The Frame–Budget Approach (FBA)

How time, dynamics, and geometry emerge from budget flows

An operational bridge between quantum mechanics and general relativity

Part IX: Cosmic Dynamics, Time Dilation & Inflation (TDI)

Dipl. Wirt.-Inf. Jens Tetzner

January 21, 2026

Contents

Part IX

Cosmic Dynamics, Time Dilation & Inflation (TDI)

IX.1 Introduction & Target Picture

IX.1.1 Motivation

This treatise develops an FBA-based description of *cosmic dynamics*, in which the observed accelerated expansion is understood as an effect of an inflation through time dilation, here called *Time–Dilation Inflation (TDI).*[1]

The central point is *not* an additional energy term, but a controlled statement about how *clock rates* and *large-scale scale evolution* are related in FBA once an external front sets the comparison scale.

TDI is therefore not an ad–hoc postulate, but follows (under the imported assumptions and in the large-scale, homogeneous/isotropic limit) from budget flows and their irreversible components (FRW coarse–graining): If part of the budget is irreversibly bound internally, then local proper time runs systematically slower relative to the externally calibrated time, and exactly this relativization is reflected in the cosmological observables.[2] [3]

IX.1.2 Logic path

We build the argument such that each new quantity is introduced only once it is clear *which measurement problem* it solves and *why* it becomes necessary from the previously fixed building blocks (in this protocol):

1. **Sequence & budget.** The ordered sequence of global frames/minimal events and the budget calculus (internal/external/irreversible) are the only basis. This fixes what counts as change at all and how “resource consumption” is accounted for operationally.
2. **Calibration.** An external front fixes the time scale and the maximal propagation rate c. Only then do statements such as “slower” or “faster” between systems become operationally comparable, because all measurement protocols share the same external tick and the same front bound.
3. **FRW coarse–graining.** The homogeneous/isotropic mean of the budget flows yields the background description $(a(t), H(t))$. This step cleanly separates background and (here only sketched) perturbations and makes clear which quantities are meaningfully observable on large scales at all.
4. **TDI factor.** From irreversible *internal* budget components a dilation factor arises

$$\chi(t) := \frac{d\tau_{\text{geo}}}{dt} \in (0, 1],$$

---

[1] An overview of all parts of the FBA treatise including download links can be found in Section IX.12 of this document.

[2] See FBA Part I: FBA – Foundations, Sections I.1–I.3 “Sequence, Budget & Calibration”.

[3] See FBA Part I: FBA – Foundations, Sections I.4–I.5 “Proper Time & GKLS/DPI”.

which couples to $a(t)$. This is the central lever: χ is not a new "substance", but the bookkeeping consequence of internal irreversibility, once t is calibrated by the front and proper time is operationally given as a clock readout.

5. **Observations.** From (a, χ) follow distance and time measures (e. g. $d_L(z)$, $H(z)$, age integrals) as well as consistency relations that link geometry (distance channel) and timing (time channels).
6. **Tests.** Because χ is tied to budget flows, overdetermined relations arise. This enables pass/fail checks against standard cosmology, instead of deviations being merely re-parameterized.

IX.1.3 Scope/delimitations

We work on a homogeneous/isotropic background (FRW symmetry) with an optional curvature parameter $k \in \{-1, 0, +1\}$. Linear perturbations (growth) are only sketched, because the core contribution of this treatise lies in the background: we first isolate the influence of time calibration on the expansion before additional degrees of freedom enter through perturbation physics. Gravity from budget flows in general is treated in later parts.[4] Scales and units follow the calibration and scale treatise.[5] This treatise focuses on the *background dynamics* and the observable consequences of the TDI coupling.

IX.1.4 Contribution relative to standard cosmology

Instead of postulating an additional dark energy density, an effective acceleration term is derived from irreversible *internal* budget use and calibrated time dilation. The gain is structural: (i) *overdetermined* consistency relations arise between $H(z)$, distance measures, and age integrals. (ii) time-based observables (e. g. redshift drift, chronometers) become direct carriers of the signature, because TDI primarily targets clock rates. (iii) the counting of degrees of freedom is reduced: $\chi(t)$ is not free, but coupled to budget flows and thus testable as a hypothesis.

IX.1.5 Reading guide

The Sections are arranged such that first the *concepts and measurement protocols* are fixed and only afterwards the cosmological statements that follow from them:

- **Section IX.2 - Preliminaries & conventions** imports the necessary basics from FBA – Foundations and sets the cosmological notation, so that later derivations do not hinge on convention questions and no new time concepts "slip in" unnoticed.[6]
- **Section IX.3 - FRW effective dynamics from budget flows** constructs the FRW effective description from budget flows. Only afterwards is it clear which quantities in FBA take the roles of $a(t)$ and $H(t)$ and how distance kinematics is to be set up at all.
- **Section IX.4 - TDI factor: deduction & coupling** introduces the TDI factor χ and couples it to the background quantities. This specifies *where* TDI acts: not as a

---

[4] See FBA Part VI: Gravity & Geometry from Budget Flows, Sec. VI.2–VI.4 "Budget–Geometry".

[5] See FBA Part VII: Constants, Scales & Renormalization, Sec. VII.1–VII.2 "Calibration & Thermal Scales".

[6] See FBA Part I: FBA – Foundations, Sections I.1–I.5 "Sequence, Budget, Calibration, Proper Time".

new “substance”, but as a timing factor of real clocks relative to the front-calibrated time.

- **Section IX.5 - Observable quantities: distances, times, drifts** derives observable quantities (distances, age integrals, redshift drift) and makes explicit which channels carry χ and which do not – thereby concrete, data-driven reconstruction equations arise.
- **Section IX.6 - Budget balances & effective equation of state (EoS)** bundles the budgetary balances and formulates an effective EoS parameter, so that comparison to the standard notation is possible without shifting meanings or assuming field equations.
- **Section IX.7 - Predictions & falsifiability (pass/fail)** collects the overdetermined predictions as null tests and pass/fail criteria, i.e. exactly the place where TDI is not “made to fit” but is checked.
- **Section IX.8 - Delimitation & comparison to standard cosmology** delimits systematically against ΛCDM/wCDM and shows how time channels break typical geometry degeneracies, while curvature is handled cleanly in the distance channel.
- **Section IX.9 - Summary & checklist** condenses everything into an operational pipeline – which data you need, what you reconstruct, and which residuals decide.
- **Section IX.10 - The state before time - intuition outside the previously spanned formal framework** supplements the formal scaffold with an intuition about the “state before the first tick”, without introducing new primitives – as a reading of why the operational order sequence → calibration → measure is central.
- **Section IX.11 - Philosophical excursus: where the “why?” began** closes with a philosophical excursus that specifies from when on a “why?” carries at all – and how one formulates pre-F_1 questions as possibility questions without overstretching physics.

IX.2 Preliminaries & Conventions (Import from Part I: FBA – Foundations)

Why an import? In Part IX we formulate cosmic background dynamics such that it follows from the same primitives as time, proper time, and irreversibility in FBA. So that TDI is not later misunderstood as an additional model assumption, sequence, budget balance, calibration (front), and proper time are already fixed here. The following Sections therefore add nothing fundamental, but only develop (i) an FRW coarse–graining of the budget flows and (ii) the resulting relativization of local proper times with respect to the calibrated front time.

Imported building blocks (unchanged)

We adopt the following building blocks from FBA – Foundations (Secs. I.2 through I.6) without redefinition:

- **Sequence of global states & minimal events:** Global states, frame sequence and minimal event (ME) as well as co-actuality and refinement invariance (cf. FBA – Foundations, Sec. I.2 including the boxes there).
- **Difference function & operational minimal difference:** Difference function and operational minimal difference as an operational comparison basis (cf. FBA – Foundations, box in Sec. I.2).
- **Budget calculus (internal/external/irreversible) & balance:** One-step budget and decomposition, balance equations, as well as refinement invariance of the balance (cf. FBA – Foundations, Sec. I.3, incl. formula box and lemma).
- **External calibration & front:** Calibration, front costs, front bound, and signal front as an operational fixing of the time scale (cf. FBA – Foundations, Sec. I.3, definition, lemma, and corollary).
- **Proper time & aging, Minkowski limit:** Proper time, aging as an irreversible component, Minkowski limit including the quadric, as well as time dilation (cf. FBA – Foundations, Sec. I.4, definitions, formula box, and lemma).
- **Admissible dynamics (CPTP/GKLS), DPI/Spohn:** Admissible channels (CPTP), Kraus/Stinespring, measurement as CPTP, GKLS generators, Spohn monotonicity, semigroup budget, as well as DPI arrow and no-recovery (cf. FBA – Foundations, Sec. I.5, definitions, formulas, lemmas, and corollary).
- **Composition, locality & no-signalling:** Symmetric-monoidal structure, budget additivity, no-wire inflation and local operations as well as causal cones and local GKLS (cf. FBA – Foundations, Sec. I.6, definition, formula, lemma, and corollary).

What is the purpose of this import in the reading path? The box is a circularity lock: In Sections IX.3 to IX.6 we will make statements about expansion, age, and time scales. So that these statements do not smuggle in new time concepts or additional dynamical postulates implicitly, the carrier concepts (budget decomposition, front calibration, proper time, irreversibility) must already be fixed. Everything that is new in Part IX is therefore explicitly marked as a coarse–graining step and as a consequence of the imported irreversibility.

With the basic concepts thus fixed, we now fix the notation with which calculations are carried out and comparisons to data are made in Part IX.

Notation & cosmological conventions

- **Times.** Calibrated time t (front-based) and local proper time τ. In the background we use the *TDI factor*

  $$\chi(t) := \frac{d\tau_{\text{geo}}}{dt} \in (0, 1]\,,$$

  where τ_{geo} denotes the geometrically reconstructed proper time in the FRW limit. *Symbol protection:* χ remains reserved exclusively for this TDI factor in Part IX.
- **FRW background.** Scale factor $a(t)$, Hubble parameter $H(t) := \dot{a}/a$ (dot means d/dt), present value $H_0 := H(t_0)$. We use the curvature sign $k \in \{-1, 0, +1\}$ *together with* a (present) curvature radius $R_k \in (0, \infty]$, so that spatial curvature is carried by the combination k/R_k^2 (for $k = 0$ we set $R_k := \infty$). Where not otherwise noted, we set $a(t_0) = 1$ for the present epoch t_0. Comoving coordinate r, conformal time $d\eta := dt/a(t)$ only where explicitly used.
- **Redshift.**

  $$1 + z := \frac{a(t_0)}{a(t_{\text{em}})} = \frac{1}{a(t_{\text{em}})}\,.$$

  In Part IX we treat z as purely geometric FRW redshift. TDI acts here primarily on time measures (e. g. age integrals, drift rates); the corresponding χ factors are each explicitly stated there.
- **Distances.** We keep c explicit so that all distances carry the dimension of a length. As a reference quantity we use the *radial comoving distance*

  $$D_{\text{C}}(z) := c \int_0^z \frac{dz'}{H(z')}\,.$$

  For $k \neq 0$ it is convenient to additionally use the transverse comoving distance $D_M(z)$. To ensure dimensional consistency (arguments of sin / sinh are dimensionless), we define

  $$D_M(z) := \begin{cases} R_k \sin\!\big(D_{\text{C}}(z)/R_k\big), & k = +1, \\ D_{\text{C}}(z), & k = 0, \\ R_k \sinh\!\big(D_{\text{C}}(z)/R_k\big), & k = -1. \end{cases}$$

  The corresponding relations to $d_A(z)$ and $d_L(z)$ are specified in Section IX.5. Important for the reading path: these geometric distance relations remain formally unchanged in the TDI framework; TDI enters only in time readouts via χ.
- **Budget balances (cosmic).** Density-like quantities arise from internal/external/irreversible flows in FRW coarse–graining; an effective EoS parameter w_{eff} serves in Section IX.6 as a comparison language to the standard notation.
- **Units.** c and k_B remain explicit; $\beta = (k_B T)^{-1}$. We distinguish

  $$t_H := H_0^{-1} \quad \text{(Hubble time, e. g. in Gyr)}\,, \qquad D_H := \frac{c}{H_0} = c\,t_H \quad \text{(Hubble distance, e. g. in Gpc)}\,.$$

IX.3 FRW Effective Dynamics from Budget Flows

The aim of this Section is the (under the imported FBA building blocks and in the large-scale FRW limit) *deductive* construction of a homogeneous/isotropic (FRW) effective description directly from the FBA budget calculus. To this end, we (i) formalize the large-scale coarse–graining, (ii) define the scale factor $a(t)$ purely kinematically from front-calibrated length measurements, and (iii) derive a cosmic background budget balance.

What matters is also what is *not* done: we assume no gravitational field equation and postulate no Einstein equations. The role of this Section in the reading path is therefore clear: it establishes the minimal FRW stage on which the TDI factor $\chi(t) = d\tau_{\mathrm{geo}}/dt$ acts in Section IX.4.

IX.3.1 Coarse–graining & FRW symmetry

Cosmology here is not the introduction of new matter fields, but a change of descriptive level: From local budget quantities (internal/external/irreversible), a background dynamics is extracted by averaging. Thus homogeneity/isotropy is not additional physics, but the statement that on sufficiently large scales only the *scalar* evolution remains relevant.

