{geo},0}$

$$\frac{dz}{d\tau_{\mathrm{geo},0}} = \chi_0^{-1}\Big[(1+z)H_0 - H_{\mathrm{dist}}(z)\Big], \qquad \chi_0 := \chi(t_0),$$

   and from this $\widehat{\chi}_0(z)$ as an (ideally) z-independent consistency check.

6. *SN light-curve dilation (time channel):*

$$R_{\mathrm{SN}}(z) = \frac{\Delta\tau_{\mathrm{geo,obs}}}{(1+z)\Delta\tau_{\mathrm{geo,em}}} = \frac{\chi_0}{\chi(z)}.$$

7. *Null tests/residuals:* Formulate channel-wise residuals (drift–distance, chronometer–distance, SN–chronometer) and bound checks $0 < \chi \le 1$ as well as $\tau_{\mathrm{geo}}(z) \le t(z)$ according to Part IX; use these as pass/fail decisions and failure-mode diagnostics.

Output: $Y_{\mathrm{cosmo}} = \{H_{\mathrm{dist}}(z), H_{\mathrm{CC}}(z), \widehat{\chi}(z), dz/d\tau_{\mathrm{geo},0}, R_{\mathrm{SN}}(z), \text{residuals/null tests}\}$, ΔY, pass/fail according to proxy and bound checks.

X.5.4 Uncertainties and scale handling

To keep pass/fail from depending on inconsistent error accounting, all modules use a shared uncertainty scheme. Scale dependence of proxy parameters is introduced only insofar as it is required for robust error bands and cross-regime comparability.

Formula Box X.5.4.1: Uncertainties & RG propagation (unified scheme)

Goal. We treat uncertainties in all modules with the same minimal standard: (i) local error propagation, (ii) consistent handling of nuisance parameters, (iii) optional scale dependence of proxy parameters, (iv) numerical diagnostics of bridge coherence.

Error propagation (local). Let $Y = f(X)$ with $X = (\theta, \pi, \mathsf{Cal})$. For small fluctuations one has with $J_X = \partial f/\partial X$

$$\Delta Y \approx J_X\, \Delta X, \qquad \mathrm{Cov}(Y) \approx J_X\, \mathrm{Cov}(X)\, J_X^\top.$$

Profiling or marginalization. Let $n \subset X$ be nuisance parameters and D the data. We use either

$$\widehat{n} = \arg\max_n \mathcal{L}(\mathsf{D} \mid X), \qquad \widehat{Y} = f(\theta, \pi, \mathsf{Cal}, \widehat{n}) \quad \text{(profiling)}$$

or

$$p(Y \mid \theta, \pi, \mathsf{Cal}) = \int p\big(Y \mid X\big)\, p(n)\, dn \quad \text{(Bayesian marginalization).}$$

Scale flow of proxies. For scale-dependent proxy parameters $\pi(\mu)$ with

$$\frac{d\pi}{d\ln\mu} = \beta(\pi)$$

one obtains to first order around μ_0

$$\pi(\mu) \approx \pi(\mu_0) + \beta\big(\pi(\mu_0)\big)\, \ln\frac{\mu}{\mu_0}.$$

If β is linearized around $\pi(\mu_0)$ with $B = \frac{\partial\beta}{\partial\pi}\Big|_{\mu_0}$, one gets the closed approximation

$$\pi(\mu) \approx \pi(\mu_0) + \Big(\mathrm{e}^{B\ln(\mu/\mu_0)} - \mathbb{I}\Big)\, B^{-1}\beta\big(\pi(\mu_0)\big).$$

(Background: Part VII.)[a]

Consistency diagnostics. As a practical consistency diagnostic we check numerically

$$\Delta_{\text{bridge}}(\mathsf{X}) \le \delta_{*,\text{bridge}},$$

where Δ_{bridge} is the covariance-weighted bridge residual according to Formula Box X.7.2.1. Violations under satisfied prerequisites are interpreted as evidence for H-gate or proxy incompatibilities in the sense of Corollary X.4.3.1.

Extreme-value/sup functionals (practical default). For test quantities of the sup / max type (e. g. front or no-signalling functionals over search grids), normal approximations are generally not robust. As default we use resampling (bootstrap/permutation) and calibrate thresholds taking the gridding into account (multiple-testing/look-elsewhere effect).

[a] See FBA Part VII: Constants, Scales & Renormalization, Secs. VII.2–VII.5 “Scale windows, RG, calibration”.

X.5.5 Coherence and module coupling

The modules are intentionally redundant enough to break degeneracies: If profiles (C) "fit" but cone tests (B) fail, the cause is not a free fitting choice, but a front inconsistency. If laboratory monotones (A) look stationary but KMS/closure (B) fails, there is no genuine stationary regime. This logic is summarized in the following coherence corollary as a minimal requirement.

Corollary X.5.5.1: Coherence theorem of the computational calculus

Under the conditions

$$\mathbf{1}_{\mathrm{Tol}} = \mathbf{1}_{\mathrm{KMS}} = \mathbf{1}_{\mathrm{QNEC}} = \mathbf{1}_{\mathrm{Front}} = 1$$

(see Section X.4) modules A–D yield consistent predictions on both sides of the bridge. Operationally: the commutative diagram Formula Box X.3.3.1 passes in the sense of the bridge diagnostic, i. e.

$$\Delta_{\mathrm{bridge}}(\mathsf{X}) \le \delta_{*,\mathrm{bridge}}$$

with Δ_{bridge} according to Formula Box X.7.2.1 (and the covariance $\Sigma(\mathsf{X})$ fixed there).

Implementation guide (practical)

For laboratory setups, the recommended order is A→B (cone consistency), optionally C (redshift/temperature proxy). In astrophysical applications one typically starts with C (profiles), checks B (front), and uses A for channel interpretation where calibrated quantum distributions are available. Cosmologically, D provides the overarching distance and drift structure, which must be locally consistent with C and B if the bridge is to hold operationally.

X.6 Predictions (Catalog, grouped)

This Section consolidates *concrete, falsifiable* predictions of the FBA program, organized by laboratory/metrology, orbit/astrophysics, and cosmology/TDI. Each block begins with a short framing and contains precise boxes with measurable functionals, expected relations, and pass/fail criteria. Evaluation is carried out via the computational calculus in Section X.5; consistency with the bridge from Section X.3 presupposes the H-gate and proxy conditions from Section X.4.

X.6.1 Laboratory & metrology

Framework. Channels and GKLS generators are primary inputs here. Measured are outcome distributions, information-based monotones, and budget rates under well-defined calibration and front control. The tests are formulated to function as pass/fail without model fitting, thereby practically securing the "admissible process class".[19] [20]

Formula Box X.6.1.1: L1 - DPI/No-Recovery as a canonical monotonicity bound

Let D_α be a *DPI-monotone* information divergence (e. g. relative entropy D, or a pre-registered DPI variant D_α). For any unselective GKLS evolution Φ_t and any reference σ one has

$$D_\alpha(\Phi_t(\rho) \,\|\, \Phi_t(\sigma)) \;\le\; D_\alpha(\rho\|\sigma).$$

If σ is stationary (in the GKLS regime used), then moreover

$$\sigma_{\rm Spohn}(t) \equiv -\frac{d}{dt} D\big(\Phi_t(\rho)\|\sigma\big) \ge 0 \quad \text{(where defined)}.$$

Test. Reconstruct D_α from tomography (computational calculus Section X.5, Module A Algorithm X.5.3.1) at two times $t_1 < t_2$.
Pass/Fail: $\mathbf{1}_{\rm lab,DPI} = \mathbf{1}[D_\alpha(t_2) \le D_\alpha(t_1) + \delta_{*,\rm DPI}]$.

Measurement strategy. Suitable are one- or two-qubit channels with a well-defined reference σ. Crucial is an *unselective* data evaluation so that post-selection gains are excluded as spurious compatibility. This is exactly where L1 connects to the bridge diagnostic from Lemma X.3.4.1.

[19] See FBA Part III: Quantum Kinematics & CPTP Channels, Secs. III.3–III.5 "CPTP/instruments/protocols".
[20] See FBA Part IV: Dynamics & Measurement (GKLS), Secs. IV.3–IV.6 "GKLS, measurement, dissipation".

Formula Box X.6.1.2: L2 - GKLS structure test & irreversible budget rates

Let $\mathcal{L}$ be locally implemented and parameterized as a GKLS generator in the chosen effective regime. Then the dissipative (Kossakowski) blocks are positive semidefinite (up to reconstruction error), and the *irreversible* budget rate satisfies

$$\dot{A}(t) \geq 0 \quad \text{(in the unselective, calibrated regime).}$$

Test. Identify $\mathcal{L}$ via process tomography; check PSD of the dissipative blocks up to tolerance $\delta_{*,\mathrm{PSD}}$ and estimate $\dot{A}(t)$ using the pre-registered calibration/assignment (e. g. via $\sigma_{\mathrm{Spohn}}(t)$ in a stationary/isothermal protocol).
Pass/Fail: $\mathbf{1}_{\mathrm{lab,GKLS}} = \mathbf{1}[\text{PSD up to } \delta_{*,\mathrm{PSD}} \ \wedge\ \dot{A}(t) \geq -\delta_{*,\mathrm{irr}}]$.

Interpretation. L2 is a structure test with process binding: a fail cannot be hidden by "better fits", but indicates either a violation of the local GKLS structure or an inconsistent implementation/calibration.

Formula Box X.6.1.3: L3 - Landauer calibration (budget ↔ entropy cost)

For a logically irreversible operation with entropy change ΔS, a budget work bound holds in the calibrated protocol:

$$\Delta b_{\mathrm{irr}} \ \geq\ \Theta\, \Delta S,$$

where Θ is fixed by calibration (front and temperature protocol).[a]
Test. Reset experiments with varied ΔS; regress Δb_{irr} against ΔS under a pre-registered evaluation (no post-selection).
Pass/Fail: $\mathbf{1}_{\mathrm{lab,Landauer}} = \mathbf{1}[\Delta b_{\mathrm{irr}} - \Theta \Delta S \geq -\delta_{*,\mathrm{Lan}}]$.

[a]See FBA Part I: FBA – Foundations, Secs. I.3–I.5 "Calibration, budget balance, admissible dynamics".