Definition IX.3.1.1: FRW–Coarse-Graining (homogeneous/isotropic)

Let $\mathcal{C}_L$ be a spatial averaging operator over comoving volumes V_L of length scale L with (i) *translational and rotational invariance*, (ii) *ergodicity* on scales $L \gg \ell_{\mathrm{corr}}$, (iii) *commutativity* with temporal refinement in the large-scale limit. For any local budget density $b(x,t)$ (internal/reversible, internal/irreversible, external) we define background fields

$$\bar{b}(t) := \mathcal{C}_L[b(\cdot, t)].$$

FRW symmetry means: $\bar{b}(t)$ is position-independent, and spatial slices are isotropic (up to the curvature class $k \in \{-1, 0, +1\}$).

With Definition IX.3.1.1 it is fixed what we even mean by “background”. The next step is then not a dynamical assumption, but a uniqueness statement: If all large-scale length measurements are isotropic, then their time evolution can proceed only through a single scalar.

Lemma IX.3.1.1: Uniqueness of the scale factor (up to normalization)

Under Definition IX.3.1.1 there exists a scalar function $a(t) > 0$ (scale factor) such that all large-scale, front-calibrated lengths can be factorized as

$$\ell_{\mathrm{phys}}(t) = a(t)\, \ell_{\mathrm{com}}$$

where ℓ_{com} are comoving lengths. The scale factor $a(t)$ is unique up to a global normalization factor.

Proof Sketch IX.3.1.1: Uniqueness of the scale factor (up to normalization)

Front-calibrated lengths are determined operationally via signal-front travel times (radar/front measurement): $\ell_{\text{phys}} \propto c\,\Delta t$ (up to a fixed calibration constant/radar factor). In the large-scale FRW limit all directions are equivalent; therefore the map from comoving distances to physical distances must not contain direction-dependent stretches.

Homogeneity moreover forbids position-dependent scalings. Thus the only remaining possibility is a time-dependent, global dilation $\ell_{\text{phys}}(t) = a(t)\ell_{\text{com}}$.

That two such factors can differ only by a constant normalization follows because a rescaling $\ell_{\text{com}} \mapsto \lambda\,\ell_{\text{com}}$ can be absorbed by the choice of the comoving length unit.

For later derivations it is useful to parametrize the background not only in time, but also in a discrete frame sequence. This is purely an index convention and is meant only to prevent an unnoticed introduction, at some point, of a "before" that does not exist in the model.

Index choice & initial boundary

We choose the global frame index such that $n = 0$ marks the *earliest resolvable frame* (initial boundary).

This is a calibration convention; it makes no statement about a "state before time".

IX.3.2 Kinematics: front-calibrated FRW relations

Once $a(t)$ is fixed as a purely kinematic carrier of scale, distance and time relations follow from the signal-front principle: Light or front signals are the reference processes by which we operationally relate lengths and travel times *at all.* In the FRW background this structure can be expressed in the usual FRW null geometry, without thereby already claiming a field equation: Null curves merely encode the statement that signal fronts propagate with maximal speed c.[7]

---

[7] See FBA Part I: FBA – Foundations, Section I.3 "External Calibration & Signal Front".

Formula Box IX.3.2.1: FRW kinematics (front-calibrated)

With $H(t) := \dot{a}/a$ and

$$1 + z := \frac{a(t_0)}{a(t_{\mathrm{em}})}$$

the comoving radial distance satisfies

$$D_{\mathrm{C}}(z) = \int_{t_{\mathrm{em}}}^{t_0} \frac{c\,dt}{a(t)} = c \int_0^z \frac{dz'}{H(z')},$$

where the second equality follows from $1 + z = a(t_0)/a(t)$ and $\dot{z} = -(1+z)H$. For $k \neq 0$ we use the (present) curvature radius R_k fixed in Section IX.2 (with $R_k = \infty$ for $k = 0$) and define the transverse comoving distance

$$D_M(z) := \begin{cases} R_k \sin\big(D_{\mathrm{C}}(z)/R_k\big), & k = +1, \\ D_{\mathrm{C}}(z), & k = 0, \\ R_k \sinh\big(D_{\mathrm{C}}(z)/R_k\big), & k = -1. \end{cases}$$

The angular-diameter and luminosity distances then read

$$d_A(z) = \frac{D_M(z)}{1+z}, \qquad d_L(z) = (1+z)^2 d_A(z) = (1+z)\,D_M(z).$$

The formulas in Formula Box IX.3.2.1 are deliberately formulated as a kinematic basis: They state how distance and redshift measurements respond to $a(t)$ and $H(t)$. Precisely for this reason it is later possible to introduce TDI as a *temporal* modification without bending the geometric kinematics.

IX.3.3 Cosmic background budget balance

We now establish the connection to the FBA budget calculus. In the FRW limit we expect a balance that cleanly separates between (i) reversible volume work through expansion, (ii) external inflow or outflow, and (iii) irreversible internal budget binding/use. The decisive point here is the direction of irreversibility: it is not a new postulate, but follows from the DPI/Spohn structure fixed in the FBA dynamics.[8]

---

[8] See FBA Part I: FBA – Foundations, Section I.5 "DPI/Spohn Monotonicity".

Definition IX.3.3.1: Background budget quantities

We describe the large-scale (FRW-averaged) budget dynamics by four time-dependent background quantities:

- $\rho_B(t)$: *internal* budget density per *physical* volume,
- $p_B(t)$: associated *reversible* work component (pressure) under expansion,
- $\sigma_B(t) \geq 0$: density of the *irreversible internal budget binding/consumption rate* (sink in the available internal budget),
- $J_{\text{ext}}(t)$: *external* injection rate per physical volume (sources/couplings).

Formula Box IX.3.3.1: FRW budget balance (continuous limit)

In the FRW background,

$$\dot{\rho}_B(t) + 3H(t)\big(\rho_B(t) + p_B(t)\big) = J_{\text{ext}}(t) - \sigma_B(t), \qquad \sigma_B(t) \geq 0.$$

Interpretation: $3H(\rho_B + p_B)$ is the reversible dilution and/or volume work, σ_B the irreversible internal sink component, J_{ext} external supply.

Proof Sketch IX.3.3.1: FRW budget balance (continuous limit)

Consider a fixed comoving volume V_{com} with physical volume $V_{\text{phys}}(t) = a(t)^3 V_{\text{com}}$. The internal budget in it is $B_{\text{int}}(t) = \rho_B(t)\, V_{\text{phys}}(t)$.

The one-step balance becomes a power balance in the continuous limit.[a]

$$\dot{B}_{\text{int}}(t) = -p_B(t)\, \dot{V}_{\text{phys}}(t) + \big(J_{\text{ext}}(t) - \sigma_B(t)\big)\, V_{\text{phys}}(t),$$

where $\sigma_B \geq 0$ encodes the irreversible internal direction (DPI/Spohn). With $\dot{V}_{\text{phys}}/V_{\text{phys}} = 3H$ and division by V_{phys}, Formula Box IX.3.3.1 follows immediately.

---

[a] See FBA Part I: FBA – Foundations, Section I.3 “Budget Calculus & Balance”.

Two immediate consequences are important because they later serve as consistency checks: First, the balance reproduces the standard adiabatic case when neither external sources nor irreversibility act. Second, the effect of σ_B has an unambiguous direction and cannot be defined away by reparameterization.

Corollary IX.3.3.1: Adiabatic limit & scaling rules

For $J_{\text{ext}} \equiv 0$ and $\sigma_B \equiv 0$ and an equation of state $p_B = w\,\rho_B$, it follows that

$$\rho_B \propto a^{-3(1+w)}.$$

In particular: "dust" $w = 0 \Rightarrow \rho_B \propto a^{-3}$, "radiation" $w = 1/3 \Rightarrow \rho_B \propto a^{-4}$.

Corollary IX.3.3.2: Irreversibility accelerates dilution

For $J_{\text{ext}} \equiv 0$ and $\sigma_B \geq 0$,

$$\dot{\rho}_B + 3H(\rho_B + p_B) \leq 0,$$

i.e.: irreversible internal processes strengthen the decrease of ρ_B relative to the adiabatic case.

Comparison to the standard FRW form

Equation Formula Box IX.3.3.1 is the kinematic-thermodynamic *budget* version of the FRW continuity equation. It requires no gravitational field equation (no Friedmann or Raychaudhuri equation) and therefore also provides no closed dynamics for $a(t)$.

In this treatise, $a(t)$ is either inserted empirically via $H(z)$ and distance data or is later closed from budget geometry.[a]

The next step in Section IX.4 is then deliberately orthogonal to this: we couple proper-time ticking via χ to observational quantities and obtain additional, testable relations in time observables.

---

[a] See FBA Part VI: Gravity & Geometry from Budget Flows, Sec. VI.2–VI.4 "Budget–Geometry".

IX.3.4 Minimal examples & checks

To conclude this Section we fix two simple models that later serve as reference cases: first the adiabatic limit as a "null test", and second a linear source/loss scheme as a minimal model for net inflow or outflow.

Adiabatic reference case

$J_{\text{ext}} = 0$, $\sigma_B = 0$, w constant $\Rightarrow$ Corollary IX.3.3.1. All standard scaling rules are reproduced; TDI effects enter observable time and distance relations only via χ in Sections IX.4 and IX.5.

Source/loss model

Let $J_{\text{ext}} = \Gamma\rho_B$, $\sigma_B = \lambda\rho_B$ with $\Gamma, \lambda \geq 0$ and w constant. Then

$$\frac{\dot{\rho}_B}{\rho_B} = -3(1+w)H + \Gamma - \lambda.$$

Sources (Γ) can compensate irreversible consumption (λ); the net dilution remains scaled by H.

IX.3.5 Classification & outlook

We have formalized FRW coarse–graining, fixed the front-calibrated kinematic distance relations, and derived the background budget balance. Thus precisely the quantities are established to which the TDI factor $\chi(t)$ couples in Section IX.4 in order to generate observable signatures (distances, age integrals, redshift drift) without new postulates. For notes on the index choice see Subsection IX.3.1.

IX.4 TDI Factor: Deduction & Coupling

This Section fixes the TDI factor $\chi(t) := \frac{d\tau_{\text{geo}}}{dt}$ as a *derived* quantity from the budget calculus and couples it to large-scale (FRW) background quantities. The operational core is a clock comparison: Observable time rates come from proper times of local systems, whereas the cosmological background description is formulated in the front-calibrated time t. So that the translation between both levels does not become a free model parameter, we tie χ directly to the budget decomposition per front-calibrated time step dt. The only conventional step is a *choice of units* for internal budget rates; we make this choice explicit via the time calibration $\kappa_\mathcal{T}$.

Core idea. Per front-calibrated time unit dt, along a comoving worldline only a *part* of the internal budget is available *reversibly* for geometric proper time. The remaining share is bound or consumed irreversibly (aging/entropy production). This split fixes χ without an extra term and generates testable scale factors in time-based observables, which appear as pass/fail relations in Sections IX.5 and IX.7.[9]

IX.4.1 Definition & budget split

We formulate the ticking as a budget identity per front-calibrated step, without additional dynamical assumptions. Essential is the time calibration $\kappa_\mathcal{T}$ (already used in the series), which translates internal budget increments into time increments. An optional normalization then fixes only the *unit* for internal rates, not the physics.

Definition IX.4.1.1: TDI factor χ (budget definition)

Along a comoving, large-scale averaged worldline, let the internal budget rate per front-calibrated time dt decompose into a reversible and an irreversible part:

$$\dot{b}^{\text{tot}}_{\text{int}}(t) = \dot{b}^{\text{rev}}_{\text{int}}(t) + \dot{b}_{\text{irr,int}}(t), \qquad \dot{b}_{\text{irr,int}}(t) \geq 0.$$

With the time calibration $\kappa_\mathcal{T}$ (budget→time) we define geometric proper time and aging by

$$d\tau_{\text{geo}} := \frac{1}{\kappa_\mathcal{T}}\, db^{\text{rev}}_{\text{int}}, \qquad dA := \frac{1}{\kappa_\mathcal{T}}\, db_{\text{irr,int}}, \qquad d\tau_{\text{tot}} = d\tau_{\text{geo}} + dA.$$

Thus

$$\chi(t) := \frac{d\tau_{\text{geo}}}{dt} = \frac{1}{\kappa_\mathcal{T}}\, \dot{b}^{\text{rev}}_{\text{int}}(t) \in (0,\infty), \qquad \vartheta(t) := \frac{dA}{dt} = \frac{1}{\kappa_\mathcal{T}}\, \dot{b}_{\text{irr,int}}(t) \geq 0,$$

and the *budget identity* holds:

$$\chi(t) + \vartheta(t) = \frac{1}{\kappa_\mathcal{T}}\, \dot{b}^{\text{tot}}_{\text{int}}(t).$$

Optional normalization (choice of units): If one measures internal rates in units of $\kappa_\mathcal{T}$, i. e. one sets $\dot{b}^{\text{tot}}_{\text{int}}(t) \equiv \kappa_\mathcal{T}$, then the identity reduces to

$$\chi(t) + \vartheta(t) = 1, \qquad \text{and hence} \qquad \chi(t) \in (0,1].$$

---

[9] See FBA Part VIII: Classical Limit, Thermodynamics & Aging, Sec. VIII.6–VIII.8 “Entropy Production & Aging in FBA”.

Calibration status of κ_τ and the optional normalization

The identity in Definition IX.4.1.1 is a bookkeeping and calibration statement: κ_τ translates internal budget increments into time increments along a worldline. The optional setting $\dot{b}^{\mathrm{tot}}_{\mathrm{int}} \equiv \kappa_\tau$ is *only* a choice of unit for internal rates (normalization), not an additional dynamical assumption. In Part IX, only the resulting overdetermined consistency relations between distance, chronometer, drift, and light-curve channels are testable, not the normalization itself.