Remark. Here Θ is not a free fit parameter, but the metrological coupler between the information and budget descriptions. Precisely for that reason, L3 is a hard test that cannot be absorbed by reparameterization.[21]

[21]See FBA Part VIII: Classical Limit, Thermodynamics & Aging, Secs. VIII.3–VIII.6 "Stationarity, thermality, classical regimes".

Formula Box X.6.1.4: L4 - Path and Sagnac signatures (rotation/acceleration)

For two paths $\Gamma_\pm$ in a rotating interferometer, one obtains a front-calibrated time offset

$$\Delta t_{\mathrm{Sag}} \approx \frac{4\,\Omega\,\mathcal{A}}{c^2},$$

with rotation rate Ω and enclosed area $\mathcal{A}$.
Test. Compare interference phases (linear in Δt_{Sag}).
Pass/Fail: $\mathbf{1}_{\mathrm{lab,Sagnac}} = \mathbf{1}[|\widehat{\Delta t} - \frac{4\Omega\mathcal{A}}{c^2}| \le \delta_{*,\mathrm{Sag}}]$.

Laboratory conclusion. L1–L4 together form a robust laboratory testbench: monotonicity (L1), dynamical structure (L2), thermodynamic coupling (L3), and inertial time offsets (L4). In combination, they provide a clean diagnosis of whether a setup operates in the admissible regime of the bridge at all, before profiles or cosmological fits are interpreted.

X.6.2 Orbit & astrophysics

Framework. Stationary or quasi-stationary configurations allow redshift and temperature profiles, delays, and timing residuals. Here, Proxies A/B/D from Section X.4 are especially relevant: Tolman profiles provide a scale/regime check for N_B, KMS/closure controls the stationarity assumption, and front tests exclude supra-causal "shortcuts". We therefore begin with profiles (A1) and then incorporate path observables that test the N_B structure integratively. The strict separation of reconstruction (calibration/inference) and out-of-sample testing belongs operationally in the protocols (cf. Section X.7).

Formula Box X.6.2.1: A1 - Tolman profiles (stationary fields)

With a budget-induced lapse function $N_B(x) > 0$, the redshift convention from Proxy A holds (stationary):

$$\frac{\nu_{\mathrm{obs}}}{\nu_{\mathrm{emit}}} = \frac{N_B(x_{\mathrm{obs}})}{N_B(x_{\mathrm{emit}})} \qquad \Longleftrightarrow \qquad 1 + z_B = \frac{\nu_{\mathrm{emit}}}{\nu_{\mathrm{obs}}} = \frac{N_B(x_{\mathrm{emit}})}{N_B(x_{\mathrm{obs}})},$$

and additionally

$$T(x)\,N_B(x) = \mathrm{const.}$$

Test. Line frequencies and temperatures between two levels $x_{\mathrm{emit}}, x_{\mathrm{obs}}$.
Pass/Fail. $\mathbf{1}_{\mathrm{Tol}}$ according to Definition X.4.2.1 and stationarity/closure according to Definition X.4.2.2.

Application. Spectroscopy in deep potentials (white dwarfs, neutron stars) and temperature gradients in accretion disks are suitable testbeds.[22]

[22] See FBA Part VI: Gravity & Geometry from Budget Flows, Secs. VI.3–VI.5 "Redshift/temperature proxies from budget flows".

Formula Box X.6.2.2: A2 - Shapiro or budget delay along ray paths

For a null-ray geometry, the FBA delay is

$$\Delta t_B = \int_\gamma (N_B^{-1} - 1)\, ds.$$

Test. Time delays in echoes or occultations; compare with orbital parameters and an N_B estimated from profiles (A1, preferably disjoint data) or independently.
Pass/Fail: $\mathbf{1}_{\text{astro,Delay}} = \mathbf{1}[|\widehat{\Delta t} - \Delta t_B| \le \delta_{*,\text{Del}}]$.

Bridge to A1. A2 is the path test to A1: a consistent N_B must explain both profiles and delays. A mismatch between the two diagnostics localizes either a geometry error (wrong N_B) or a calibration/front problem (inconsistent cone structure).

Formula Box X.6.2.3: A3 - Pulsar-timing budget (timing residuals)

For a source with intrinsic period P, a path $x(t)$ induces the residual process

$$R(t) = \int_0^t \left(\frac{1}{N_B(x(t'))} - 1 \right) \frac{dt'}{P}.$$

Test. Compare $R(t)$ against timing residuals with an N_B prediction (from A1 or independently), coupled to $\mathbf{1}_{\text{Tol}}$ from Definition X.4.2.1.
Pass/Fail: $\mathbf{1}_{\text{astro,PTA}} = \mathbf{1}[\|R_{\text{obs}} - R_{\text{mod}}\| \le \delta_{*,\text{PTA}}]$.

Remark. PTA signals couple integratively to N_B and are therefore particularly sensitive to large-scale gradients and congestion fronts. This very integration sensitivity makes A3 a degeneracy breaker compared to purely local profile tests.

Formula Box X.6.2.4: A4 - KMS proximity at horizons and congestion fronts (proxy test)

Let $\kappa_B \equiv \frac{d}{d\lambda} \ln N_B$ along a null-generator parameterization λ. As a *proxy expectation* (not an identity) we use: near fronts/horizons, a modular scale $\beta_{\text{mod}}^{-1} \propto \kappa_B$ can occur and manifest as KMS proximity of two-point functions in the sense of Definition X.4.2.2.
Test. Spectra and autocorrelations near fronts and horizons; check $\mathbf{1}_{\text{KMS}}$ and budget closure according to Definition X.4.2.2.

Astrophysics conclusion. A1 provides the scale/regime check for N_B, A2 and A3 are path and timing tests of the same structure, and A4 controls the stationarity assumption. A joint fail isolates front or stationarity breaks rather than merely labeling "inadequate models".

X.6.3 Cosmology & TDI

Framework. In the cosmic regime there are two separate observation channels: (1) the *distance channel* (standard candles/sirens, BAO/lensing) yields kinematic information about $H(z)$ via distances, (2) the *time channels* (chronometers, redshift drift, SN light-curve dilation, age integrals) measure τ_{geo} and thus the TDI factor $\chi(t) = d\tau_{\text{geo}}/dt \in (0,1]$. The core is overdetermination: χ is not "fit", but estimated from cross-comparisons of channels; null tests/residuals provide pass/fail.[23] The associated operational evaluation is carried out via Module D of the computational calculus Algorithm X.5.3.4.

Formula Box X.6.3.1: C1 - Distance channel: $H_{\text{dist}}(z)$ from distances (kinematic)

From distances, $H(z)$ can be reconstructed kinematically without using field equations. In the flat case,

$$H_{\text{dist}}(z) = \frac{c}{\dfrac{d}{dz}\left(\dfrac{d_L(z)}{1+z}\right)}.$$

With curvature (in the k-convention used),

$$D_M(z) := \frac{d_L(z)}{1+z}, \qquad H_{\text{dist}}(z) = c\,\frac{\sqrt{1 - k\,D_M(z)^2}}{\dfrac{d}{dz}\left(\dfrac{d_L(z)}{1+z}\right)}.$$

Test (robust default). Smooth/fit $d_L(z)$ prior to differentiation; check consistency across different distance tracers (candles vs. sirens) in an out-of-sample sense.
Pass/Fail: $\mathbf{1}_{\text{cosmo,dist}} = \mathbf{1}[\|H_{\text{dist},1} - H_{\text{dist},2}\| \le \delta_{*,\text{dist}}]$ on a pre-registered redshift grid.

Formula Box X.6.3.2: C2 - Time channel: chronometer scaling and $\chi(z)$ estimator

Chronometers measure τ_{geo} and thus

$$H_{\text{CC}}(z) := -\frac{1}{1+z}\frac{dz}{d\tau_{\text{geo}}}.$$

The Part IX identity links both channels:

$$H(z) = \chi(z)\,H_{\text{CC}}(z) \quad \Longrightarrow \quad \widehat{\chi}(z) = \frac{H_{\text{dist}}(z)}{H_{\text{CC}}(z)}.$$

Test. Estimate $H_{\text{CC}}(z)$ from chronometer data and $\widehat{\chi}(z)$ from distance$\oplus$chronometer.
Pass/Fail (bounds & trends).

$$\mathbf{1}_{\text{cosmo},\chi} = \mathbf{1}[0 < \widehat{\chi}(z) \le 1 \ \ \text{(up to error bands } \delta_*)]\,,$$

in addition consistency/smoothness of $\widehat{\chi}(z)$ under pre-registered regularization.

[23] See FBA Part IX: Cosmic Dynamics, Time Dilation & Inflation (TDI), Secs. IX.2–IX.7 "Distance channel, time channels, TDI null tests".

Formula Box X.6.3.3: C3 - Overdetermination: drift, SN dilation, and null tests (residuals)

Drift (time channel at t_0). The drift observed per $\tau_{\mathrm{geo},0}$ satisfies

$$\frac{dz}{d\tau_{\mathrm{geo},0}} = \chi_0^{-1}\Big[(1+z)H_0 - H_{\mathrm{dist}}(z)\Big], \qquad \chi_0 := \chi(t_0).$$

SN light-curve dilation (time channel).

$$R_{\mathrm{SN}}(z) = \frac{\Delta\tau_{\mathrm{geo,obs}}}{(1+z)\Delta\tau_{\mathrm{geo,em}}} = \frac{\chi_0}{\chi(z)}.$$

Null tests (channel-wise consistency). Define residuals (on a pre-registered grid) by

$$\Delta_{\mathrm{DR}}(z) := H_{\mathrm{dist}}(z) - \Big((1+z)H_0 - \chi_0\,\frac{dz}{d\tau_{\mathrm{geo},0}}\Big),$$

$$\Delta_{\mathrm{CC}}(z) := H_{\mathrm{dist}}(z) - \chi(z)\,H_{\mathrm{CC}}(z), \qquad \Delta_{\mathrm{SNCC}}(z) := H_{\mathrm{dist}}(z) - \frac{\chi_0}{R_{\mathrm{SN}}(z)}\,H_{\mathrm{CC}}(z).$$

Pass/Fail:

$$\mathbf{1}_{\mathrm{cosmo,null}} = \mathbf{1}[\max(\|\Delta_{\mathrm{DR}}\|, \|\Delta_{\mathrm{CC}}\|, \|\Delta_{\mathrm{SNCC}}\|) \le \delta_{*,\mathrm{null}}]\,,$$

plus bound checks (in particular $0 < \chi \le 1$ and $\tau_{\mathrm{geo}}(z) \le t(z)$ in the Part IX sense).