The definition in Definition IX.4.1.1 does not make χ a new substance, but a bookkeeping quantity: χ is the (calibrated) share of the internal rate that may be counted as geometric proper time per dt.

Formula Box IX.4.1.1: Aging coupling & EP calibration

Write the local total proper time as the sum of a geometric part and irreversible aging,

$$d\tau_{\mathrm{tot}} = d\tau_{\mathrm{geo}} + dA.$$

Then by definition

$$\frac{dA}{dt} = \vartheta(t), \qquad A(t) = \int^t \vartheta(s)\, ds.$$

Under isothermal calibration with $\beta = (k_B T)^{-1}$ and internal entropy production Σ_{int} (unselected), we use the series calibration

$$\frac{dA}{dt} = \frac{\beta^{-1}(t)}{\kappa_\tau}\, \dot{\Sigma}_{\mathrm{int}}(t),$$

so that ϑ is directly operationally accessible via $\dot{\Sigma}_{\mathrm{int}}$. In FRW coarse–graining, ϑ couples to the large-scale irreversible sink density σ_B from Formula Box IX.3.3.1.

Cross references

- **Time calibration κ_τ / proper-time split:** Imported from the series foundations (budget→time; $\tau_{\text{tot}} = \tau_{\text{geo}} + A$).[a] [b]
- **Thermal calibration β:** Scale/unit calibration (incl. thermal scales) in the series reference.[c]
- **Entropy production and aging A:** DPI/Spohn, entropy production, and aging in FBA.[d]
- **Large-scale irreversibility σ_B:** Formula Box IX.3.3.1.

---

[a] See FBA Part I: FBA – Foundations, Section I.4 "Proper Time & Minkowski Limit".

[b] See FBA Part VIII: Classical Limit, Thermodynamics & Aging, Sec. VIII.6–VIII.8 "Entropy Production & Aging".

[c] See FBA Part VII: Constants, Scales & Renormalization, Sec. VII.1–VII.2 "Calibration & Thermal Scales".

[d] See FBA Part VIII: Classical Limit, Thermodynamics & Aging, Sec. VIII.6–VIII.8 "DPI/Spohn, Entropy Production & Aging".

IX.4.2 Coupling to FRW quantities: what changes where?

The kinematic distance formulas from Formula Box IX.3.2.1 remain formally unchanged because they are formulated in the front-calibrated time t. The entry point of TDI is where data analysis uses proper time as a proxy for t: Time-based observables measure τ_{geo} and therefore carry explicit χ factors.

We work out the coupling in three steps. We begin with chronometers because they reconstruct $H(z)$ directly from a measured time derivative. Precisely there, the distinction between t and τ_{geo} is not cosmetic, but structurally determining.

Lemma IX.4.2.1: Chronometer scaling

The standard identity $H(z) = -\frac{1}{1+z}\frac{dz}{dt}$ corresponds, for chronometer evaluation with τ_{geo}, to:

$$H(z) = \chi(z)\, H_{\text{CC}}(z), \qquad H_{\text{CC}}(z) := -\frac{1}{1+z}\,\frac{dz}{d\tau_{\text{geo}}}\,.$$

The statement is, at its core, a chain rule: Chronometers deliver a derivative with respect to proper time, while H is by definition a derivative with respect to t.

Proof Sketch IX.4.2.1: Chronometer scaling

From Definition IX.4.1.1 it follows that $d\tau_{\text{geo}} = \chi\, dt$, hence $dt = d\tau_{\text{geo}}/\chi$. Thus

$$H = -\frac{1}{1+z}\frac{dz}{dt} = -\frac{1}{1+z}\frac{dz}{d\tau_{\text{geo}}/\chi} = \chi\, H_{\text{CC}}.$$

Second, we consider the redshift drift. It is particularly suitable because it measures a temporal change of a directly observable quantity and thereby makes unavoidable the question in which time this change is parameterized.

Formula Box IX.4.2.1: Redshift drift with χ (Sandage–Loeb)

The theoretical drift per observer time t_0 is $\dot{z} = \frac{dz}{dt_0} = (1+z)H_0 - H(z)$. Experimentally, however, the drift is recorded per observer proper time:

$$\frac{dz}{d\tau_{\text{geo},0}} = \chi_0^{-1}\left[(1+z)H_0 - H(z)\right], \qquad \chi_0 := \chi(t_0).$$

Finally, we use a third, independent class of time measurements: Light-curve widths (e. g. SN Ia) are not derivatives as for chronometers, but directly compare durations between emission and observation. Precisely for that reason, χ appears here as the ratio $\chi_{\text{obs}}/\chi_{\text{em}}$.

Formula Box IX.4.2.2: Light-curve dilation (e. g. SN Ia)

Let $\Delta\tau_{\text{geo,em}}$ be the proper duration at the emitter and $\Delta\tau_{\text{geo,obs}}$ the measured observer proper time. From the standard relation $\Delta t_{\text{obs}} = (1+z)\Delta t_{\text{em}}$ for the front-calibrated time and $d\tau_{\text{geo}} = \chi\, dt$ it follows that

$$\Delta\tau_{\text{geo,obs}} = (1+z)\,\frac{\chi_{\text{obs}}}{\chi_{\text{em}}}\,\Delta\tau_{\text{geo,em}}.$$

Without TDI ($\chi \equiv 1$) this reduces to $\Delta\tau_{\text{obs}} = (1+z)\Delta\tau_{\text{em}}$.

IX.4.3 Effective acceleration without a postulate

Why can $\chi < 1$ look like an additional acceleration even though Formula Box IX.3.2.1 remains kinematically unchanged? Because part of the standard inference relies precisely on the identification "measured clock time $\approx t$". If this identification is systematically violated, joint fits of distance and time observables must be adjusted consistently.

Heuristic: how χ generates "acceleration"

Chronometers, redshift drift, and light-curve widths interpret proper time as cosmic time. If $\chi(z) < 1$, then:

- Chronometers deliver $H_{\text{CC}}(z)$, while geometry requires $H(z)$. By Lemma IX.4.2.1 it follows immediately that $H(z) = \chi(z)\, H_{\text{CC}}(z)$.
- Distance integrals remain tied to $H(z)$ (cf. Formula Box IX.3.2.1), while drift and stretch rates carry explicit χ factors (cf. Formula Boxes IX.4.2.1 and IX.4.2.2).

Joint evaluation therefore leads to overdetermined consistency relations that reproduce standard FRW for $\chi \equiv 1$ and otherwise display a TDI signature.

IX.4.4 Minimal dynamics for χ from the balance

An "equation for χ" is not an additional hypothesis but a translation: $\vartheta = \frac{dA}{dt}$ is the irreversible share per dt along a worldline, whereas σ_B in Formula Box IX.3.3.1 appears as an irreversible sink density per physical volume. In the homogeneous background, a once-fixed reference cell therefore suffices to identify both quantities consistently. What matters is that the conversion is fixed by the reference cell and calibration ($\kappa_\mathcal{T}$) and introduces no new degree of freedom.

Formula Box IX.4.4.1: Background coupling $\chi \leftrightarrow \sigma_B$

Consider a reference cell with fixed comoving volume V_{com} and physical volume $V_{\mathrm{phys}}(t) = a(t)^3 V_{\mathrm{com}}$. Then the irreversible share of the internal rate in this cell is

$$\dot{b}_{\mathrm{irr,int}}(t) = \sigma_B(t)\, V_{\mathrm{phys}}(t) = a(t)^3\, V_{\mathrm{com}}\, \sigma_B(t).$$

With $\vartheta = \frac{1}{\kappa_\mathcal{T}} \dot{b}_{\mathrm{irr,int}}$ from Definition IX.4.1.1 it follows the dimensionally consistent coupling

$$\vartheta(t) = \alpha\, \sigma_B^{\mathrm{com}}(t), \qquad \sigma_B^{\mathrm{com}}(t) := a(t)^3\, \sigma_B(t), \qquad \alpha := \frac{V_{\mathrm{com}}}{\kappa_\mathcal{T}} > 0.$$

Under the optional normalization $\dot{b}_{\mathrm{int}}^{\mathrm{tot}} \equiv \kappa_\mathcal{T}$ (so that $\chi + \vartheta = 1$) equivalently

$$\chi(t) = 1 - \alpha\, \sigma_B^{\mathrm{com}}(t).$$

Here α is *not* free physics, but fixed by (i) the choice of reference cell and (ii) the calibration $\kappa_\mathcal{T}$.

The form above shows explicitly where the choice of units sits: If one changes V_{com} or $\kappa_\mathcal{T}$, α rescales accordingly without changing the content of the theory.

Corollary IX.4.4.1: Bounds & monotonicity statements

From $\sigma_B \geq 0$ it follows that $\vartheta \geq 0$ and hence

$$\chi(t) \leq \chi(t) + \vartheta(t) = \frac{1}{\kappa_\mathcal{T}}\, \dot{b}_{\mathrm{int}}^{\mathrm{tot}}(t).$$

Under the optional normalization $\dot{b}_{\mathrm{int}}^{\mathrm{tot}} \equiv \kappa_\mathcal{T}$ in particular $0 < \chi \leq 1$.

If large-scale irreversibility decreases (e. g. $\sigma_B^{\mathrm{com}} \searrow 0$ at late t), this drives $\chi \nearrow 1$.

Strong irreversibility pushes $\chi \searrow$ and strengthens the TDI signature in time observables.

IX.4.5 Classification & outlook

We have fixed χ as a derived quantity: budget identity Definition IX.4.1.1, aging and EP coupling Formula Box IX.4.1.1, and the embedding into the large-scale balance Formula Boxes IX.3.3.1 and IX.4.4.1. In Section IX.5 we apply this coupling systematically to distance and time measures and derive concrete consistency relations and observer formulas. Section IX.6 integrates χ into an effective equation-of-state language, and Section IX.7 collects pass/fail tests from it.

IX.5 Observable quantities: distances, times, drifts

This Section bundles the observable consequences of the coupling $\chi(t) = \frac{d\tau_{\mathrm{geo}}}{dt}$ from Section IX.4 with the FRW kinematics from Section IX.3. The guiding idea is a clean division of labor: Distance relations are geometric and remain, in the front-calibrated time t, formally as in Formula Box IX.3.2.1.[1] Time observables, by contrast, parametrize changes and durations in proper time and therefore carry χ factors; cf. Lemma IX.4.2.1 and Formula Boxes IX.4.2.1 and IX.4.2.2.[2–7] From this separation arise consistent reconstruction equations and overdetermined tests.

IX.5.1 Distance-based reconstruction of $H(z)$

Distance data allow a reconstruction of a *geometric* expansion history $H(z)$ without involving χ.[1] Precisely for this reason, distance relations serve as a reference channel against which time-based channels can be benchmarked.

Formula Box IX.5.1.1: $H(z)$ from distance data (flat $k = 0$)

In the flat case $D_M(z) = D_{\mathrm{C}}(z)$ and thus $d_L(z) = (1 + z)\, D_{\mathrm{C}}(z)$. Hence

$$H_{\mathrm{dist}}(z) = \frac{c}{\dfrac{d}{dz}\left(\dfrac{d_L(z)}{1+z}\right)} .$$

For curved spaces it is practical to first define the transverse comoving distance, because it is extracted directly from d_L.

Formula Box IX.5.1.2: $H(z)$ from distance data (general k)

Define as in Formula Box IX.3.2.1

$$D_M(z) := \frac{d_L(z)}{1+z}\,.$$

With the (present) curvature radius R_k from Section IX.2, in the FRW background

$$D_M(z) = \begin{cases} R_k\,\sin\bigl(D_\mathrm{C}(z)/R_k\bigr), & k=+1,\\ D_\mathrm{C}(z), & k=0,\\ R_k\,\sinh\bigl(D_\mathrm{C}(z)/R_k\bigr), & k=-1. \end{cases}$$

Set $u(z) := D_\mathrm{C}(z)/R_k$ (dimensionless) and

$$S_k(u) := \begin{cases} \sin(u), & k=+1,\\ u, & k=0,\\ \sinh(u), & k=-1. \end{cases}$$

Then $D_M(z) = R_k\,S_k(u(z))$ and thus

$$\frac{dD_M}{dz} = S_k'(u)\,\frac{dD_\mathrm{C}}{dz}, \qquad S_k'(u) = \begin{cases} \cos(u), & k=+1,\\ 1, & k=0,\\ \cosh(u), & k=-1, \end{cases}$$

and equivalently (with the nonnegative root as the physical branch)

$$S_k'(u) = \sqrt{1-k\,S_k(u)^2} = \sqrt{1-k\left(\frac{D_M(z)}{R_k}\right)^2}\,.$$

Thus from $H_\mathrm{dist}(z) = c\bigl(dD_\mathrm{C}/dz\bigr)^{-1}$ the reconstruction formula follows

$$H_\mathrm{dist}(z) = c\,\frac{S_k'(u(z))}{\dfrac{dD_M}{dz}} = c\,\frac{\sqrt{1-k\left(\dfrac{D_M(z)}{R_k}\right)^2}}{\dfrac{d}{dz}\left(\dfrac{d_L(z)}{1+z}\right)}\,.$$

For $k=0$ (thus $R_k=\infty$) this reduces to Formula Box IX.5.1.1.

Practicality

- In data analyses it is advisable to determine $H_{\mathrm{dist}}(z)$ via smooth fits or Gaussian-process reconstructions of $d_L(z)$ before differentiating.
- Etherington's duality $d_L = (1+z)^2 d_A$ remains valid in the TDI framework as long as photon number and beam geometry are treated as in Formula Box IX.3.2.1.[8]
- Without absolute distance calibration (e. g. an absolute SN scale), $d_L(z)$ in practice primarily yields the *shape* $H(z)/H_0$; absolute normalizations must then be added consistently from H_0 information or an external distance ladder.[1]
- For $k \neq 0$, R_k is an additional geometry parameter (equivalently curvature); in practical fits it can be estimated as a nuisance parameter jointly with H_{dist} or fixed by external curvature constraints.

IX.5.2 Time-based observables with χ

Time observables are the natural place of TDI: They do not measure t but τ_{geo}. Thus the same physical background yields two evaluation languages, which match only if χ is correctly taken into account.

We record the experimentally accessible forms so that later estimators and consistency tests can be read off directly.

Formula Box IX.5.2.1: Chronometer identity (with χ)

$$H(z) = \chi(z)\, H_{\mathrm{CC}}(z), \qquad H_{\mathrm{CC}}(z) := -\frac{1}{1+z}\,\frac{dz}{d\tau_{\mathrm{geo}}}\,.$$

Formula Box IX.5.2.2: Redshift drift per observer proper time

$$\left.\frac{dz}{d\tau_{\mathrm{geo}}}\right|_0 = \chi_0^{-1}\,\big[(1+z)H_0 - H(z)\big], \qquad \chi_0 := \chi(t_0).$$

Formula Box IX.5.2.3: Light-curve dilation (emitter/observer)

$$\Delta\tau_{\mathrm{geo,obs}} = (1+z)\,\frac{\chi_{\mathrm{obs}}}{\chi_{\mathrm{em}}}\,\Delta\tau_{\mathrm{geo,em}}\,.$$

IX.5.3 Consistency relations & χ estimators

The central advantage of the separation in Subsections IX.5.1 and IX.5.2 is overdetermination: From distance data one obtains $H_{\mathrm{dist}}(z)$ without χ, while chronometers, drift, and light curves carry χ at different points. Thus χ can be estimated directly, and the same quantity is tested via multiple channels.

Corollary IX.5.3.1: χ from distance $\oplus$ chronometers

With Formula Boxes IX.5.1.1, IX.5.1.2 and IX.5.2.1 it follows that

$$\widehat{\chi}_{\mathrm{CC}}(z) = \frac{H_{\mathrm{dist}}(z)}{H_{\mathrm{CC}}(z)}\,.$$

For $\chi \equiv 1$ we have $\widehat{\chi}_{\mathrm{CC}} \equiv 1$; systematic deviations signal TDI or violated assumptions in the channel.[2, 3]

The drift channel additionally isolates the present ticking χ_0. This is useful because χ_0 enters as a common normalization for several observer formulas.

Corollary IX.5.3.2: χ_0 from redshift drift $\oplus$ distance

Insert in Formula Box IX.5.2.2 the geometric history $H(z) = H_{\mathrm{dist}}(z)$. Then for each z one obtains

$$\widehat{\chi}_0(z) = \frac{(1+z)\,H_0 - H_{\mathrm{dist}}(z)}{\left.\dfrac{dz}{d\tau_{\mathrm{geo}}}\right|_{0,\mathrm{obs}}}\,.$$

Consistency requires z-independence: $\widehat{\chi}_0(z) \approx \mathrm{const.}$[4, 5]

Light curves do not provide $H(z)$, but a ratio of tick rates. Precisely for that reason they are an independent cross-check against chronometers and drift.

Corollary IX.5.3.3: χ ratio from light-curve dilation

For source families with known or internally standardized $\Delta\tau_{\mathrm{geo,em}}$, Formula Box IX.5.2.3 yields

$$R_{\mathrm{SN}}(z) := \frac{\Delta\tau_{\mathrm{geo,obs}}}{(1+z)\,\Delta\tau_{\mathrm{geo,em}}} = \frac{\chi_{\mathrm{obs}}}{\chi_{\mathrm{em}}}\,.$$

With $\chi_{\mathrm{obs}} = \chi_0$ and $\chi_{\mathrm{em}} = \chi(z)$ it follows that

$$R_{\mathrm{SN}}(z) = \frac{\chi_0}{\chi(z)} \qquad \Rightarrow \qquad \widehat{\chi}_{\mathrm{SN}}(z) = \frac{\chi_0}{R_{\mathrm{SN}}(z)}\,.$$

[6, 7]

Joint evaluation & overdetermination

The three approaches $\widehat{\chi}_{\mathrm{CC}}(z)$, $\widehat{\chi}_0(z)$, and $\widehat{\chi}_{\mathrm{SN}}(z) = \chi_0/R_{\mathrm{SN}}(z)$ share $H_{\mathrm{dist}}(z)$ from distances (Formula Boxes IX.5.1.1 and IX.5.1.2) and are thus overdetermined.

- **Pass:** all channels are consistent within empirical tolerances δ_* and yield compatible $\chi(z)$ and χ_0.
- **Fail:** at least one channel shows robust, systematic inconsistencies after controlling the respective systematics ⇒ TDI falsified *or* one of the input assumptions (calibration, photon number, curvature assumption, astrophysical model) violated.

IX.5.4 Age integrals & time scales

Age and duration observables are particularly sensitive because they represent integrals over time rates. In the TDI framework it is therefore important to consistently distinguish between the front-calibrated cosmic age t and the geometric proper-time component τ_{geo}.[1]

Formula Box IX.5.4.1: Geometric age & cosmic age

Let z_* denote the earliest resolvable initial boundary of the sequence in the coarse–graining used (cf. Subsection IX.3.1). We then define

$$t(z) = \int_z^{z_*} \frac{dz'}{(1+z')\,H(z')}\,, \qquad \tau_{\mathrm{geo}}(z) = \int_z^{z_*} \chi(z')\,\frac{dz'}{(1+z')\,H(z')}\,.$$

Note: In standard notation $z_* = \infty$ is often used as shorthand. Chronometers measure differences in τ_{geo}; comparison to $t(z)$ generates χ signatures.

IX.5.5 Minimal parametrization & example

For first data comparisons it is often helpful to test $\chi(z)$ with a smooth ansatz that (under the optional normalization used in Part IX) respects the bounds $0 < \chi \leq 1$ and approaches the standard case at $z = 0$.

Simple χ model

Set $\chi(z) = 1 - \varepsilon_\chi\, u(z)$ with $0 \leq \varepsilon_\chi \ll 1$ and smooth $u(0) = 0$, $u'(0) > 0$ (e. g. $u(z) = \frac{z}{1+z}$). Then

$$\widehat{\chi}_{\mathrm{CC}}(z) \approx 1 - \varepsilon_\chi\, u(z), \qquad \widehat{\chi}_0(z) \approx \chi_0 = 1, \qquad R_{\mathrm{SN}}(z) = \frac{\chi_0}{\chi(z)} \approx 1 + \varepsilon_\chi\, u(z).$$

Consistent linear trends across all channels are a pass; opposing or channel-dependent trends a fail.

IX.5.6 Systematics & robustness

Overdetermination is informative only if systematics do not act unnoticed as a χ signal. We mark the most important control points that should be explicitly varied or modeled in a practical analysis.

Control points

- **Chronometers:** metallicity and star-formation history, aging models, and population synthesis.[2, 3]
- **SN dilation:** standardization, stretch and color corrections, K-corrections, selection effects.[6, 7]
- **Redshift drift:** instrumental stability and calibration, peculiars and local accelerations.[4, 5]
- **Distance derivative:** smoothing and regularization before differentiation, model choice for $k \neq 0$.

IX.5.7 Classification & outlook

We have formulated three complementary χ approaches (distance $\oplus$ chronometers, drift $\oplus$ distance, light-curve dilation) as well as age integrals. In Section IX.6 we integrate χ into the background balance Formula Box IX.3.3.1 and into an effective equation-of-state language w_{eff}. Section IX.7 bundles pass/fail tests and consistent evaluation paths for data analyses derived from it.

IX.6 Budget balances & effective equation of state (EoS)

In this Section we combine the large-scale budget flows from Formula Box IX.3.3.1 with the TDI coupling from Definition IX.4.1.1 and Formula Boxes IX.4.1.1 and IX.4.4.1 into a compact effective description. The goal is twofold: (i) an *effective* equation of state w_{eff} as a pure reparameterization of the budget balance (without gravitational field equations) and (ii) kinematic characteristics that precisely express how χ shifts time-based inference quantities relative to geometry.

For the remainder of Part IX we work under the normalization identified in Definition IX.4.1.1 as a choice of units, such that $0 < \chi \leq 1$ and $1-\chi = \vartheta \geq 0$ holds. Without this normalization, the following relations remain valid unchanged if one replaces $1-\chi$ by ϑ everywhere.

IX.6.1 Effective equation of state from the budget balance

The purpose of w_{eff} is not to postulate a new dynamics, but to bring the balance Formula Box IX.3.3.1 into a form that is directly comparable with the usual scaling rules. This makes transparent which parts of the dilution come from reversible expansion and which from irreversible or external contributions.

Definition IX.6.1.1: Effective equation of state w_{eff} (budget reparameterization)

Let $H \neq 0$ and $\rho_B > 0$. Given the background quantities from Definition IX.3.3.1 we define w_{eff} by the identity

$$\dot{\rho}_B + 3H(1 + w_{\text{eff}})\rho_B \equiv 0.$$

Thus Formula Box IX.3.3.1 is equivalent to

$$w_{\text{eff}} = \frac{p_B}{\rho_B} + \frac{\sigma_B - J_{\text{ext}}}{3H\,\rho_B}\,.$$

This definition is chosen such that the adiabatic limit immediately reappears when $\sigma_B = J_{\text{ext}} = 0$. Everything that goes beyond the standard dilution is then contained in a single correction term.

Formula Box IX.6.1.1: Decomposition: $w_{\text{eff}} = w_{\text{rev}} + \Delta w_{\text{irr/ext}}$

With

$$w_{\text{rev}} := \frac{p_B}{\rho_B}, \qquad \Delta w_{\text{irr/ext}} := \frac{\sigma_B - J_{\text{ext}}}{3H\,\rho_B}$$

we have

$$w_{\text{eff}} = w_{\text{rev}} + \Delta w_{\text{irr/ext}}\,.$$

In the expansion regime $H > 0$ (and $\rho_B > 0$) this implies: irreversibility ($\sigma_B \geq 0$) raises w_{eff}, external injection ($J_{\text{ext}} > 0$) lowers it – both purely budgetary, without gravitational interpretation.

Classification via the adiabatic reference case

For $\sigma_B = J_{\text{ext}} = 0$, Definition IX.6.1.1 reduces to the usual scaling rules, cf. Corollary IX.3.3.1.

IX.6.2 Kinematics: acceleration parameter and χ

Time-based data products often reconstruct acceleration indicators from derived quantities such as $H(z)$. To meaningfully compare these indicators with the geometric background description at all, we consistently separate derivatives with respect to t and with respect to τ_{geo}. This leads to two acceleration parameters, both of which are purely definitional.

So that time-based data products (chronometers) and geometric reconstructions do not silently use the same symbols for different derivative notions, we fix the two common acceleration parameters as *purely definitional* quantities.

Definition IX.6.2.1: Acceleration parameters q_{geo} and q_{CC}

We define

- the *geometric* acceleration parameter (derivative with respect to front-calibrated time t)

$$q_{\text{geo}} := -1 - \frac{\dot{H}}{H^2}, \qquad \dot{(\cdot)} := \frac{d}{dt},$$

- the *chronometer* acceleration parameter (derivative with respect to geometric proper time τ_{geo})

$$q_{\text{CC}} := -1 - \frac{1}{H_{\text{CC}}^2}\frac{dH_{\text{CC}}}{d\tau_{\text{geo}}}, \qquad H_{\text{CC}} := \frac{H}{\chi}$$

  with $\chi = \frac{d\tau_{\text{geo}}}{dt}$ from Definition IX.4.1.1 and the chronometer relation from Lemma IX.4.2.1.

Both parameters are purely kinematic definitions and assume no field equations.

The relation between the two is then not interpretation, but follows from the chain rule and is formulated in Formula Box IX.6.2.1 as an explicit χ shift.

Formula Box IX.6.2.1: Difference of the acceleration parameters

Between q_{CC} and q_{geo} one has (for $H \neq 0$ and a smooth background)

$$q_{\text{CC}} = q_{\text{geo}} + \frac{\dot{\chi}}{H\,\chi} = q_{\text{geo}} + \frac{d\ln\chi}{d\ln a}\,,$$

where the second equality uses only the identity $d\ln a = H\,dt$.

Proof Sketch IX.6.2.1: Difference of the acceleration parameters

From Lemma IX.4.2.1 we have $H_{\mathrm{CC}} = H/\chi$, and from Definition IX.4.1.1 we have $d/d\tau_{\mathrm{geo}} = \chi^{-1} d/dt$. Thus

$$\frac{dH_{\mathrm{CC}}}{d\tau_{\mathrm{geo}}} = \frac{1}{\chi}\frac{d}{dt}\left(\frac{H}{\chi}\right) = \frac{1}{\chi}\left(\frac{\dot H}{\chi} - \frac{H\dot\chi}{\chi^2}\right) = \frac{\dot H}{\chi^2} - \frac{H\dot\chi}{\chi^3}.$$

Moreover $H_{\mathrm{CC}}^2 = H^2/\chi^2$, hence

$$\frac{1}{H_{\mathrm{CC}}^2}\frac{dH_{\mathrm{CC}}}{d\tau_{\mathrm{geo}}} = \frac{\chi^2}{H^2}\left(\frac{\dot H}{\chi^2} - \frac{H\dot\chi}{\chi^3}\right) = \frac{\dot H}{H^2} - \frac{\dot\chi}{H\chi}.$$

Inserting into $q_{\mathrm{CC}} = -1 - (dH_{\mathrm{CC}}/d\tau_{\mathrm{geo}})/H_{\mathrm{CC}}^2$ yields the claim.