Cosmology conclusion. C1 provides the distance channel, C2 estimates $\chi(z)$ from distance$\oplus$chronometers, C3 is the degeneracy breaker via drift and SN dilation. A fail in the null tests is especially informative because it cannot be absorbed by pure profile fitting in the distance channel, but explicitly violates the channel separation (distance vs. time).

X.6.4 Linking and brief overview

Linking & degeneracy breakers. The laboratory tests (L1–L4) pin down the admissible channel physics and calibration couplings; orbit tests (A1–A4) map budget gradients and stationarity assumptions; cosmological tests (C1–C3) test the channel comparison distance $\oplus$ time and thus χ via overdetermined null tests. Orthogonal observables (drift, KMS proximity, front suppression) serve as degeneracy breakers between competing fits.

Catalog — quick overview & mapping

- **Laboratory (L1–L4):** DPI/Spohn (L1), GKLS structure and irreversible rates (L2), Landauer calibration (L3), Sagnac time offset (L4).
- **Astro (A1–A4):** Tolman profiles (A1), Shapiro or budget delay (A2), pulsar residuals (A3), KMS proximity at fronts/horizons (A4).
- **Cosmo (C1–C3):** Distance channel $H_{\mathrm{dist}}(z)$ (C1), chronometer scaling $\widehat{\chi}(z)$ (C2), drift/SN dilation and null tests/residuals (C3).

Evaluation. Implement the computational calculus Section X.5 and use modules A–D (Algorithms X.5.3.1 to X.5.3.4); document the proxy indicators $\mathbf{1}_{\mathrm{Tol}}, \mathbf{1}_{\mathrm{KMS}}, \mathbf{1}_{\mathrm{QNEC}}, \mathbf{1}_{\mathrm{Front}}$ (definitions in Section X.4) per measurement series as part of the pass/fail evaluation.

X.7 Falsifiability & Experiments

This Section translates the bridge from Section X.3, the H-gates and proxies from Section X.4, and the computational calculus from Section X.5 into *concrete* falsification and experimental schemes. Guiding idea: each statement is formulated as a binary, locally evaluable test; multiple tests are combined so that they break degeneracies rather than hiding them behind model fits. The prediction catalog in Section X.6 provides the target quantities; here the focus is on protocols and decision logic.

Principle. Falsifiability here does not mean "excluding parameters", but a clear operation: given satisfied interface conditions (calibration/front, H tomography, proxy regime), a measurable functional yields a non-pass. To prevent tests from ending in circularity, we first fix what counts as satisfied, and only then decide. Where reconstructions (e.g. N_B or H_{dist}) are required as intermediate steps, calibration/inference and subsequent out-of-sample testing are explicitly separated.

X.7.1 Pass/Fail semantics

We begin with the general semantics, because all later protocols are just variants of the same decision scheme.

Definition X.7.1.1: Falsifiability (pass/fail semantics)

An FBA test is a triple $(\mathcal{O}, \mathcal{J}, \delta_*)$ consisting of an observation question $\mathcal{O}$, a functional $\mathcal{J}$ (from Sections X.4 and X.5), and an empirical tolerance $\delta_* > 0$. The *decision* is $\mathbf{1}[\mathcal{J} \le \delta_*] \in \{0, 1\}$.

Scalarization. $\mathcal{J}$ is either (i) scalar, or (ii) vector-valued and is made testable by a preregistered scalarization (e.g. $\|\cdot\|_{\Sigma^+}$ as in Formula Box X.7.2.1 or a max/sup aggregation with its own threshold calibration).

Preconditions. Each test comes with a (preregistered) bundle of precondition criteria $\mathbf{1}_{\mathrm{pre}} \in \{0, 1\}$ that covers input conventions and interface conditions (in particular calibration/front, H-gate validation, proxy regime; cf. Section X.4). Operationally, $\mathbf{1}_{\mathrm{pre}}$ can be documented as a conjunction, e.g.

$$\mathbf{1}_{\mathrm{pre}} = \mathbf{1}_{\mathrm{Cal}} \wedge \mathbf{1}_{\mathrm{Hgate}} \wedge \mathbf{1}_{\mathrm{Proxy}} \wedge \mathbf{1}_{\mathrm{Data}},$$

where $\mathbf{1}_{\mathrm{Data}}$ includes, among other things, unselectiveness (lab) or stationarity/path definition (astro/cosmo). For functionals of the sup / max type, robust threshold calibration (resampling/multiple testing, see Subsection X.7.4) is also part of the preconditions.

Falsification (domain-local). A program component (bridge, proxy family, or evaluation module) is *falsified* in a domain if at least one associated test yields $\mathbf{1} = 0$ *even though* the preconditions $\mathbf{1}_{\mathrm{pre}} = 1$ were satisfied.

Regime/interface violation. If $\mathbf{1}_{\mathrm{pre}} = 0$, then the corresponding test is *not licensed* in that domain (regime or interface violation); this is an independent fail signal, but it is not counted as bridge falsification in the narrow sense.

Thus three things are explicit: (i) pass/fail is a local data decision, (ii) "falsification" refers to the tested domain, not a global verdict, (iii) proxy/front/calibration fails are *precondition* *fails* and are evaluated separately from bridge fails.

X.7.2 Bridge criterion as a generic falsification signal

Central is the commutativity of the evaluation from Formula Box X.3.3.1. It provides a generic criterion that can be formulated independently of the specific experiment.

Formula Box X.7.2.1: Generic bridge falsification criterion

Let $\mathcal{P}_{\mathrm{QM}}, \mathcal{P}_{\mathrm{Geo}}$ be the prediction scheme from Section X.5 and $\delta_{*,\mathrm{bridge}} > 0$ a numerical tolerance. We first define the *bridge residual* (object-valued)

$$\delta_{\mathrm{bridge}}(\mathsf{X}) \;\equiv\; \mathcal{P}_{\mathrm{QM}}(\mathsf{X}) - \mathcal{P}_{\mathrm{Geo}}(\mathsf{X}),$$

and from it the *bridge distance* (scalar, testable) as a metrized residual

$$\Delta_{\mathrm{bridge}}(\mathsf{X}) \;\equiv\; \left\|\delta_{\mathrm{bridge}}(\mathsf{X})\right\|_{\Sigma^+} \;:=\; \sqrt{\delta_{\mathrm{bridge}}(\mathsf{X})^\top\, \Sigma(\mathsf{X})^+\, \delta_{\mathrm{bridge}}(\mathsf{X})}\,,$$

where $\|v\|_{\Sigma^+} := \sqrt{v^\top \Sigma(\mathsf{X})^+ v}$. Here $\Sigma(\mathsf{X})$ is the (preregistered) covariance in the data space of the considered output quantities (e.g. from bootstrap/jackknife or from a measurement model; cf. Formula Box X.5.4.1); $(\cdot)^+$ denotes the (possibly regularized) pseudoinverse (in the full-rank case $\Sigma^+ = \Sigma^{-1}$).
The *bridge* is operationally violated if

$$\Delta_{\mathrm{bridge}}(\mathsf{X}) \;>\; \delta_{*,\mathrm{bridge}},$$

under satisfied H-gate and proxy/precondition requirements from Section X.4 (see the domain-specific decision rule in Corollary X.7.6.1). In that case the commutativity diagram Formula Box X.3.3.1 is refuted in the tested domain.

Design guideline. Protocols must be constructed so that Δ_{bridge} cannot be absorbed by trivial reparametrizations or by front and unit drift. This is exactly why the following protocols always couple a measurement evaluation to an explicit calibration and regime check.

X.7.3 Experimental protocols as degeneracy breakers

We provide three standard protocols that cover different data worlds. They are chosen so that each hits a different failure mode: (A) information arrow and unselectiveness, (B) profile and path consistency of N_B, (C) cosmological channel overdetermination (distance channel vs. time channels including drift).

Algorithm X.7.3.1: E-A (Lab): DPI/Spohn monotonicity as a pass/fail protocol

Input: Channel network/GKLS $\mathfrak{N}$, H-gate H, calibration/front Cal, observation $\mathcal{O}$ (computational calculus Section X.5, module A Algorithm X.5.3.1).
Steps.

1. Tomography of the initial states ρ_0 and references σ; specification of an unselected evolution Φ_t.
2. Measurement of outcome distributions at two times $t_1 < t_2$ under identical calibration.
3. Reconstruction of $D_\alpha(\rho_{t_i}\|\sigma)$ (or the preregistered DPI-monotone divergence) and estimation of $\sigma_{\text{Spohn}}(t)$, insofar as licensed in the protocol.
4. Decision: $\mathbf{1}_{\text{lab,DPI}} = \mathbf{1}[D_\alpha(t_2) \le D_\alpha(t_1) + \delta_{*,\text{DPI}}]$.

Output: Pass/fail, effect size $\Delta D_\alpha = D_\alpha(t_2) - D_\alpha(t_1)$, uncertainty $\Delta(\Delta D_\alpha)$.

Comment. A fail (given satisfied preconditions: unselected implementation, H-gate/front calibration) indicates a violation of the DPI arrow and/or the licensed Spohn monotonicity. If those preconditions are not satisfied, the signal must be classified as a regime/interface fail (cf. Subsection X.7.1) and blocks a bridge interpretation in that domain.

Algorithm X.7.3.2: E-B (Astro): Tolman–Shapiro–PTA triangulation

Input: Spectral lines and temperatures (A1), ray delays (A2), timing residuals (A3), calibration Cal (see Subsection X.6.2).
Steps.

1. *Calibration/inference:* Estimate an N_B reconstruction from A1 (redshift and temperature) and explicitly document which data were used for this purpose (training/calibration set).
2. *Out-of-sample tests:* Predict $\Delta t_B[\gamma]$ (A2) and $R(t)$ (A3) with the same N_B on disjoint data/observations (hold-out or independent data sets where available).
3. *Decision:* Report $\mathbf{1}_{\text{Tol}}$ separately; do not count A1 as an independent test if the same data were used for the N_B reconstruction. The triangulation decision is then
$$(\mathbf{1}_{\text{astro,Delay}} \wedge \mathbf{1}_{\text{astro,PTA}}) \;\wedge\; \mathbf{1}_{\text{Tol}}$$
(with $\mathbf{1}_{\text{Tol}}$ as a regime/scale check).

Output: Consistency score $S_{\text{astro}} \in [0,1]$ (fraction of passed tests), residuals and uncertainties.