Interpretation

χ shifts time-based acceleration estimators relative to geometry: If χ grows with a, then $d\ln\chi/d\ln a > 0$ and thus $q_{\mathrm{CC}} > q_{\mathrm{geo}}$. If χ decreases, then $q_{\mathrm{CC}} < q_{\mathrm{geo}}$.

This shift is pure kinematics and is in direct accord with Formula Boxes IX.4.2.1 and IX.5.2.1.

IX.6.3 Closed coupling: ρ_B balance with χ

The balance Formula Box IX.3.3.1 is formulated in $\rho_B, p_B, \sigma_B, J_{\mathrm{ext}}$. Via Formula Box IX.4.4.1 the irreversible contribution can alternatively be expressed in terms of χ. This makes visible how changes in ticking show up in a pure balance language.

Formula Box IX.6.3.1: Budget balance with χ

Under the normalization used in this part (so that $1-\chi = \vartheta$), from Formula Box IX.4.4.1 one has $1 - \chi = \alpha\,\sigma_B^{\mathrm{com}}$ with $\sigma_B^{\mathrm{com}} := a^3\sigma_B$, thus

$$\sigma_B(t) = \frac{1-\chi(t)}{\alpha\,a(t)^3}.$$

Hence Formula Box IX.3.3.1 becomes

$$\dot\rho_B + 3H(\rho_B + p_B) = J_{\mathrm{ext}} - \frac{1-\chi}{\alpha\,a^3}.$$

Accordingly, Definition IX.6.1.1 can be written as

$$w_{\mathrm{eff}} = \frac{p_B}{\rho_B} + \frac{1}{3H\rho_B}\left(\frac{1-\chi}{\alpha\,a^3} - J_{\mathrm{ext}}\right)$$

The point is not that χ replaces an “energy component”, but that the same irreversible contribution contained in σ_B appears in time-based observables as a ticking factor. The two representations are consistently linked via reference cell and calibration.

Corollary IX.6.3.1: Bounds & trends

Let $\alpha > 0$, $H > 0$, and $\rho_B > 0$. From $0 < \chi \leq 1$ it follows that $0 \leq (1-\chi)/(\alpha a^3) \leq 1/(\alpha a^3)$ and hence

$$-\frac{J_{\rm ext}}{3H\rho_B} \leq \Delta w_{\rm irr/ext} \leq \frac{1/(\alpha a^3) - J_{\rm ext}}{3H\rho_B}.$$

If $\sigma_B^{\rm com} \searrow 0$, then $\chi \nearrow 1$ and thus $w_{\rm eff} \to w_{\rm rev} - \frac{J_{\rm ext}}{3H\rho_B}$.

IX.6.4 Comparison quantity: mapping to $w(z)$ fits

For comparison with standard analyses (e. g. wCDM) it is helpful to define a purely algebraic comparison curve $w_{\rm cmp}(z)$. It is explicitly *not* claimed that any gravitational dynamics holds in FBA: $w_{\rm cmp}(z)$ is only a translation tool that embeds a geometry $H_{\rm dist}(z)$ reconstructed from distances into the familiar GR notation.

Comparison parameterization $w_{\rm cmp}(z)$

Given $H_{\rm dist}(z)$ and reference parameters $(\Omega_{m0}, \Omega_{r0}, \Omega_{k0})$ (standard GR fit language), one formally defines

$$\rho_X(z) := \frac{3H_{\rm dist}(z)^2}{8\pi G_N} - \rho_m(z) - \rho_r(z) - \rho_k(z),$$

where G_N denotes Newton's constant in the comparison notation (not an FBA coupling). One then sets

$$\frac{d\ln\rho_X}{d\ln(1+z)} = 3(1 + w_{\rm cmp}(z)).$$

Note: $w_{\rm cmp}(z)$ is a comparison quantity; in FBA, $H(z)$ does not follow from a field equation.

IX.6.5 Minimal models & examples

Finally, a small kinematic check: even a weak, smooth deviation from $\chi \equiv 1$ shifts $q_{\rm CC}$ relative to $q_{\rm geo}$ by a controlled amount. This later becomes a direct handle for tests, because it is formulated independently of a gravitational equation.

Weak TDI: $\chi(a) = 1 - \varepsilon_\chi\, a^\nu$, $\varepsilon_\chi \ll 1$

Then for $a \in (0, 1]$

$$\frac{d\ln\chi}{d\ln a} = \frac{a}{\chi}\frac{d\chi}{da} = -\frac{\varepsilon_\chi\,\nu\,a^\nu}{1 - \varepsilon_\chi\,a^\nu} \approx -\varepsilon_\chi\,\nu\,a^\nu.$$

Thus from Formula Box IX.6.2.1

$$q_{\rm CC} \approx q_{\rm geo} - \varepsilon_\chi\,\nu\,a^\nu.$$

IX.6.6 Classification & outlook

We have established a budgetary effective description: w_{eff} as a pure reparameterization of the balance (Definition IX.6.1.1 and Formula Boxes IX.6.1.1 and IX.6.3.1) and the kinematic shift $q_{\text{CC}} - q_{\text{geo}} = d \ln \chi / d \ln a$ (Formula Box IX.6.2.1). Section IX.7 bundles pass/fail tests of overdetermined combinations of distances, chronometers, redshift drift, and light curves, while Section IX.8 takes over the systematic comparison to ΛCDM/wCDM.

IX.7 Predictions & falsifiability (pass/fail)

We bundle the *model-independent* identities between distance and time observables that follow from Sections IX.3 to IX.6. *Model-independent* here means: without a gravitational field equation, but under the kinematic FRW stage (incl. Etherington duality, photon number/beam geometry) and consistent calibration. The logic is deliberately overdetermined: Distances provide a geometric $H_{\rm dist}(z)$, while time channels measure χ-weighted rates. If TDI is correct, all channels must yield a *consistent* $\chi(z)$ structure. If not, robust inconsistencies arise, provided the respective systematics are controlled.

IX.7.1 Core identities (without dynamical assumptions)

We combine the geometric $H_{\rm dist}(z)$ from distances (Formula Boxes IX.5.1.1 and IX.5.1.2) with the time channels chronometers, redshift drift, and light curves (Formula Boxes IX.5.2.1 to IX.5.2.3). Each of the following relations is either (i) a null test in the standard mode $\chi \equiv 1$ or (ii) a consistency condition in the TDI mode.

Formula Box IX.7.1.1: Drift–distance identity (null test N_1)

From Formula Box IX.5.2.2 and $H(z) = H_{\rm dist}(z)$ it follows for each z:

$$H_{\rm dist}(z) = (1+z)\,H_0 - \chi_0 \left.\frac{dz}{d\tau_{\rm geo,0}}\right|_{z,\rm obs}, \qquad \chi_0 := \chi(t_0).$$

In standard mode one sets $\chi_0 = 1$ and tests the same equation as a null test.

N_1 is particularly sharp because it pits a direct time change ($dz/d\tau_{\rm geo,0}$) against the distance reconstruction of $H(z)$. Thus the test depends sensitively on the common calibration of H_0 and the drift measurement, but not on a dynamical model.

Formula Box IX.7.1.2: Chronometer–distance identity (null test N_2)

With Formula Box IX.5.2.1 and $H(z) = H_{\rm dist}(z)$,

$$H_{\rm dist}(z) = \chi(z)\,H_{\rm CC}(z), \qquad H_{\rm CC}(z) := -\frac{1}{1+z}\,\frac{dz}{d\tau_{\rm geo}}.$$

In standard mode one sets $\chi(z) = 1$ and expects $H_{\rm dist}(z) = H_{\rm CC}(z)$.

N_2 is the most direct channel for $\chi(z)$: Chronometers reconstruct a time derivative, but with respect to $\tau_{\rm geo}$ rather than t. Precisely this translation is χ.

Formula Box IX.7.1.3: SN dilation $\oplus$ chronometers (null test N_3)

Define the directly measurable stretch factor

$$R_{\mathrm{SN}}(z) := \frac{\Delta\tau_{\mathrm{geo,obs}}}{(1+z)\,\Delta\tau_{\mathrm{geo,em}}} = \frac{\chi_{\mathrm{obs}}}{\chi_{\mathrm{em}}} = \frac{\chi_0}{\chi(z)}\,,$$

cf. Formula Box IX.5.2.3. Then from Formula Box IX.5.2.1 and $H(z) = H_{\mathrm{dist}}(z)$ it follows that

$$H_{\mathrm{dist}}(z) = \frac{\chi_0}{R_{\mathrm{SN}}(z)}\,H_{\mathrm{CC}}(z).$$

In standard mode $R_{\mathrm{SN}}(z) = 1$ and thus again $H_{\mathrm{dist}}(z) = H_{\mathrm{CC}}(z)$.

N_3 couples two classes of time measurement: Chronometers provide a rate, light curves a duration relation. Precisely this combination makes the test robust against channel-typical degeneracies, because merely bending $H(z)$ cannot replace the explicit χ ratios in $R_{\mathrm{SN}}(z)$.

IX.7.2 χ estimators & internal consistency

The core identities yield immediate estimators. The decisive point is not that one can fit χ, but that χ must be reconstructed compatibly from *different* channels.

Corollary IX.7.2.1: Estimators $\widehat{\chi}_{\mathrm{CC}}(z)$, $\widehat{\chi}_0(z)$, and $\widehat{\chi}_{\mathrm{SN}}(z)$

From Formula Box IX.7.1.2:

$$\widehat{\chi}_{\mathrm{CC}}(z) = \frac{H_{\mathrm{dist}}(z)}{H_{\mathrm{CC}}(z)}\,.$$

From Formula Box IX.7.1.1:

$$\widehat{\chi}_0(z) = \frac{(1+z)H_0 - H_{\mathrm{dist}}(z)}{\left.\dfrac{dz}{d\tau_{\mathrm{geo},0}}\right|_{z,\mathrm{obs}}}\,.$$

From Formula Box IX.7.1.3:

$$\widehat{\chi}_{\mathrm{SN}}(z) = \frac{\chi_0}{R_{\mathrm{SN}}(z)} \qquad \text{(in practice: } \widehat{\chi}_{\mathrm{SN}}(z) = \widehat{\chi}_0 \,/\, R_{\mathrm{SN}}(z)\text{)}.$$

Consistency requirements:

$$\widehat{\chi}_{\mathrm{CC}}(z) \approx \widehat{\chi}_{\mathrm{SN}}(z) \text{ for all } z, \qquad \widehat{\chi}_0(z) \approx \text{const.}$$

Physical bounds

From Corollary IX.4.4.1 it follows (under the normalization indicated in Definition IX.4.1.1) that $0 < \chi \le 1$. In fits, $\widehat{\chi}_{\mathrm{CC}}(z)$ and $\widehat{\chi}_{\mathrm{SN}}(z)$ are to be constrained to $(0, 1]$ (hard priors).

IX.7.3 Pass/fail criteria (operational)

So that “pass” and “fail” do not become matters of interpretation, we formulate all tests as residuals. In standard mode one sets $\chi \equiv 1$ and checks whether the residuals vanish. In TDI mode, χ_0 and $\chi(z)$ become reconstructible, and one additionally checks bounds and estimator consistency.

Formula Box IX.7.3.1: Residuals & decision

Define residuals

$$\Delta_{\mathrm{DR}}(z) := H_{\mathrm{dist}}(z) - (1+z)H_0 + \chi_0 \left.\frac{dz}{d\tau_{\mathrm{geo},0}}\right|_{z,\mathrm{obs}},$$

$$\Delta_{\mathrm{CC}}(z) := H_{\mathrm{dist}}(z) - \chi(z)\, H_{\mathrm{CC}}(z), \qquad \Delta_{\mathrm{SNCC}}(z) := H_{\mathrm{dist}}(z) - \frac{\chi_0}{R_{\mathrm{SN}}(z)}\, H_{\mathrm{CC}}(z).$$

Normalized residuals:

$$\mathcal{N}_1(z) := \frac{\Delta_{\mathrm{DR}}(z)}{H_0}, \quad \mathcal{N}_2(z) := \frac{\Delta_{\mathrm{CC}}(z)}{H_{\mathrm{dist}}(z)}, \quad \mathcal{N}_3(z) := \frac{\Delta_{\mathrm{SNCC}}(z)}{H_{\mathrm{dist}}(z)}.$$

Pass (empirical): $|\mathcal{N}_i(z)| \leq \delta_{*,i}(z)$ for all z (within error bars and after systematic checks).
Fail: stable, systematic deviation $|\mathcal{N}_i| > \delta_{*,i}$ over z intervals $\Rightarrow$ TDI falsified or assumptions violated (photon number, calibration, curvature).

Overdetermination & degeneracies

ΛCDM/wCDM degeneracies primarily affect $H(z)$ and distance measures; the time channels carry independent information about χ. The simultaneity of tests N_1 to N_3 breaks typical $w(z)$ degeneracies: If H_{dist} is fitted by a $w(z)$ curve, drift, chronometers, and SN must consistently deliver the same χ structure. That is the pass/fail lever.

IX.7.4 Minimal parametrization & fit strategy

For data comparisons, a smooth, positively constrained parameterization of $\chi(z)$ is recommended, so that the bounds $0 < \chi \leq 1$ do not have to be repaired only a posteriori.

Example ansätze for $\chi(z)$

- *Linear-fractional:* $\chi(z) = 1 - \varepsilon_\chi \frac{z}{1+z}$, $\varepsilon_\chi \in [0,1)$.
- *Log basis:* $\chi(z) = 1 - \sum_{k=1}^{K} a_k\, [\ln(1+z)]^k$ with $0 \leq \sum_{k=1}^{K} a_k [\ln(1+z)]^k < 1$.
- *GP reconstruction:* $\chi(z) = \mathrm{sigmoid}(g(z))$ with a GP on g; the sigmoid ensures $0 < \chi \leq 1$.

Formula Box IX.7.4.1: Joint likelihood (sketched)

With data channels $\mathcal{D} = \{d_L, H_{\mathrm{CC}}, (dz/d\tau)_0, \Delta\tau\}$ and model $\{H_{\mathrm{dist}}[d_L; R_k],\ \chi(z;\theta),\ \chi_0\}$:

$$-2\ln\mathcal{L} = \sum_z \sum_{i=1}^{3} \frac{\mathcal{N}_i(z;\theta,\chi_0)^2}{\sigma_i(z)^2} + \mathrm{Priors}[\chi \in (0,1],\ R_k\ (\text{or } \Omega_{k0}),\ H_0].$$

IX.7.5 Secondary predictions

Besides the direct null tests, there are two particularly robust consequences: (i) a purely kinematic shift in acceleration parameters (Formula Box IX.6.2.1) and (ii) an age corset from Formula Box IX.5.4.1 that delimits τ_{geo} against t.

Corollary IX.7.5.1: Acceleration difference as a χ derivative

$$q_{\mathrm{CC}}(z) - q_{\mathrm{geo}}(z) = \frac{d\ln\chi}{d\ln a}\,.$$

Monotone $\chi \nearrow 1 \Rightarrow q_{\mathrm{CC}} > q_{\mathrm{geo}}$ at small z; conversely for $\chi \searrow$.

Corollary IX.7.5.2: Age corset

Let z_* denote the earliest resolvable initial boundary of the sequence in the coarse–graining used (cf. Subsection IX.3.1). With $H(z) = H_{\mathrm{dist}}(z)$ and Formula Box IX.5.4.1,

$$\tau_{\mathrm{geo}}(z) = \int_z^{z_*} \chi(z')\,\frac{dz'}{(1+z')H_{\mathrm{dist}}(z')} \ \le\ t(z),$$

with equality only for $\chi \equiv 1$. Chronometers measure $\Delta\tau_{\mathrm{geo}}$; combining with $t(z)$ yields a lower bound on χ.

IX.7.6 Systematics & robustness

The control points marked in Subsection IX.5.6 are an integral part of any pass/fail judgment. Three additional points enter here, because they feed directly into N_1 to N_3.

Additional points for robust pass/fail judgments

- **Curvature:** Treat explicitly via R_k (equivalently Ω_{k0}) in Formula Box IX.5.1.2 and marginalize with priors or fits.
- **Calibration H_0:** Null test N_1 is sensitive; a consistent H_0 calibration across data sets is mandatory.
- **Photon-number conservation:** Etherington duality is assumed; test possible violations separately before interpreting χ.

IX.7.7 Summary (short checklist)

The following points are the compact decision rules as they can be implemented in an analysis pipeline:

Pass/fail - TDI at a glance

- **Null tests:** N_1 to N_3 data-compatible (cf. Formula Boxes IX.7.1.1 to IX.7.1.3).
- **Estimator consistency:** $\widehat{\chi}_{\mathrm{CC}}(z) \approx \widehat{\chi}_{\mathrm{SN}}(z) = \chi_0/R_{\mathrm{SN}}(z)$ and $\widehat{\chi}_0(z)$ z-constant (Corollary IX.7.2.1).
- **Bounds:** $0 < \chi \leq 1$ (Corollary IX.4.4.1) and $\tau_{\mathrm{geo}}(z) \leq t(z)$ (Corollary IX.7.5.2).
- **Acceleration:** $q_{\mathrm{CC}} - q_{\mathrm{geo}} = d \ln \chi / d \ln a$ (Corollary IX.7.5.1).

IX.8 Demarcation & comparison to standard cosmology

This Section contrasts the TDI effective description of FBA with standard cosmology (ΛCDM / wCDM). Core idea: TDI introduces *no* additional energy density, but modifies the *time readout* of real clocks via $\chi = d\tau_{\rm geo}/dt$ (Section IX.4). This yields testable shifts in *time observables* while leaving FRW kinematics unchanged (Section IX.3). The central comparison points are parameter counting, degeneracies, curvature-dependent effects, and null tests (Section IX.7).

IX.8.1 Parameter counting & degrees of freedom

So that the comparison does not fail due to conceptual mixing, we strictly separate between (i) *geometry* (what distances fix) and (ii) *ticking* (what time channels additionally measure).

Minimal parameter set (conceptual)

- **Geometry (distance channel):** $H_{\rm dist}(z)$ from $d_L(z)$ (or $d_A(z)$) according to Formula Boxes IX.5.1.1 and IX.5.1.2; this channel contains no χ information.
- **Ticking (time channels):** $\chi(z) \in (0, 1]$ with present χ_0 and the relative deviation $\chi(z)/\chi_0$, reconstructible via chronometers/drift/SN (Sections IX.5 and IX.7).

Comparison: wCDM parameterizes geometry via a dynamical $w(z)$ curve in a field-equation logic; TDI parameterizes *not* the geometry, but the *time readout* of clocks via χ in the time channels.

IX.8.2 Degeneracies & their breaking

Distances determine $H_{\rm dist}(z)$ (Formula Boxes IX.5.1.1 and IX.5.1.2) – but they are blind to the question whether a measured time derivative was taken with respect to t or with respect to $\tau_{\rm geo}$. This blindness is broken only by time channels.

Formula Box IX.8.2.1: Geometry–time degeneracy & breaking

From Formula Box IX.5.2.1 it follows that $H(z) = \chi(z)\, H_{\rm CC}(z)$. Thus

$$\underbrace{H_{\rm dist}(z)}_{\text{distance}} = \underbrace{\chi(z)}_{\text{ticking}} \cdot \underbrace{H_{\rm CC}(z)}_{\text{chronometers}},$$

and the joint evaluation of distance and time data breaks the (H, χ) degeneracy (null test Formula Box IX.7.1.2).

The direction of the statement matters: Not "χ explains any deviation", but conversely – a consistent $\chi(z)$ must *simultaneously* explain multiple time channels, while $H_{\rm dist}(z)$ is held fixed geometrically.

Corollary IX.8.2.1: Why pure $H(z)$ tuning cannot simultaneously rescue the time channels

Assume the distance channel fixes $H_{\text{dist}}(z)$ robustly, and time data consistently suggest $\chi(z) \neq 1$. Then an approach that changes *only* $H(z)$ (e. g. via $w(z)$ in wCDM) cannot reproduce the time channels simultaneously, without additionally changing at least one of the input assumptions:

- drift carries χ_0^{-1} in Formula Box IX.5.2.2,
- light curves carry a χ ratio in Formula Box IX.5.2.3,
- chronometers carry $\chi(z)$ in Formula Box IX.5.2.1.

These factors cannot be absorbed by a pure reshaping of $H(z)$ as long as geometry (distance inversion) and time readouts (proper time) remain separately calibrated.

IX.8.3 Curvature, dualities & robustness

Curvature k affects distance inversion and thus $H_{\text{dist}}(z)$, but it does *not* enter the χ factors of the time channels. Methodologically this is convenient: k is handled in the distance channel and then checked jointly with χ for consistency.

Role of curvature

In Formula Box IX.5.1.2 k is explicit; χ appears only in Formula Boxes IX.5.2.1 to IX.5.2.3. Etherington's duality (photon-number conservation) remains valid; violations of this assumption must be tested separately (cf. Subsection IX.5.6).

IX.8.4 Mapping to wCDM (only as a comparison quantity)

The $w_{\text{cmp}}(z)$ introduced in Section IX.6 is a useful translation object: It states which effective $w(z)$ curve would mimic the *geometrically* reconstructed $H_{\text{dist}}(z)$. But it is deliberately not a statement about time channels.

Reading of $w_{\text{cmp}}(z)$

$w_{\text{cmp}}(z)$ is a comparison plot: it describes a GR notation that would lead to the same geometry $H_{\text{dist}}(z)$. TDI signatures, by contrast, lie where time channels at identical H_{dist} consistently suggest or require $\chi(z) \neq 1$ (Section IX.7).

IX.8.5 Acceleration diagnostics

If acceleration is estimated from time data, it must be clear which time is in play. Exactly here, the difference of the acceleration parameters isolates a pure χ derivative (Section IX.6).

Formula Box IX.8.5.1: Kinematic diagnostic

$$q_{\mathrm{CC}}(z) - q_{\mathrm{geo}}(z) \;=\; \frac{d\ln\chi}{d\ln a},$$

cf. Formula Box IX.6.2.1. A consistent sign and trend behavior over z is a sharp TDI signature.

IX.8.6 Growth (sketched)

Without field equations, the growth of linear perturbations cannot be fully determined. TDI nevertheless provides a clear hint as to *where* a bias can arise: Many growth analyses use derivatives with respect to a "cosmic time" that in practice is approximated via proper times.

Growth indicator (qualitative)

Analyses of the growth index γ often use derivatives with respect to "cosmic time". For $\chi \neq 1$, such derivatives shift relative to proper time, which can generate inference bias. For dynamical growth predictions, budget geometry from Part VI must be used.[a] The null tests from Section IX.7 are unaffected by this, because they test only geometry (distances) against time readouts (clocks).

---

[a] See FBA Part VI: Gravity & Geometry from Budget Flows, Secs. VI.2–VI.4 "Budget–Geometry".

IX.8.7 Model comparison (practical)

For a data-driven comparison, an information-criterion comparison is suitable, built directly on the null-test residuals. This avoids implicitly parameterizing field equations "through the back door".

Formula Box IX.8.7.1: BIC/AIC on null-test residuals

Take residuals $\mathcal{N}_{1,2,3}(z)$ from Formula Box IX.7.3.1. For a χ model with parameter vector θ and N data points:

$$\mathrm{AIC} = 2k - 2\ln\mathcal{L}(\theta), \qquad \mathrm{BIC} = k\ln N - 2\ln\mathcal{L}(\theta),$$

with $-2\ln\mathcal{L} = \sum_i \sum_z \mathcal{N}_i(z;\theta)^2/\sigma_i(z)^2$ (cf. Formula Box IX.7.4.1). ΛCDM corresponds to the special case $\chi \equiv 1$.

IX.8.8 Short checklist (comparison)

The following points bundle the demarcation in compact form.

TDI vs. standard - at a glance

- **Geometry:** $H_{\text{dist}}(z)$ from distances (independent of χ; Formula Boxes IX.5.1.1 and IX.5.1.2).
- **Time channels:** χ scales chronometers/drift/light curves (Formula Boxes IX.5.2.1 to IX.5.2.3); ΛCDM sets $\chi \equiv 1$.
- **Null tests:** $\mathcal{N}_{1,2,3}$ (Section IX.7; Formula Boxes IX.7.1.1 to IX.7.1.3) $\Rightarrow$ overdetermined test.
- **Acceleration:** $q_{\text{CC}} - q_{\text{geo}} = d\ln\chi/d\ln a$ (Formula Box IX.8.5.1).
- **Curvature:** affects distance inversion, not the χ factors; marginalize separately (Formula Box IX.5.1.2).

IX.8.9 Classification & outlook

TDI differs conceptually from wCDM: χ acts in time observables as a ticking factor, not as an additional energy term in geometry. Thus Sections IX.7 and IX.8 provide a basis for data analyses that rely on null tests, estimators, and information criteria – without assumptions about gravitational field equations. Section IX.9 closes with a summarizing checklist and editorial notes for implementation.

IX.9 Summary & checklist

This Section bundles the central results from Sections IX.3 to IX.8 into a compact, operationally usable checklist. The purpose is not repetition, but an unambiguous *instruction* for data analyses: First reconstruct the geometry from distances, then determine the ticking from time channels, and only then test both sides against each other via overdetermined identities.

The core statements connect FRW kinematics (Formula Box IX.3.2.1), the budgetary background balance (Formula Box IX.3.3.1), the deductively defined TDI factor $\chi = d\tau_{\mathrm{geo}}/dt$ (Definition IX.4.1.1 and Formula Boxes IX.4.1.1 and IX.4.4.1) and the observable time channels chronometers, redshift drift, and light curves (Formula Boxes IX.5.2.1 to IX.5.2.3). The null tests from Section IX.7 (Formula Boxes IX.7.1.1 to IX.7.1.3 and IX.7.3.1) provide the pass/fail criteria; Section IX.8 provides the conceptual demarcation from standard approaches.

IX.9.1 Core statements (condensed)

The following points are the pivot statements used in this treatise. They are phrased so that it remains clear *where* TDI can enter at all: not in distance kinematics, but in the time readout.