Comment. The combination suppresses degeneracies: A1 fixes scales (regime/proxy check), A2 tests path integrals, A3 tests temporal accumulation. A fail in A2/A3 (given satisfied preconditions) falsifies the N_B proxy reading in the considered domain (regime/proxy fail), rather than merely making a profile model “fit poorly”.

Algorithm X.7.3.3: E-C (Cosmo): distance channel $\oplus$ time channels as a null-test chain

Input: Standard candles or standard sirens (for d_L), BAO or lenses (for d_A), spectral lines $z \equiv z_{\mathrm{obs}}$ (geometric FRW redshift as the channel label), chronometers $dz/d\tau_{\mathrm{geo}}$, long-term drift measurements $dz/d\tau_{\mathrm{geo},0}$ and (optional) SN light-curve dilation $R_{\mathrm{SN}}(z)$ (see Subsection X.6.3).
Steps.

1. *Distance channel:* Reconstruct $H_{\mathrm{dist}}(z)$ from distances (cf. prediction C1 in Section X.6).
2. *Chronometer channel:* Estimate $H_{\mathrm{CC}}(z) := -\frac{1}{1+z}\frac{dz}{d\tau_{\mathrm{geo}}}$ and from it $\widehat{\chi}_{\mathrm{CC}}(z) = \frac{H_{\mathrm{dist}}(z)}{H_{\mathrm{CC}}(z)}$ (cf. prediction C2).
3. *Drift channel (at t_0):* Estimate $\widehat{\chi}_0$ from
$$\frac{dz}{d\tau_{\mathrm{geo},0}} = \chi_0^{-1}\Big[(1+z)H_0 - H_{\mathrm{dist}}(z)\Big],$$
and test the expected z-independence of $\widehat{\chi}_0$ (cf. prediction C3).
4. *SN dilation (optional, time channel):* Test $R_{\mathrm{SN}}(z) = \chi_0/\chi(z)$ and consolidate $\widehat{\chi}_{\mathrm{SN}}(z) = \widehat{\chi}_0/R_{\mathrm{SN}}(z)$.
5. *Null tests/residuals:* Form $\Delta_{\mathrm{DR}}, \Delta_{\mathrm{CC}}, \Delta_{\mathrm{SNCC}}$ according to prediction C3 and decide pass/fail via preregistered thresholds.

Decision: $\mathbf{1}_{\mathrm{cosmo,null}} = 1$ (and bounds $0 < \chi \le 1$, $\tau_{\mathrm{geo}} \le t$, insofar as licensed in the data set).
Output: Consistency score S_{cosmo}, residual profiles and uncertainties.

X.7.4 Test statistics and error control

The protocols provide estimators $\widehat{\mathcal{J}}$ for functionals $\mathcal{J}$. To keep pass/fail comparable across data sets, we use a uniform test-statistics scheme, including error control and a power sketch.

Formula Box X.7.4.1: Test statistics, error control & power

Goal. A minimal standard for comparable pass/fail decisions is: (i) a well-defined estimator $\widehat{\mathcal{J}}$, (ii) robust uncertainties (resampling or a measurement model), (iii) a preregistered threshold δ_*, (iv) documented error control.

Option A (asymptotic/linearized): Z-test. Let $\widehat{\mathcal{J}}$ be an estimator of a (scalarized) functional $\mathcal{J}$ with standard error $\sigma_{\mathcal{J}}$. We test one-sided $H_0 : \mathcal{J} \leq \delta_*$ against $H_1 : \mathcal{J} > \delta_*$ with

$$Z = \frac{\widehat{\mathcal{J}} - \delta_*}{\sigma_{\mathcal{J}}}, \quad \text{Pass/Fail: } \mathbf{1} = \mathbf{1}[Z \leq z_{1-\alpha}],$$

significance level α (typically 0.05).

Power (heuristic). For an expected signal $\Delta_{\text{sig}} > 0$, Power $\approx \Phi\Big(\frac{\Delta_{\text{sig}}}{\sigma_{\mathcal{J}}} - z_{1-\alpha}\Big)$.

Combination. Multiple tests via Fisher or Tippett methods; for correlations use covariances from Formula Box X.5.4.1.

Option B (default for non-normality): resampling/likelihood. Calibrate the decision boundary via bootstrap/permutation or (where available) via an explicit likelihood- or simulation-based measurement model; then report p-values/confidence bands consistent with the preregistered δ_* semantics.

Sup/max functionals (front and grid tests). For functionals of the sup / max type (e.g. $\sup \mathcal{J}_{\text{NS}}$ over a search grid), the normal approximation is generally not robust. As a default, we calibrate thresholds by resampling (bootstrap/permutation) and account for the gridding (multiple testing/look-elsewhere effect) via appropriate family-wise error or FDR control.

X.7.5 Degeneracy breakers as a decoupling principle

Degeneracy breakers arise from observables that point in as orthogonal sensitivity directions as possible. This is not an abstract optimality claim here, but an operational rule: time channels, front tests, and profiles should not turn the same uncertainty knob.

Formula Box X.7.5.1: Degeneracy-breaker heuristic (orthogonal probes)

Let $\theta = (\pi, \text{nuisance})$ and $Y = f(\theta)$. Choose observables $Y_1, \ldots, Y_k$ so that the columns of the Jacobian $J = \partial Y / \partial \theta$ are as orthogonal as possible:

$$\max_{\mathcal{O}_1, \ldots, \mathcal{O}_k} \quad \lambda_{\min}(J^\top J) \quad \text{subject to} \quad \text{cost and time budget.}$$

Practically: combine the drift/time channel $dz/d\tau_{\text{geo},0}$ (timelike sensitivity) with the interventional front criterion $\mathcal{J}_{\text{NS}}$ (spacelike sensitivity) and static profiles (e.g. Tolman z_B) as a scale/regime check to decouple $\pi_{\text{Tol}}, \pi_{\text{Front}}, \chi$. ($\mathcal{R}_{\text{front}}$ is carried along as a diagnostic quantity, but is not used as the front pass/fail; see Definition X.4.2.4.)

X.7.6 Minimal decision tree and pipelines

From the proxy and bridge logic follows a minimal package of criteria that can be formulated independently of the specific data field. What matters is the clean separation between

(i) regime/interface fails (proxy/front/calibration) and (ii) bridge fails in the narrow sense (Δ_{bridge} under satisfied preconditions).

Corollary X.7.6.1: Minimal decision tree (proxy fail vs. bridge fail)

(A) Regime/proxy fail. If $\mathbf{1}_{\text{pre}} = 0$, then the evaluation must be classified as a *precondition fail*; a bridge test according to Formula Box X.7.2.1 is not licensed in that domain. In particular: if at least one of the indicators $\mathbf{1}_{\text{Tol}}, \mathbf{1}_{\text{KMS}}, \mathbf{1}_{\text{QNEC}}, \mathbf{1}_{\text{Front}}$ is zero (see Subsection X.4.4), then the corresponding proxy family (or the assumed regime/interface bundle) is violated in the tested domain.

(B) Bridge falsification (narrow sense). If $\mathbf{1}_{\text{pre}} = 1$ *and* all proxy indicators from (A) are one *and* additionally $\Delta_{\text{bridge}}(\mathsf{X}) > \delta_{*,\text{bridge}}$ according to Formula Box X.7.2.1, then the bridge scheme from Section X.3 is falsified in the tested domain.

The following example flows illustrate how protocols are typically coupled to data sources without requiring any post-hoc adjustment of the decision criteria.

Experimental pipelines (examples)

Lab: E-A → (optional) Landauer calibration → Sagnac module, each with front calibration and H-gate tomography.

Astro: E-B with spectroscopy (A1 as a scale/regime check) → VLBI delays (A2) → PTA residuals (A3), with calibration/inference (A1) and subsequent out-of-sample tests (A2/A3) explicitly documented as separate steps; front/no-signalling checks via $\mathbf{1}_{\text{Front}}$ (see Definition X.4.2.4).

Cosmo: E-C with sirens or candles → BAO or lenses → long-term drift (time channel); feedback of local proxies (Tolman, front) for low z.

X.7.7 Roadmap and documentation standard

Finally we provide a prioritization by technical maturity and expected evidential strength. What matters is not the calendar year, but the sequence: first hard lab and front diagnostics, then triangulated profiles, then drift/null tests as the cosmological degeneracy breaker.

Roadmap & prioritization

- *Short term (1–3 years):* Lab E-A (DPI/Spohn) on ion traps or spin qubits; Sagnac signatures; astro A1/A2 on existing data sets.
- *Mid term (3–5 years):* Full E-B triangulation with PTA data; first consistent N_B maps in selected systems; cosmological distance-channel reconstructions $H_{\text{dist}}(z)$ with improved standard sirens.
- *Long term (>5 years):* Precise drift/time-channel measurements $dz/d\tau_{\text{geo},0}$ and overdetermined null tests/residuals (distance $\oplus$ time channels); large-scale consistency checks $\{H_{\text{dist}}, H_{\text{CC}}, \widehat{\chi}, \text{Drift}, R_{\text{SN}}\}$ with local proxy feedback; systematic bridge stress tests via Δ_{bridge}.

Documentation. Each experiment publishes X according to the input format Subsection X.5.2, the raw and output data $Y, \Delta Y$, and the binary indicators.

Note. For all protocols: c remains explicitly calibrated (no setting c=1 without a front protocol). The pass/fail thresholds δ_* are set before data inspection to avoid post-hoc threshold adjustment.

X.8 Contextualization & Comparison with Standard QM/GR/ΛCDM

This Section positions the FBA bridge from Section X.3, the H-gates and proxies from Section X.4, and the computational calculus from Section X.5 with respect to the standard approaches of quantum mechanics, general relativity, and the cosmological ΛCDM reference. The guiding questions are: Where is FBA equivalent in the appropriate regime? What is additional content in the form of measurable proxies? What produces deviations, and how are they diagnosed as pass/fail (see Sections X.6 and X.7)?

Framework. We separate three levels: (i) core equivalences in the appropriate limit (flat, local, stationary), (ii) additional structure via budget and front proxies, (iii) deviation regimes (non-stationarity, strong gradients, TDI). This separation is necessary so that "additional structure" is not confused with "deviation": in the equivalence regime the proxies are not meant to claim new physics, but to mark precisely when the standard shortcuts are operationally licensed.

Core equivalences (flat, local, stationary)

- **QM (open systems):** Local GKLS dynamics, CPTP channels, and POVMs coincide with the lab module A of the computational calculus (Algorithm X.5.3.1). DPI and Spohn monotonicities are compatible with cone causality (Lemma X.3.4.1 and Corollary X.3.6.1). *Source (overview):* Secs. IV.3–IV.6.
- **GR (local Minkowski limit):** Light cones and proper time follow from the budget quadric and front calibration; Tolman relations (Proxy A, Definition X.4.2.1) reproduce gravitational redshift in a stationary field. *Source (overview):* Secs. VI.3–VI.5.
- **Cosmology (without TDI):** For $\chi \equiv \text{const.}$ (and in particular $\chi \equiv 1$ after calibration), the distance channel and time channels coincide: $H_{\text{dist}}(z) = H_{\text{CC}}(z) = H(z)$, the geometric redshift is $1 + z = a_0/a$, and Etherington duality holds in the licensed regime (cf. Subsection X.6.3).

The core equivalences are the reference point for everything that follows: only when it is clear that the standard regimes are reproduced do proxy fails and TDI signatures acquire the correct meaning, namely as targeted, localizable violations of regime assumptions.

Kinematics. We start with Minkowski kinematics because it provides the common language for laboratory and near-field tests and at the same time explains why front calibration in FBA is not optional, but the operational fixation of the light-cone structure.

Formula Box X.8.1: Lorentz kinematics from the budget quadric (Minkowski limit)

In the flat (Minkowski) limit with front calibration,

$$d\tau_{\rm geo}^2 \;=\; -\eta_{\mu\nu}\, dx^\mu dx^\nu / c^2, \qquad \gamma = \frac{dt}{d\tau_{\rm geo}} \;=\; \frac{1}{\sqrt{1 - v^2/c^2}},$$

where $d\tau_{\rm geo}$ is the reversible internal budget contribution (notation and decomposition in Section X.2). The total proper time $\tau_{\rm tot} = \tau_{\rm geo} + A$ separates irreversible aging $A \geq 0$.

Consequence. Time dilation γ and the light-cone structure agree with standard SR. Irreversible contributions appear as observable rates and entropy productions, not as a change of the cone geometry.

This also makes clear how FBA positions standard concepts: "geometry" means the cone- and proper-time-related structure from $b_{\rm int}$, while irreversibility is carried as its own, measurable channel in $b_{\rm irr}$. This separation is crucial later because it prevents dissipative signatures from being relabeled as "geometric effects".

Microcausality. Standard QFT formulates microcausality as (approximate) commutativity of spacelike separated operators. In the FBA context, the relevant question is operational: Can an additional operation placed spacelike affect the evaluation in a measurable way? Exactly this (no-signalling) statement is made testable as a data criterion by the front proxy.

Formula Box X.8.2: Microcausality operationally: no-signalling front proxy + correlation as diagnostics

For two spacelike separated regions A, B ($|\Delta x| > c|\Delta t|$) we define an interventional no-signalling functional $\mathcal{J}_{\rm NS}(\Delta x, \Delta t)$ (cf. Proxy D in Definition X.4.2.4):

$$\mathcal{J}_{\rm NS}(\Delta x, \Delta t) = \sup_{\mathcal{I}_A \in \mathfrak{I}_A,\ \mathcal{M}_B \in \mathfrak{M}_B} \big\| p(b \mid \mathcal{I}_A, \mathcal{M}_B, t + \Delta t) - p(b \mid \mathrm{id}, \mathcal{M}_B, t + \Delta t) \big\|_1,$$

where $\mathcal{I}_A$ denotes interventions/additional local operations on A, $\mathcal{M}_B$ a fixed test POVM on B, and b its outcomes.

Pass/Fail (Proxy D). $\mathbf{1}_{\rm Front} = 1$ if and only if the no-signalling criterion (including threshold/tolerance) specified in Definition X.4.2.4 holds on the preregistered test grid.

Context. This criterion is the operational form of "no spacelike influence" and is compatible with the standard intuition of microcausality as a prohibition of signalling/influence.

Diagnostics. In addition, spacelike correlations via $\mathcal{R}_{\rm front}(\Delta x, \Delta t)$ (see Definition X.4.2.4) can be reported, but $\mathcal{R}_{\rm front}$ is *not* a no-signalling quantity: non-vanishing correlations can arise from common causes/entanglement without any signalability.

The advantage of this formulation is that it does not replace the standard requirement, but makes it tangible as a measurement protocol: microcausality then is not a purely axiomatic statement, but a pass/fail criterion with explicit error control and a clear intervention semantics.

Translation glossary. So that comparisons do not fail due to symbolism, we fix a short dictionary. It is intentionally brief: it is meant to enable the comparison without introducing a second world of notation.

FBA ↔ standard quantities (dictionary)

- $N_B(x)$ (budget-induced lapse) ↔ gravitational redshift factor / Tolman lapse (Proxy A, Definition X.4.2.1).
- Δb_{irr}, $\dot{A}$ ↔ entropy production / irreversibility measure (lab coupling via Landauer, Formula Box X.6.1.3; production rates via DPI/Spohn, Formula Box X.6.1.1).
- H (H-gate) ↔ tomographically fixed cross-basis/isometry between the FBA and Hilbert-space representations (Definition X.4.1.1).
- $\mathcal{J}_{\text{NS}}$ or $\mathbf{1}_{\text{Front}}$ ↔ operational no-signalling/front test (Proxy D, Definition X.4.2.4); $\mathcal{R}_{\text{front}}$ as an additional diagnostic quantity (correlations, not signalability).
- χ (TDI factor) ↔ effective time-scale modulation that is not introduced via a "z redefinition", but is testable via *channel comparison*:

$$\widehat{\chi}(z) = \frac{H_{\text{dist}}(z)}{H_{\text{CC}}(z)}$$

  (distance channel H_{dist} vs. chronometer channel H_{CC}, plus drift/null tests; see Section X.7).

Stationary fields. In the stationary regime, FBA should not "deviate", but reproduce the standard relations. The additional content here is that stationarity and redshift are not merely assumed, but can be checked via proxies as regime conditions.

Corollary X.8.1: Tolman and KMS equivalence in the stationary regime

If $\mathbf{1}_{\text{Tol}} = \mathbf{1}_{\text{KMS}} = 1$ (see Definitions X.4.2.1 and X.4.2.2), then FBA reproduces the standard relations for stationary fields:

$$1 + z_B = \frac{N_B(x_{\text{obs}})}{N_B(x_{\text{emit}})}, \qquad T(x)N_B(x) = \text{const.}$$

Moreover, two-point spectra in the observed range are KMS-compatible in the sense of the proxy definition.

This makes clear what "added value" means here: not new equations in the stationary limit, but the possibility to establish the limit regime itself as pass/fail.

Deviation regimes. In the FBA context, deviations are not ex post free parameters, but arise as consequences of clearly identified regime violations (proxy fails) or as a large-scale coupling (TDI). The following map serves as a diagnostic aid because it separates failure modes from one another.

Deviations & signatures (diagnostic map)

Non-stationarity or strong gradients. Violation of $\mathbf{1}_{\mathrm{KMS}}$ (Proxy B): non-thermal spectra, drifting effective temperatures, asymmetries in autocorrelations (see Definition X.4.2.2).

Front-near dynamics. Violation of $\mathbf{1}_{\mathrm{Front}}$ (Proxy D) in the interventional sense: the no-signalling functional $\mathcal{J}_{\mathrm{NS}}$ lies above the tolerance preregistered in Definition X.4.2.4; this suggests revising local generators and/or front protocols (see Definition X.4.2.4 and Section X.7). $\mathcal{R}_{\mathrm{front}}$ is, in this context, an additional diagnostic quantity, but not a signalability decision.

Budget leaks (missing closure). Violation of the closure condition in Proxy B: discrepant Shapiro or timing delays despite otherwise matching Tolman profiles; typically an indication of underestimated external net flows (see Definition X.4.2.2 and the astro protocols in Section X.7).

TDI coupling (channel overdetermination instead of z re-labeling). $\chi \not\equiv$ const. appears operationally as a *divergence* of distance and time channels:

$$\widehat{\chi}(z) = \frac{H_{\mathrm{dist}}(z)}{H_{\mathrm{CC}}(z)} \neq 1 \quad \text{(after calibration).}$$

Additional orthogonality is provided by the drift/time channel at the observer:

$$\frac{dz}{d\tau_{\mathrm{geo},0}} \overset{\text{(TDI)}}{\approx} \chi_0^{-1}\Big[(1+z)H_0 - H_{\mathrm{dist}}(z)\Big],$$

where a z-independent χ_0 estimator (and consistent $\widehat{\chi}(z)$) serves as a null-test chain (cf. Section X.7).

Examples and checks. The most important practical question is not whether a single observable deviates, but whether several logically coupled probes are mutually consistent. The following triple is therefore a standard check: profiles fix N_B, delays and timing test the same structure as a path- and accumulation quantity, respectively.

Formula Box X.8.3: Coherence check: profiles ↔ delays ↔ timing

With $N_B(x)$ from profiles, for ray paths γ we have

$$\Delta t_B[\gamma] = \int_\gamma (N_B^{-1} - 1)\, ds, \quad R(t) = \int_0^t \left(\frac{1}{N_B(x(t'))} - 1\right) \frac{dt'}{P}.$$

Criterion. Pass of all three (profiles, delays, timing) ⇒ consistent N_B in the tested range. Fail in one ⇒ localizable proxy violation with experimental logic from Section X.7.

Cosmological level. Without TDI, the comparison with ΛCDM is a limit check. With TDI, FBA provides additional, orthogonal tests because drift and time channels cannot be absorbed by the same parametrization as distance fits.

Formula Box X.8.4: ΛCDM limit and TDI signatures (channel comparison)

Without TDI (limit case). $\chi \equiv$ const. (and after calibration $\chi \equiv 1$) $\Rightarrow$ distance channel and time channels agree:

$$H_{\mathrm{dist}}(z) = H_{\mathrm{CC}}(z) = H(z), \qquad \frac{dz}{d\tau_{\mathrm{geo},0}} = (1+z)H_0 - H(z).$$

Etherington duality holds in the licensed regime (cf. Subsection X.6.3).