Core results at a glance

- **FRW kinematics remains formal:** Distances follow Formula Box IX.3.2.1; $H_{\mathrm{dist}}(z)$ is geometrically reconstructible from $d_L(z)$ (Formula Boxes IX.5.1.1 and IX.5.1.2).
- **TDI is derived, not an extra term:** Under the normalization indicated in Definition IX.4.1.1 we have $\chi + \vartheta = 1$ and $\vartheta = \frac{\beta^{-1}}{\kappa_\tau}\dot{\Sigma}_{\mathrm{int}}$ (Definition IX.4.1.1 and Formula Box IX.4.1.1); on large scales (in reference-cell coarse–graining) $1 - \chi = \alpha\,\sigma_B^{\mathrm{com}}$ with $\sigma_B^{\mathrm{com}} := a^3\sigma_B$ (Formula Box IX.4.4.1).
- **Time channels carry χ:** Chronometers, drift, and light curves scale according to Formula Boxes IX.5.2.1 to IX.5.2.3.
- **Overdetermined tests:** Null tests Formula Boxes IX.7.1.1 to IX.7.1.3 and residuals Formula Box IX.7.3.1 test $\chi \equiv 1$ against $\chi \neq 1$.
- **Kinematic diagnostics:** $q_{\mathrm{CC}} - q_{\mathrm{geo}} = d\ln\chi/d\ln a$ isolates a pure χ derivative (Formula Box IX.6.2.1).

IX.9.2 Pass/fail checklist (operational)

The tests are formulated so that they work without gravitational field equations and use only observable quantities. The important reading is: A **fail** means either that TDI does not hold, or that at least one of the input assumptions (calibration, curvature, dualities, systematics) is violated. Precisely for that reason, the criteria are formulated as separate residuals and bounds.

Formula Box IX.9.2.1: Test catalog & decision

1. **(N_1) Drift–distance null test:** $\Delta_{\mathrm{DR}}(z) \overset{!}{=} 0$ from Formula Box IX.7.1.1. *Fail:* systematic deviation over z.
2. **(N_2) Chronometer–distance null test:** $\Delta_{\mathrm{CC}}(z) \overset{!}{=} 0$ from Formula Box IX.7.1.2. *Fail:* coherent offset or trend.
3. **(N_3) SN dilation $\oplus$ chronometers:** $\Delta_{\mathrm{SNCC}}(z) \overset{!}{=} 0$ from Formula Box IX.7.1.3. *Fail:* inconsistent $R_{\mathrm{SN}}(z)$.
4. χ **bounds:** $\widehat{\chi}_{\mathrm{CC}}(z) \in (0,1]$ and $\widehat{\chi}_{\mathrm{SN}}(z) \in (0,1]$ as hard priors (cf. Corollary IX.4.4.1).
5. **Age corset:** $\tau_{\mathrm{geo}}(z) \le t(z)$ (Formula Box IX.5.4.1 and Corollary IX.7.5.2). *Fail:* violation beyond errors.
6. **Acceleration difference:** Trend of $q_{\mathrm{CC}} - q_{\mathrm{geo}} = d\ln\chi/d\ln a$ (Formula Box IX.6.2.1). *Fail:* pattern incompatible with the χ fits.

Decision: Use the normalized residuals $\mathcal{N}_{1,2,3}$ (Formula Box IX.7.3.1) with channel-specific tolerance bands $\delta_{*,i}(z)$.
Pass if all $|\mathcal{N}_i(z)| \le \delta_{*,i}(z)$ and all bounds are satisfied; otherwise **fail** (or assumptions violated).

IX.9.3 Minimal data set & evaluation

For a complete test, a data set suffices that makes geometry and time readout separately accessible. The central practical point is the order: first H_{dist}, then χ, then null tests.

Data & pipeline (minimal)

Input data:

- **Distances:** $d_L(z)$ (or $d_A(z)$) $\Rightarrow H_{\mathrm{dist}}(z)$ via Formula Boxes IX.5.1.1 and IX.5.1.2.
- **Chronometers:** $H_{\mathrm{CC}}(z) = -\frac{1}{1+z}\frac{dz}{d\tau_{\mathrm{geo}}}$ (Formula Box IX.5.2.1).
- **Redshift drift:** $(dz/d\tau_{\mathrm{geo}})|_{0,\mathrm{obs}}$ (Formula Box IX.5.2.2), together with H_0.
- **Light curves:** $\Delta\tau_{\mathrm{geo,obs}}/((1+z)\Delta\tau_{\mathrm{geo,em}}) \Rightarrow R_{\mathrm{SN}}(z)$ (Formula Box IX.5.2.3).

Reconstruction & checks:

- Reconstruct $H_{\mathrm{dist}}(z)$ (smooth, then differentiate; cf. Subsection IX.5.1).
- Determine $\widehat{\chi}_{\mathrm{CC}}(z) = H_{\mathrm{dist}}(z)/H_{\mathrm{CC}}(z)$, $\widehat{\chi}_0$ from drift, and $\widehat{\chi}_{\mathrm{SN}}(z) = \widehat{\chi}_0/R_{\mathrm{SN}}(z)$ (Subsection IX.7.2).
- Evaluate null tests and residuals $\mathcal{N}_{1,2,3}$ from Formula Box IX.7.3.1.

IX.9.4 Robustness & typical pitfalls

The null tests are only as good as the control of the systematics entering the respective channels. The following list is deliberately short and refers to the places where the effects were introduced in the reading path.

Systematics (short list)

- **Calibration H_0:** enters directly in Formula Box IX.7.1.1; drift tests require consistent H_0 calibration.
- **Curvature k:** affects H_{dist} via Formula Box IX.5.1.2; fit or marginalize separately (cf. Subsection IX.5.1).
- **Photon-number conservation:** Etherington duality is assumed; test possible violations separately (cf. Subsection IX.5.6).
- **Chronometer models:** stellar-population systematics enter Formula Box IX.5.2.1; without robust model control, $\widehat{\chi}_{\text{CC}}(z)$ is not reliable.
- **SN standardization:** stretch and K-corrections enter Formula Box IX.5.2.3; $R_{\text{SN}}(z)$ is only as stable as the standardization.

IX.9.5 Concluding remark

The TDI test logic strictly separates geometry (from distances) and ticking (from time channels) and brings them together in overdetermined identities. This provides a clear, field-equation-free pass/fail mechanism that contains ΛCDM ($\chi \equiv 1$) as a special case. Any deviation $\chi \neq 1$ must manifest as a consistent pattern in the null tests Formula Boxes IX.7.1.1 to IX.7.1.3 and the bounds Corollaries IX.4.4.1 and IX.7.5.2.

IX.10 The state before time - intuition outside the formal framework set up so far

This Section sketches a non-formal intuition within the framework of the FBA and the Time-Dilation Inflation (TDI) derived from it. It introduces no new primitives and replaces no derivations. Its role is a narrative complement: it makes vivid *why* the separation built up in Sections IX.3 to IX.8 between geometry (distance channel) and ticking (time channels via χ) starts exactly at the point where classical origin narratives otherwise talk back "before time".

Time as an order of differences

In FBA, time is not a background flow, but the order of *actualized* differences. Clocks count minimal events, and the global frame sequence is the bookkeeping that something has become *distinguishably different* at all.[10] This is a sober statement, but it has a sharp consequence: Where there are no actualized differences, there is operationally also no "before" and "after".

Limiting picture before the first tick

Cosmological retrospection often narrates the beginning as approaching a limit point "$a \to 0$". In FBA we also encounter a descriptive boundary, but for a different reason: If time is the order of actualized differences, then the "beginning" is not first a value of a, but the first step in which a difference is actually *actualized* at all.

We can read this as a transition $F_0 \to F_1$: With the first actualized minimal difference, a global state becomes distinguishable from the preceding one for the first time. *From* this moment on, sequence, duration, and comparison acquire operational meaning. Before that it is not "earlier", but simply: no difference yet, hence nothing countable.

Zero-time field

"Zero-time field" (ZTF) denotes the logical descriptive boundary at which no actualized differences are present: no places, no directions, no operational times or densities. The ZTF is not a physical medium "before" time, but the name for the limiting case "nothing counted yet".

The usefulness of this term is methodological: It prevents us from retrospectively applying already calibrated quantities (time, volume, density) to a situation in which these quantities have no operational carrier at all.

First tick and start of the sequence

As soon as a minimal difference is actualized, not only does an "after" exist, but also that to which calibration can attach: comparison of travel times, front protocols, thus the introduction of a maximal propagation rate c as a calibration anchor. Only then does the mere possibility of geometry become a geometry that is actually measurable.

---

[10] See FBA Part I: FBA – Foundations, Secs. I.2–I.4 "Frame sequence, minimal events, proper time".

Formula Box IX.10.1: Operational start criterion of the sequence

The sequence starts exactly when a minimal difference is actualized:

$$F_n \to F_{n+1} \iff \exists \text{ ME with } \Delta \geq \delta_{\min}.$$

Without an actualized difference there is nothing to count – no frames, no sequence, no duration.

In the formal development this is not an additional postulate, but a reformulation of what "frame sequence" and "minimal event" already mean: Frames differ only by actualized, operationally accessible differences, and precisely these differences are the countable material of time. ($\delta_{\min}$ is here the operational minimal difference as a model primitive; empirical tolerance bands are denoted in Part IX as δ_*.)

No singularity by backward rhetoric

With this reading it becomes clear why classical singularity rhetoric loses its operational object in FBA: Quantities like density or temperature are quotients that presuppose notions of measure and volume. But these notions arise only *after* sequence and calibration.

The operational order is therefore not "take a limit and then interpret", but: first sequence, then calibration, then measure and volume, and only afterwards the derived quotients. If the first resolvable step has a finite front span $\ell_* = c\,\Delta t$, then the first volume anchor is finite as well. Thus the densities formed from it are finite, without having to invoke any field equation at this point.

Formula Box IX.10.2: Start order instead of an initial-value problem

Sequence ⇒ calibration ⇒ measure/volume ⇒ density/temperature.
First calibrated volume $V_1 \sim \ell_*^3$ and first internal budget allocation $\Delta b_{\text{int},1}$ are finite
$\Rightarrow \;\; \rho_1 = \Delta b_{\text{int},1}/V_1 < \infty.$

Classification: This does not claim that every mathematical singularity of idealized continuum limits "disappears". The point is simpler: We model the origin not as a geodesic limit point of a smooth continuum, but as the first, finitely calibrated step. Statements on curvature, backreaction, and the classification of classical singularity theorems are treated within the gravity elaboration.[11]

Initial dynamics: drift rather than bang

If one reads the beginning as the first finite step, the intuition shifts automatically: Not "a bang out of nothing", but a beginning as a *continuation* under maximal symmetry.

In an almost homogeneous/isotropic initial situation, scaling is the only global direction that does not immediately break that symmetry. Many local triggers can moreover be co-actualized into the same global step. In such a situation it is plausible that the budget initially flows preferentially into the spatial channel: scale growth relieves the system,

---

[11] See FBA Part VI: Gravity & Geometry from Budget Flows, Secs. VI.2–VI.4 "Budget–Geometry".

increases state capacity, and makes continuation of the sequence “easier” (least-jam intuition). Macroscopically, this appears as a gentle initial drift of the scale factor with real duration – not an impulsive singularity.

In the vocabulary of the previous Sections this means: A large spatial share per step corresponds to small geometric proper time per front-calibrated time, i.e. to $\chi \ll 1$ in Definition IX.4.1.1.

What is classically introduced as an “inflationary role” via an extra field (inflaton) appears here as a collective budget mode that produces a time-dilation phase. With progressive dilution, the budget flow shifts from the spatial channel to internal degrees of freedom, χ increases, and the drift mode fades out. It is precisely this shift that is later testable in the time channels as a TDI signature (Section IX.7).

Ticking and aging

Two quantities are deliberately decoupled here: The front-calibrated time t is the counting of global steps, while real clocks along a worldline measure the geometric proper time τ_{geo}. TDI is exactly the statement that these two quantities systematically diverge:

$$d\tau_{\text{geo}} = \chi\, dt, \qquad 0 < \chi \leq 1$$

(Definition IX.4.1.1).

In parallel, the irreversible bookkeeping runs as aging A in Formula Box IX.4.1.1:

$$\frac{dA}{dt} = 1 - \chi.$$

In the normalization of the TDI definition, this is a complete split per step: What does not “tick” as geometric proper time is booked as irreversible internal binding. Early, in the drift mode, χ is small, hence τ_{geo} grows slowly per dt and A relatively fast. Later χ increases, and the relative share of proper time per step increases.

Observable consequences

This intuition is more than rhetoric only if it can be anchored in the channels that actually see χ. That is exactly why the formal construction separates geometry and ticking: Geometry (distances) remains FRW-kinematic (Formula Box IX.3.2.1), while time channels (chronometers, redshift drift, light curves) explicitly carry $\chi = d\tau_{\text{geo}}/dt$ (Sections IX.5 and IX.7). The drift picture therefore predicts consistent, overdetermined signatures in the time channels – precisely the null tests and estimators from Section IX.7.

Consequences & test hooks

- Early smoothing via co-actuality in an almost homogeneous/isotropic initial situation; a simple, symmetry-compatible continuation is scaling rather than structuring.
- No initial singularity as an operational constraint: notions of measure and volume arise only after sequence and calibration; first step finitely calibrated ⇒ derived quantities finite.
- Drift ⇒ TDI: expansion as collective slowing of proper time without an extra field; testable in time channels via the overdetermined null tests in Section IX.7.