With TDI (additional tests).

$$\widehat{\chi}(z) = \frac{H_{\mathrm{dist}}(z)}{H_{\mathrm{CC}}(z)}, \qquad \frac{dz}{d\tau_{\mathrm{geo},0}} \approx \chi_0^{-1}\Big[(1+z)H_0 - H_{\mathrm{dist}}(z)\Big].$$

Diagnostics. (1) $\widehat{\chi}(z)$ is a direct, data-driven channel comparison; (2) the drift/time channel provides an orthogonal consistency condition via χ_0. Together they form degeneracy breakers against purely distance or rate fits (see Section X.7).

Interpretation. FBA does not replace any standard theory. It grounds standard regimes from sequence and budget principles, provides an operational bridge, and adds a testable regime and consistency structure. Deviations are therefore not “free”, but tied to concrete proxy violations or to the cosmic channel overdetermination (TDI) and are thus addressable as pass/fail.

Checklist (comparison & practice)

- **Equivalences:** SR kinematics (Formula Box X.8.1), local GKLS and DPI/Spohn (Lemma X.3.4.1), stationary redshift and KMS (Proxy A/B, Definitions X.4.2.1 and X.4.2.2), ΛCDM limit for $\chi \equiv$ const. via channel equality (Formula Box X.8.4).
- **Added value:** Operational proxies (Tolman, KMS/closure, front, QNEC-near) and bridge coherence $\mathcal{P}_{\mathrm{QM}} = \mathcal{P}_{\mathrm{Geo}}$ as pass/fail tests (Formula Box X.7.2.1).
- **Deviations:** Localizable via proxy fails and cosmic channel null tests (distance channel vs. time channels plus drift; Formula Box X.8.4); degeneracy breaking via time channels and front tests (see Section X.7).
- **Workflow:** Apply the dictionary (Section X.8) $\rightarrow$ couple profiles and delays (Formula Box X.8.3) $\rightarrow$ test lab monotonicities (Formula Boxes X.6.1.1 and X.6.1.2) $\rightarrow$ cosmic channel overdetermination as the finale (distance $\oplus$ time channels $\oplus$ drift; Section X.7).

X.9 Case Studies & Replication Blueprints (including TDI & cosmic dynamics)

This Section turns the bridge (Section X.3), the H-gates and proxies (Section X.4), and the computational calculus (Section X.5) into concrete case studies. The goal is a reproducible blueprint from data acquisition all the way to the pass/fail decision. The logic is deliberately always the same: (1) input according to the grammar in Subsection X.5.2, (2) evaluation via the modules in Section X.5, (3) decision via the indicators from Subsection X.4.4 plus bridge diagnosis.

We begin in the lab (maximal control over unselectiveness and calibration), move to orbital and astrophysical applications (profiles plus path integrals), and conclude with cosmological TDI signatures (distance ladder plus drift). At the end we provide a minimal data-package format, a systematics map, and a continuous bridge quality measure.

X.9.1 Laboratory case study: unselected two-level decoherence

Narrative frame. A one- or two-qubit setup (ion trap or superconducting qubit) is the most direct place where the bridge becomes operationally "sharp": DPI/Spohn monotonicities are not a matter of interpretation here, but a hard pass/fail bound under unselected evaluation. The procedure uses lab module A (Algorithm X.5.3.1) and decides via Formula Box X.6.1.1 as well as the structure check Formula Box X.6.1.2. *Source (overview):* Secs. III.3–III.5; Secs. IV.3–IV.6.

Algorithm X.9.1.1: Blueprint L: DPI/Spohn on a two-level system

Input: Channel network $\mathfrak{N}$ (amplitude or phase damping), H-gate H (Definition X.4.1.1), calibration Cal, observation $\mathcal{O}$ (outcome distributions, monotones).
Procedure.

1. *Tomography:* Reconstruct ρ_0 and references σ; explicitly document that the evaluation is unselected (no post-selection).
2. *Evolution:* Implement Φ_t for two times $t_1 < t_2$ under identical front and time calibration.
3. *Measurement:* Acquire $\{p_{t_i}(j)\}$ in two complementary bases (cross-basis via H, see Formula Box X.4.3.1).
4. *Evaluation:* Estimate $D_\alpha(\rho_{t_i}\|\sigma)$ (in the preregistered DPI variant) and $\sigma_{\mathrm{Spohn}}(t)$ insofar as licensed in the protocol (module A, Algorithm X.5.3.1).
5. *Decision:* $\mathbf{1}_{\mathrm{lab,DPI}}$ according to Formula Box X.6.1.1 and $\mathbf{1}_{\mathrm{lab,GKLS}}$ according to Formula Box X.6.1.2.

Output: ΔD_α, $\sigma_{\mathrm{Spohn}}(t)$, $\mathbf{1}_{\mathrm{lab,DPI}}, \mathbf{1}_{\mathrm{lab,GKLS}}$, uncertainties according to Formula Box X.5.4.1.

Measurement notes. Front calibration stabilizes the time base. If an additional metrological coupling between budget and informational work is required, add reset experiments and use Formula Box X.6.1.3.

X.9.2 Astrophysical case study: Tolman–Shapiro–PTA triangulation

Narrative frame. In the astrophysical regime, the goal is not a "better fit", but the consistency of the same N_B structure across three logically distinct probes: profiles (local, stationary), delays (a path integral along null rays), and timing (temporal accumulation). This combination is a degeneracy breaker because it tests N_B against itself through different data projections. Operationally, we strictly distinguish between (i) reconstruction/calibration of N_B and (ii) subsequent out-of-sample tests. *Source (overview):* Secs. VI.3–VI.5.

Algorithm X.9.2.1: Blueprint A: consistent N_B from profiles, delays, timing

Input: Spectral lines and temperatures (Tolman), ray paths γ (echoes, occultation, VLBI, lensing), timing data t_k (pulsar/residuals), calibration Cal.
Procedure.

1. *Calibration/inference (profile step):* Reconstruct an N_B estimate from Formula Box X.6.2.1 between $(x_{\text{emit}}, x_{\text{obs}})$. Explicitly document which data are used for this purpose (training/calibration set), and record $\mathbf{1}_{\text{Tol}}$ (regime/scale check).
2. *Out-of-sample delay test:* Compute $\Delta t_B[\gamma]$ via Formula Box X.6.2.2 using the same N_B, and compare against *disjoint* delay data (hold-out or independent events/sources, where available).
3. *Out-of-sample timing test:* Compute $R(t)$ via Formula Box X.6.2.3 using the same N_B, and compare against *disjoint* timing residuals (hold-out or independent data sets).
4. *Decision:* Report $\mathbf{1}_{\text{Tol}}$ separately; do not count profiles as an independent test if the same profile data were used for the N_B reconstruction. The triangulation decision is then

$$(\mathbf{1}_{\text{astro,Delay}} \wedge \mathbf{1}_{\text{astro,PTA}}) \;\wedge\; \mathbf{1}_{\text{Tol}}$$

(with $\mathbf{1}_{\text{Tol}}$ as a regime/scale check).

Output: Consistency score S_{astro}, residuals $\{\widehat{\Delta t} - \Delta t_B,\, R_{\text{obs}} - R_{\text{mod}}\}$, (optional, in-sample/calibration-related) $\{\widehat{z}_B - z_B\}$, uncertainties according to Formula Box X.5.4.1.

Diagnostics. A fail is particularly informative here because it marks a concrete inconsistency: a fail of $\mathbf{1}_{\text{Tol}}$ points to Tolman/stationarity or calibration problems (regime/proxy fail). A fail in $\mathbf{1}_{\text{astro,Delay}}$ points to path/geometry inconsistency or front/calibration issues. A fail in $\mathbf{1}_{\text{astro,PTA}}$ is sensitive to large-scale gradients or "traffic-jam" fronts (an integrative test). (Optional) In front-near regimes, $\mathbf{1}_{\text{QNEC}}$ can additionally be reported as a null-flux/focusing check (Definition X.4.2.3). If interventional front/no-signalling checks are available, report $\mathbf{1}_{\text{Front}}$ according to Definition X.4.2.4 as a separate cross-check.

X.9.3 Cosmological case study: distance channel ⊕ time channels ⊕ drift (TDI)

Narrative frame. Cosmologically, the TDI factor $\chi(t)$ is only meaningfully testable if it cannot "disappear" into calibration freedoms. Hence the chain is deliberately closed: the distance channel yields $H_{\text{dist}}(z)$ (C1), the chronometer channel yields $H_{\text{CC}}(z)$ and thus $\widehat{\chi}(z)$ (C2), and the drift channel (C3) provides an orthogonal consistency condition via χ_0 and null tests/residuals. *Source (overview):* Secs. IX.3–IX.6.

Algorithm X.9.3.1: Blueprint C: distance $\oplus$ chronometers $\oplus$ drift (null-test chain)

Input: Distance data $d_L(z)$ (candles/sirens) and, if available, $d_A(z)$ (BAO/lenses), chronometer data $dz/d\tau_{\text{geo}}$, drift measurements $dz/d\tau_{\text{geo},0}$, (optional) SN light-curve dilation $R_{\text{SN}}(z)$, calibration Cal (including c, H_0, and possibly k in the chosen convention).

Procedure.

1. *Distance channel:* Reconstruct $H_{\text{dist}}(z)$ from $d_L(z)$ (module D, Algorithm X.5.3.4; prediction C1, Formula Box X.6.3.1) and report $\mathbf{1}_{\text{cosmo,dist}}$ according to Formula Box X.6.3.1.
2. *Chronometer channel:* Estimate $H_{\text{CC}}(z) := -\frac{1}{1+z}\frac{dz}{d\tau_{\text{geo}}}$, form $\widehat{\chi}(z) = \frac{H_{\text{dist}}(z)}{H_{\text{CC}}(z)}$, and report $\mathbf{1}_{\text{cosmo},\chi}$ according to Formula Box X.6.3.2.
3. *Drift/time channel & null tests:* Use $dz/d\tau_{\text{geo},0}$ (and optionally $R_{\text{SN}}(z)$) to form the residuals $\Delta_{\text{DR}}, \Delta_{\text{CC}}, \Delta_{\text{SNCC}}$ and decide $\mathbf{1}_{\text{cosmo,null}}$ according to Formula Box X.6.3.3.

Output: $\{H_{\text{dist}}(z), H_{\text{CC}}(z), \widehat{\chi}(z)\}$, null-test residuals $\{\Delta_{\text{DR}}, \Delta_{\text{CC}}, \Delta_{\text{SNCC}}\}$, consistency score S_{cosmo}, uncertainties according to Formula Box X.5.4.1.

Low-z cross-check. To avoid calibration aliasing, combine cosmological evaluations in the low-z regime with local proxies (Tolman and front, Definitions X.4.2.1 and X.4.2.4).

X.9.4 Data package & reproducibility

Narrative frame. To ensure replication does not fail on pipeline details, we define a minimal package format that contains inputs, raw data, covariances, and decisions in a way that allows recomputation of the bridge diagnosis and proxy decisions from exactly the same files.

Data package FBAKIT (minimal)

Contents.

- `setup.yaml`: $\mathsf{X} = (\mathcal{S}, \mathfrak{N}, \mathsf{H}, \pi, \mathsf{Cal}, \mathcal{O})$ according to the input format Subsection X.5.2.
- `observations.csv`: Raw observables $\{y_i\}$ with time and location stamps, binning, and units from Cal.
- `covariance.npy`: Covariance matrix $\mathrm{Cov}(Y)$ or block structure plus metadata, as used in Formula Box X.5.4.1.
- `proxies.json`: Tolerances ε, estimated proxy functionals $\widehat{\mathcal{J}}_{\bullet}$, and the binary indicators $\mathbf{1}_{\mathrm{Tol}}, \mathbf{1}_{\mathrm{KMS}}, \mathbf{1}_{\mathrm{QNEC}}, \mathbf{1}_{\mathrm{Front}}$ (definitions: Section X.4).
- `bridge.json`: $\delta_{\mathrm{bridge}}(\mathsf{X}) = \mathcal{P}_{\mathrm{QM}}(\mathsf{X}) - \mathcal{P}_{\mathrm{Geo}}(\mathsf{X})$, $\Sigma(\mathsf{X})$ (covariance in the data space of the considered output quantities; cf. Formula Box X.7.2.1), $\Delta_{\mathrm{bridge}}(\mathsf{X}) = \|\delta_{\mathrm{bridge}}(\mathsf{X})\|_{\Sigma^+}$ (with Σ^+ a possibly regularized pseudoinverse; in the full-rank case $\Sigma^+ = \Sigma^{-1}$), as well as $\varepsilon_{\mathrm{bridge}}$ from Formula Box X.7.2.1.

Goal. Recomputation of $\delta_{\mathrm{bridge}}(\mathsf{X})$, $\Delta_{\mathrm{bridge}}(\mathsf{X})$, and all proxy decisions without proprietary intermediate steps.

X.9.5 Bridge quality & systematics

Narrative frame. Binary decisions are sufficient for falsification, but too coarse for diagnosis. Therefore we add a continuous quality measure that (i) evaluates the size of the bridge deviation relative to errors and (ii) can serve as a comparison value across data sets.

Formula Box X.9.5.1: Bridge quality measure $\mathcal{G}_{\mathrm{bridge}}$ (continuous)

Let

$$\delta_{\mathrm{bridge}}(\mathsf{X}) \;=\; \mathcal{P}_{\mathrm{QM}}(\mathsf{X}) - \mathcal{P}_{\mathrm{Geo}}(\mathsf{X})$$

be the (component-wise) bridge residual in the data space of the considered output quantities. Let $\Sigma(\mathsf{X})$ be the covariance of $\delta_{\mathrm{bridge}}(\mathsf{X})$ (from error propagation according to Formula Box X.5.4.1 or from bootstrap/jackknife); if Σ is not full rank, use a (possibly regularized) pseudoinverse Σ^+. (In the full-rank case $\Sigma^+ = \Sigma^{-1}$.)
We define the covariance-weighted bridge distance

$$\Delta_{\mathrm{bridge}}(\mathsf{X}) \;:=\; \|\delta_{\mathrm{bridge}}(\mathsf{X})\|_{\Sigma^+} \;=\; \sqrt{\delta_{\mathrm{bridge}}(\mathsf{X})^{\top}\,\Sigma(\mathsf{X})^{+}\,\delta_{\mathrm{bridge}}(\mathsf{X})}$$

and from it the continuous quality measure

$$\mathcal{G}_{\mathrm{bridge}}(\mathsf{X}) \;=\; \exp\Big(-\tfrac{1}{2}\,\Delta_{\mathrm{bridge}}(\mathsf{X})^2\Big) \;\in\; (0,1]\,.$$

Interpretation. $\mathcal{G}_{\mathrm{bridge}} \approx 1$ corresponds to high coherence (residual small relative to errors); small values correspond to a statistically relevant bridge deviation.

A low score is only diagnostically useful if typical systematics are carried along as cross-checks. The following map summarizes the most common failure sources and their fastest cross-check.

Systematics & cross-checks

Front or time drift: Re-check calibration Cal repeatedly; ToF checks plus Sagnac as a counter-anchor (Formula Box X.6.1.4).

No-signalling/front violation: Check the interventional front test $\mathcal{J}_{\mathrm{NS}}$ and $\mathbf{1}_{\mathrm{Front}}$ (Definition X.4.2.4); interpret $\mathcal{R}_{\mathrm{front}}$ only diagnostically.

Selection artefacts: Enforce and document unselectiveness (lab blueprint, step 1; DPI test Formula Box X.6.1.1).

Path uncertainties (astro): Vary the γ bundle, use multiple echoes for Δt_B (Formula Box X.6.2.2).

Photometric scales (cosmo): Cross-calibrate candles and sirens; report chronometer and drift null tests jointly (Formula Boxes X.6.3.2 and X.6.3.3).

Null-direction/front proximity (optional): For front-near data, document QNEC-like null-flux diagnostics (Proxy C) as a separate regime check (Definition X.4.2.3).

X.9.6 Checklist (operational)

Narrative frame. The following list summarizes the minimal steps for a complete bridge check in each domain. It is structured so that regime conditions (proxies) are checked first and bridge coherence second.

Pass/fail checklist per domain

Lab: $\mathbf{1}_{\mathrm{lab,DPI}}$ and $\mathbf{1}_{\mathrm{lab,GKLS}}$ (optional: Landauer coupling via Formula Box X.6.1.3).

Astro: $\mathbf{1}_{\mathrm{Tol}}$ as a regime/scale check (profile/Tolman step; cf. Subsection X.9.2), and the out-of-sample tests $\mathbf{1}_{\mathrm{astro,Delay}}$ and $\mathbf{1}_{\mathrm{astro,PTA}}$ (triangulation via path and timing data, Subsection X.9.2). (optional) $\mathbf{1}_{\mathrm{Front}}$ if interventional measurement is available (Definition X.4.2.4); (optional) $\mathbf{1}_{\mathrm{QNEC}}$ in front-near regimes (Definition X.4.2.3).

Cosmo: $\mathbf{1}_{\mathrm{cosmo,dist}}$, $\mathbf{1}_{\mathrm{cosmo},\chi}$, $\mathbf{1}_{\mathrm{cosmo,null}}$ (null-test chain distance $\oplus$ chronometers $\oplus$ drift, Formula Boxes X.6.3.1 to X.6.3.3; blueprint Algorithm X.9.3.1).

Bridge: The bridge test is only licensed under satisfied preconditions $\mathbf{1}_{\mathrm{pre}} = 1$ (in particular satisfied proxy/front conditions; see pass/fail semantics and decision tree in Subsection X.7.1 and Corollary X.7.6.1). Then: $\Delta_{\mathrm{bridge}}(\mathsf{X}) \leq \varepsilon_{\mathrm{bridge}}$ (see Formula Box X.7.2.1) and a high score $\mathcal{G}_{\mathrm{bridge}}(\mathsf{X})$ (see Formula Box X.9.5.1).

Outlook. For real data sets, we recommend attaching `FBAKIT` (Subsection X.9.4) and always reporting $\mathcal{G}_{\mathrm{bridge}}$ jointly with the binary indicators. This keeps both hard falsification decisions and diagnostic information reproducibly available.

X.10 Conclusion: bridge status, decision criteria & open problems

This concluding Section fixes the *status* of the bridge as an operational test scaffold and makes the *decision logic* explicit: What is evaluated in binary terms as passed/failed, which continuous quantities are carried along as diagnostics, and which open points determine the next measurement priority? In doing so, the Section closes the loop from the formal bridge (Section X.3) via H-gates/proxies (Section X.4) to the computational calculus (Section X.5) and the prediction and protocol modules (Sections X.6, X.7 and X.9).

X.10.1 Core results & bridge status

To conclude, we summarize the supporting results and locate them within the structure of Sections X.3 to X.7 and X.9. The narrative thread is deliberately concise: first the three pillars of the bridge, then the operational building blocks that turn it into a pass/fail structure.

Pillars. Three internal statements carry the bridge: DPI and cone compatibility (Lemma X.3.4.1), the bridge theorem (existence and residual freedom up to isometry) (Lemma X.3.5.1) and GKLS–cone compatibility in the Minkowski limit (Corollary X.3.6.1). H-gates (Definition X.4.1.1 and Lemma X.4.1.1) fix the representational freedom, proxies (Definitions X.4.2.1 to X.4.2.4) provide the transition and regime conditions (for Proxy D: pass/fail via the interventional criterion $\mathcal{J}_{\mathrm{NS}}$; $\mathcal{R}_{\mathrm{front}}$ is a diagnostic quantity, see Definition X.4.2.4).

Executive Summary (technical, concise)

- **Bridge.** Functors $\mathcal{Q}$ (FBA→QM) and $\mathcal{G}$ (FBA→Geo) exist, are unique up to local isometries, and commute operationally within the validity range of H-gates and proxies (Lemma X.3.5.1 and Formula Box X.3.3.1).
- **Monotonicity & cones.** DPI and Spohn monotonicities are compatible with cone causality (Lemma X.3.4.1); in the flat limit there are no superluminal effective channels (Corollary X.3.6.1).
- **Operational.** Proxies A–D define pass/fail rules (Subsection X.4.4); algorithms from Section X.5 provide observables and uncertainties; predictions are catalogued in Section X.6 and made replication-robust in Sections X.7 and X.9 as protocols and blueprints.

X.10.2 Decision criteria: binary and continuous

The overall assessment combines a binary bridge test with a continuous quality measure. The binary test is the minimal falsification criterion. The score is a diagnostic and comparison quantity, so that different data sets can be assessed not only as pass/fail, but also with respect to the size and structure of their bridge deviation.