Conclusion

Told this way, the beginning of the world shifts from a bang to a drift. The core remains modest: sequence, budget, calibration. Everything operational and testable is in Sections IX.3 to IX.8 – this Section provides only the narrative view that retrospectively illuminates the formal scaffold “from the inside”.

IX.11 Philosophical excursus: Where the »Why?« began

IX.11.1 Questions upon questions

Children are naturally curious. Their curiosity drives them to question things and sometimes pushes us adults to our limits. Honestly, that makes them the best scientists: Nothing is simply accepted; there is always a »Why?« in the room. This occasionally leaves us at a loss for explanations, but for precisely that reason it should also be a reason for us to keep questioning the world anew.

And many of us know it all too well: Children ask tirelessly for the »Why«. As adults we want to give a plausible answer to each of these questions. Yet even the smartest answer is often immediately followed by the next »Why?«. The question–answer game pushes the level of explanation one step deeper each time. Even if we theoretically had the patience to continue it infinitely, we inevitably reach a point where not only does no »Therefore!« come to mind anymore. Rather, the question word itself loses its ground.

In FBA, this boundary point can be named without mystifying it. We introduced it as a descriptive boundary: the *zero-time field* (ZTF) – a name for the situation in which no realized differences are present (Section IX.10). A causality-implying »Why?« does not carry there, because there is no operational »before«. Causality is not violated; it is simply not yet constituted.

IX.11.2 F_1: The birth of the »Why«

Only at the transition from maximal homogeneity to a state distinguishable from it does the simply possible occur: a first, minimal difference becomes real. We call the first distinguishable state, the first frame in the infinitely long film of time, F_1.

With F_1, registers, pointers, and causal references become possible in the first place. In short: an order in which something can follow something else. Only from here on does it make sense to look for reasons, to formulate laws, and to check the guardrails that we fixed in the formal construction as front, budget, and irreversibility (Sections IX.3 and IX.4). Before F_1 we ask about *conditions of possibility.* From F_1 onward we ask about *causes, dynamics, and consequences.*

In this sense, F_1 is the birth of the »Why«: the point at which the question–answer game can be driven one last step deeper. Further back, causality does not reach, because the concept of »back« loses its carrier.

IX.11.3 How to ask good questions anyway

Before we keep asking »Why?« endlessly, we should check *where* this word can carry at all. As long as no difference is real yet, the question has no ground beneath it. Only with F_1 is that ground poured. That is why it helps to sort one's curiosity into three modes of speech:

Before F_1 – conditions instead of reasons. Do not ask for causes, but for conditions of possibility. The appropriate tone is: »Under what condition is X definable at all?« Time needs stable pointers, place needs distinguishable positions, causality needs a sequence. Without these building blocks, »Why?« is only an empty echo.

At the transition – admissibility of the first difference. Here the question is not »Who caused this?«, but: »What makes a first difference admissible?« What counts are the constitutive guardrails: a finite causal front (no signals faster than c), closed budgets (no leaks in the balance), and *Append-only* (what is realized is not booked back). If this is satisfied, the sequence may begin (Formula Box IX.10.1).

From F_1 onward – the home of why-questions. Now classical why- and by-what questions are meaningful. We speak about laws, initial data, and equations of state, and in FBA language about budgets, monotonicities, and access cuts that bind the evolution. In short: only here does the question for reasons have a home.

IX.11.4 Two translations of why reflexes

One can apply this distinction immediately in everyday life by reformulating typical reflex questions.

First: »Why did the ZTF exist?« – more apt is: »Why is ZTF the right description as long as no difference is real?« Because any additional pre-structure would be ungrounded information. Without differences, nothing is distinguished and thus nothing can be meaningfully preferred.

Second: »Why did the expansion begin?« – made appropriate, it becomes: »Which internal balance, which scale evolution $a(t)$, and which ticking $\chi(t)$ describe the observed kinematics after F_1?« In standard vocabulary, accelerated expansion is often coded as dark energy or $w \approx -1$. In the TDI vocabulary, the guiding question instead is how geometry (distance channel) and time readout (time channels) fit together consistently (Sections IX.5, IX.7 and IX.8).

IX.11.5 The question of uniqueness remains

When it comes to our uniqueness or singularity, we humans have often been badly wrong in the past. Precisely for that reason, we should not only ask the following questions, but also sort them cleanly and answer them from an FBA perspective.

Guiding questions

- Should one expect that the ZTF was a one-time affair?
- Could there have been multiple ZTF states, could there still be some now, or could there be some in the future?
- Are different starting points conceivable in the sense of a many-worlds theory?
- Does the ZTF perhaps continue to exist as a kind of »outside« and possibly feed dynamics?
- Can our universe generate ZTF-like conditions, for instance in the interior of black holes?

Answers in the light of FBA

Q1: Is the ZTF unique? Short answer (within the operational definition): yes. The ZTF is not something *in* time, but the logical zero mark »no realized difference«. With the first minimal difference (F_1), sequence and causality begin in the first place (Formula Box IX.10.1). A later reappearance would be a reset of already booked differences and would contradict the irreversible bookkeeping fixed in the formal framework as No-Recovery and monotonicity.[12] [13]

Q2: Multiple ZTF starts as many-worlds starting points? Short answer: no – not as multiple global resets. What in some interpretations looks like »branches« can be read in FBA rather as decoherence-separated record sectors of *one* world: practically inaccessible, but not as second ontologies. Multiple ZTF starts would be multiple global resets and would run against the Append-only logic of irreversible bookkeeping.

Q3: ZTF as an »outside« with inflow of new dynamics? Short answer: no. The ZTF is not a medium, not a reservoir, and not an outside. »Feeding in« already presupposes a causal relation. Precisely this relation is not defined before F_1, because there is no operational »before«. The question of cosmic dynamics is therefore an internal question of the sequence after F_1, i.e. a question about geometry and budget flows within the calibrated description (Sections IX.3 and IX.4).

Q4: Does the ZTF drive accelerated expansion? Short answer: no. In standard cosmology, one would discuss acceleration via a field-equation dynamics and effective states. In the TDI framework, the decisive lever lies elsewhere: The observable shifts arise in time channels via $\chi = d\tau_{\text{geo}}/dt$, while distance kinematics remains formally FRW (Sections IX.5, IX.7 and IX.8). An ZTF »engine« would be an additional assumption without an operational role.

Q5: ZTF-like conditions in the universe (e. g. in black holes)? Short answer (in the same sense of "ZTF"): no. Black holes are not ZTF-like, but extreme, highly record-carrying configurations. Their horizon physics is distinguished precisely by bundling many states into a single geometric quantity. A »local ZTF« in the interior would erase already realized differences and thus break the Append-only logic. Even hypothetical, causally decoupled domains would at most be new domains of the sequence – but not ZTF/F_1 restarts.

---

[12] See FBA Part I: FBA – Foundations, Sec. I.5 "DPI arrow & No-Recovery".

[13] See FBA Part VIII: Classical Limit, Thermodynamics & Aging, Secs. VIII.6–VIII.8 "Spohn monotonicity, entropy production & aging".

Rule of thumb (bullet-style)

- ZTF marks the condition under which »why« questions make sense (Formula Box IX.10.1).
- ZTF is unique and global because a reset would break irreversible records (Append-only).
- Diversity = decoherence-separated sectors within one world, not multiple ZTF restarts.
- Expansion and dynamics = internal physics under front, budgets, and irreversibility, testable via time channels (Section IX.7).

IX.11.6 Coda: »So there *is* room for God after all?«

Our framework describes the how from F_1 onward. The view before F_1 allows no further »why«, because it knows no operational »before«. Those who do not wish to be satisfied with the statement »In the ZTF there is no why because there is no before« and nevertheless seek a reason for the existence of the origin state leave the domain of physics and enter philosophy or faith. But even there one can impose order without bending the physics:

- *Metaphor or first ground:* The minimality of the ZTF is read as »meaning« or »ground«. This is compatible, but it generates no new predictions.
- *Lawgiver (deistic):* The guardrails are »set«. Irrelevant for physics as long as no additional consequences are asserted.
- *Agent (interventionist):* Meaningful only with empirical signatures. If an intervention breaks front, budget, or Append-only, it is incompatible. If it respects everything, it remains practically indistinguishable from rare natural processes.

Short conclusion. There is room for interpretation, but no necessity derived from FBA.

IX.12 Appendix: Overview of the FBA Series (Parts I–X)

Click the title to download the PDF

1. ***Part I*: FBA-Foundations: Ordering, Budget, Proper Time & Arrows.** *Goal:* Provide the base layer: ordering, budget, proper time/aging, front and the operational arrow of time (DPI); Minkowski limit from the budget quadric; admissible dynamics and locality/no-signalling. *Import:* – (reference for all subsequent parts). *Extension:* interface contracts, pass/fail checklists, reading guide.
2. ***Part II*: Time, Proper Time & Minkowski Geometry.** *Goal:* Capture proper time/quadric operationally and derive geodesics. *Import:* foundations (ordering, budget, proper time, front/DPI). *Extension:* smooth limit, variational principle on worldlines, calibration κ_τ.
3. ***Part III*: Quantum Kinematics & CPTP Channels.** *Goal:* State spaces and channels (CPTP) including composition. *Import:* foundations (budget, channel viewpoint, composition). *Extension:* concrete divergences/cost functionals $\mathcal{C}$, measurements, and classical registers.
4. ***Part IV*: Dynamics, Measurement & GKLS (Open Systems).** *Goal:* Continuous open dynamics (GKLS) and the operational arrow of time. *Import:* channels/DPI. *Extension:* Spohn monotonicity, stationary/NESS references, flows $b^{\mathrm{rev}}, b^{\mathrm{irr}}, b^{\mathrm{ext}}$.
5. ***Part V*: Spacetime, Light Cones & Local Field Theory.** *Goal:* Local field equations under front/locality. *Import:* front, composition, no-signalling. *Extension:* local GKLS generators, Lieb–Robinson-type bounds, effective light cones.
6. ***Part VI*: Gravity & Geometry from Budget Flows.** *Goal:* Geometrization of budget flows. *Import:* budget quadric/proper time. *Extension:* effective metrics from calibrations (κ_t, κ_x) and internal stresses; coupling to curvature.
7. ***Part VII*: Constants, Scales & Renormalization.** *Goal:* Scale running of the calibration theorems. *Import:* $c = \kappa_t/\kappa_x$, κ_τ. *Extension:* flow equations for $\kappa_t, \kappa_x, \kappa_\tau$; stability of c.
8. ***Part VIII*: Classical Limit, Thermodynamics & Aging.** *Goal:* Macroscopic behavior from $A[\gamma]$ (aging) and DPI. *Import:* proper time/aging, Spohn. *Extension:* entropy production, Euler–Lagrange forms for irreversible flows, effective transport equations.
9. ***Part IX*: Cosmic Dynamics, Time Dilation & Inflation (TDI).** *Goal:* Cosmic ordering & calibration flow. *Import:* budget, proper time/front. *Extension:* budget equations on large-scale slices; time-dilation inflation as calibration dynamics.
10. ***Part X*: Predictions, Falsifiability & Bridge FBA → QM ↔ GR.** *Goal:* Testable differences and bridges FBA ↔ QM/GR. *Import:* all foundational building blocks. *Extension:* protocols, limiting-case tests, overdetermined consistency relations (pass/fail).

References

[1] D. W. Hogg. “Distance Measures in Cosmology”. In: (1999). arXiv:astro-ph/9905116. arXiv: astro-ph/9905116 `[astro-ph]`.

[2] R. Jimenez and A. Loeb. “Constraining Cosmological Parameters Based on Relative Galaxy Ages”. In: *The Astrophysical Journal* 573.1 (2002), pp. 37–42. DOI: 10.1086/340549. arXiv: astro-ph/0106145 `[astro-ph]`.

[3] M. Moresco, A. Cimatti, R. Jimenez, et al. “Improved Constraints on the Expansion Rate of the Universe up to $z \sim 1.1$ from the Spectroscopic Evolution of Cosmic Chronometers”. In: *Journal of Cosmology and Astroparticle Physics* 2012.08 (2012), p. 006. DOI: 10.1088/1475-7516/2012/08/006. arXiv: 1201.3609 `[astro-ph.CO]`.

[4] A. Sandage. “The Change of Redshift and Apparent Luminosity of Galaxies Due to the Deceleration of Selected Expanding Universes”. In: *The Astrophysical Journal* 136 (1962), pp. 319–333. DOI: 10.1086/147463.

[5] A. Loeb. “Direct Measurement of Cosmological Parameters from the Cosmic Deceleration of Extragalactic Objects”. In: *The Astrophysical Journal* 499.2 (1998), pp. L111–L114. DOI: 10.1086/311364. arXiv: astro-ph/9802122 `[astro-ph]`.

[6] B. Leibundgut. “Time Dilation in the Light Curve of the Distant Type Ia Supernova SN 1995K”. In: *The Astrophysical Journal* 466 (1996), pp. L21–L24. arXiv: astro-ph/9605134 `[astro-ph]`.

[7] G. Goldhaber, D. E. Groom, A. Kim, et al. “Timescale Stretch Parameterization of Type Ia Supernova B-Band Light Curves”. In: *The Astrophysical Journal* 558 (2001), pp. 359–368. DOI: 10.1086/322460. arXiv: astro-ph/0104382 `[astro-ph]`.

[8] I. M. H. Etherington. “On the Definition of Distance in General Relativity”. In: *The London, Edinburgh, and Dublin Philosophical Magazine and Journal of Science*. 7th ser. 15 (1933), pp. 761–773. DOI: 10.1080/14786443309462220.