Formula Box X.10.2.1: Bridge decision rule (binary) & quality measure (continuous)

Binary. The bridge is considered passed in the tested domain $\mathcal{D}$ if

$$\mathbf{1}_{\text{pre}}^{(\mathcal{D})}(\mathsf{X}) = 1 \quad \text{and} \quad \Delta_{\text{bridge}}(\mathsf{X}) \leq \varepsilon_{\text{bridge}},$$

where $\Delta_{\text{bridge}}(\mathsf{X})$ is the covariance-weighted bridge distance from Formula Box X.7.2.1 (with preregistered $\Sigma(\mathsf{X})$, and possibly pseudoinverse/regularization) and the evaluation identity is conceptually fixed by Formula Box X.3.3.1.

Here $\mathbf{1}_{\text{pre}}^{(\mathcal{D})}(\mathsf{X})$ is the domain-specific *precondition indicator*:

$$\mathbf{1}_{\text{pre}}^{(\mathcal{D})}(\mathsf{X}) := \bigwedge_{p \in \mathcal{P}(\mathcal{D})} \mathbf{1}_p(\mathsf{X}),$$

where $\mathcal{P}(\mathcal{D})$ selects the proxy/setup indicators that are *applicable* in domain $\mathcal{D}$ (according to the pass/fail rules in Subsection X.4.4 and the domain blueprints in Section X.9). Non-applicable indicators are documented as `N/A` and do not enter the conjunction.

Continuous. In addition, report the score $\mathcal{G}_{\text{bridge}}(\mathsf{X}) \in (0, 1]$ from Formula Box X.9.5.1.

Practical reading. The binary decision rule is the "hard edge" (falsification under satisfied preconditions). The continuous measure $\mathcal{G}_{\text{bridge}}$ is the diagnosis-oriented complement: it allows one to distinguish between "barely passed", "comfortably passed", and "clearly violated" without weakening the pass/fail logic.

X.10.3 Reviewer flow (replication)

For peer review and third-party replication, a clear, reproducible workflow is needed that brings together the data package, proxy decisions, and bridge diagnosis. The flow is chosen so that it first checks regime and calibration consistency and only then evaluates the bridge itself. This avoids misinterpreting bridge deviations when proxy preconditions are already violated.

Algorithm X.10.3.1: Reviewer flow (minimal, reproducible)

Input: Data package `FBAKIT` (Subsection X.9.4), algorithms from Section X.5, proxy thresholds from Section X.4.
Steps.

1. *Integrity.* Validate `setup.yaml` and `observations.csv`; reconstruct and plausibilize the covariances (scheme from Formula Box X.5.4.1).
2. *Proxies & preconditions.* Evaluate the proxy indicators (including explicit `N/A` marking) according to Definitions X.4.2.1 to X.4.2.4 as well as all domain-specific setup indicators from Section X.9. From this, determine the domain-specific precondition indicator $\mathbf{1}_{\mathrm{pre}}^{(\mathcal{D})}(\mathsf{X})$ (see Formula Box X.10.2.1).
3. *Bridge.* Compute $\mathcal{P}_{\mathrm{QM}}(\mathsf{X})$ and $\mathcal{P}_{\mathrm{Geo}}(\mathsf{X})$, from them $\delta_{\mathrm{bridge}}(\mathsf{X})$, $\Sigma(\mathsf{X})$, and $\Delta_{\mathrm{bridge}}(\mathsf{X})$ (criterion Formula Box X.7.2.1) as well as $\mathcal{G}_{\mathrm{bridge}}(\mathsf{X})$ (Formula Box X.9.5.1).
4. *Decision.* Perform the binary test via $\mathbf{1}_{\mathrm{pre}}^{(\mathcal{D})}(\mathsf{X})$ and $\Delta_{\mathrm{bridge}}(\mathsf{X})$ and report the score (see Formula Box X.10.2.1); diagnose deviations along the proxy channels and assign them to the appropriate protocols from Sections X.7 and X.9.

Output: Pass/fail, $\mathcal{G}_{\mathrm{bridge}}(\mathsf{X})$, error budget, and diagnosis path.

X.10.4 Open problems & research agenda

The bridge makes explicitly testable assumptions visible. What remains open is not *whether* one can test, but *how robust* the tests are in boundary regimes and which measurement strategies keep proxy and bridge decisions stable there.

Open problems & research questions

(O1) Non-Markov regimes. Robustness of Lemma X.3.4.1 under strongly time-nonlocal dynamics with only piecewise CPTP evolution; operationalization of "unselected" in the presence of nontrivial record/memory channels.

(O2) Strong fronts and horizons. Calibration and proxy determination near strong "traffic-jam" fronts; consistent choice of N_B under large gradients, including controlled path uncertainties in delay integrals.

(O3) QNEC proxy. Noise-robust estimators for S_k'' (or surrogate entropies) and calibration-stable determination of κ in Definition X.4.2.3; separating estimation errors from genuine null-flux violation.

(O4) H-gate cross-bases. Uniqueness beyond local isometries in strongly coupled many-body systems, and controllable tomography schemes for Definition X.4.1.1 under limited measurement resolution.

(O5) TDI decoupling. Joint fit of $\{H_\chi, d_L, d_A, \dot{z}_\chi\}$ with local proxies; systematic degeneracy breakers in the low-z regime and robust separation of photon losses/front issues vs. genuine χ dynamics (see Formula Boxes X.6.3.2 and X.6.3.3 and Definition X.4.2.4).

X.10.5 Roadmap (next milestones)

From the open points and the experimental Sections, a prioritized sequence with clear deliverables follows. The common denominator is always the same: a published input specification, pass/fail indicators, and a documented bridge diagnosis.

Roadmap - next milestones

- **Lab (first).** DPI and Spohn tests with strict unselectiveness; GKLS structure check (PSD); H-gate cross-check via Formula Box X.4.3.1; (optional) Landauer calibration Θ on two platforms for metrological robustness.
- **Astro (then).** N_B triangulation: profile $\to$ delay $\to$ timing (out-of-sample, Section X.9); KMS/closure as a stationarity anchor; (optional) QNEC-like checks in front-near regimes.
- **Cosmo (finally, once available).** Joint reporting of $\{H_\chi, d_L, d_A, \dot{z}_\chi\}$ including duality and drift residuals; aggregation of $\mathcal{G}_{\text{bridge}}(\mathsf{X})$ across data sets with an explicit low-z proxy cross-check (Tolman/front).

X.10.6 Meta-coherence & concluding remark

Within the validity range of the proxies and for small bridge deviation, the bridge reproduces the standard relations. Deviations are then not interpretational leeway, but precisely localizable consequences: either proxy violations (a regime problem) or a genuine bridge inconsistency (breakdown of commutativity).

Corollary X.10.6.1: Meta-coherence theorem (programmatic)

If $\mathbf{1}^{(\mathcal{D})}_{\text{pre}}(\mathsf{X}) = 1$ and $\Delta_{\text{bridge}}(\mathsf{X}) \le \varepsilon_{\text{bridge}}$, then the FBA evaluation in the tested regime is empirically equivalent to standard QM and standard GR. The additional FBA structure (budgets and proxies) then acts as testable diagnostics, without producing contradictions with standard predictions.

Conclusion. The bridge FBA $\to$ QM $\leftrightarrow$ GR is formulated here not as an interpretation, but as a computational and testing architecture: $\mathcal{P}$ generates observables, proxies license regimes, and Δ_{bridge} decides commutativity in a domain-specific way. This makes clear what "progress" means in the program: not more parametrization, but *more independent, orthogonal* pass/fail tests that separate failure modes and enable replication.

X.11 Appendix: Overview of the FBA Series (Parts I–X)

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1. ***Part I*: FBA-Foundations: Ordering, Budget, Proper Time & Arrows.** *Goal:* Provide the base layer: ordering, budget, proper time/aging, front and the operational arrow of time (DPI); Minkowski limit from the budget quadric; admissible dynamics and locality/no-signalling. *Import:* – (reference for all subsequent parts). *Extension:* interface contracts, pass/fail checklists, reading guide.
2. ***Part II*: Time, Proper Time & Minkowski Geometry.** *Goal:* Capture proper time/quadric operationally and derive geodesics. *Import:* foundations (ordering, budget, proper time, front/DPI). *Extension:* smooth limit, variational principle on worldlines, calibration κ_τ.
3. ***Part III*: Quantum Kinematics & CPTP Channels.** *Goal:* State spaces and channels (CPTP) including composition. *Import:* foundations (budget, channel viewpoint, composition). *Extension:* concrete divergences/cost functionals $\mathcal{C}$, measurements, and classical registers.
4. ***Part IV*: Dynamics, Measurement & GKLS (Open Systems).** *Goal:* Continuous open dynamics (GKLS) and the operational arrow of time. *Import:* channels/DPI. *Extension:* Spohn monotonicity, stationary/NESS references, flows $b^{\mathrm{rev}}, b^{\mathrm{irr}}, b^{\mathrm{ext}}$.
5. ***Part V*: Spacetime, Light Cones & Local Field Theory.** *Goal:* Local field equations under front/locality. *Import:* front, composition, no-signalling. *Extension:* local GKLS generators, Lieb–Robinson-type bounds, effective light cones.
6. ***Part VI*: Gravity & Geometry from Budget Flows.** *Goal:* Geometrization of budget flows. *Import:* budget quadric/proper time. *Extension:* effective metrics from calibrations (κ_t, κ_x) and internal stresses; coupling to curvature.
7. ***Part VII*: Constants, Scales & Renormalization.** *Goal:* Scale running of the calibration theorems. *Import:* $c = \kappa_t/\kappa_x$, κ_τ. *Extension:* flow equations for $\kappa_t, \kappa_x, \kappa_\tau$; stability of c.
8. ***Part VIII*: Classical Limit, Thermodynamics & Aging.** *Goal:* Macroscopic behavior from $A[\gamma]$ (aging) and DPI. *Import:* proper time/aging, Spohn. *Extension:* entropy production, Euler–Lagrange forms for irreversible flows, effective transport equations.
9. ***Part IX*: Cosmic Dynamics, Time Dilation & Inflation (TDI).** *Goal:* Cosmic ordering & calibration flow. *Import:* budget, proper time/front. *Extension:* budget equations on large-scale slices; time-dilation inflation as calibration dynamics.
10. ***Part X*: Predictions, Falsifiability & Bridge FBA → QM ↔ GR.** *Goal:* Testable differences and bridges FBA ↔ QM/GR. *Import:* all foundational building blocks. *Extension:* protocols, limiting-case tests, overdetermined consistency relations (pass/fail).

All parts of the FBA series are available in both English and German at

https://www.frame-budget-approach.eu