The Frame–Budget Approach (FBA)

How time, dynamics, and geometry emerge from budget flows

An operational bridge between quantum mechanics and general relativity

Part VII: Constants, Scales & Renormalization

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Constants, Scales & Renormalization

VII.1 Introduction & Target Picture

VII.1.1 Motivation and objectives

In the preceding parts[1] it has already become clear that central quantities such as c are not needed as metaphysical constants, but appear as *calibration results* from operational protocols: fronts fix a maximal propagation rate, geometric proper time arises as the integrated internal budget contribution, and effective gravitation couples, in the appropriate limit, to a calibrated strength.[2] [3] If one takes this perspective seriously, the next natural question is no longer *which* constants exist, but *how* normalizations remain stable, how one mediates between descriptive regimes, and what notion of scale is meant in the first place.

Target picture. In the FBA we trace the fundamental constants c, $\hbar$, and G back, as *regime calibrations*, to a small number of operational calibration protocols (normalization of time, space, and proper time). If these protocols are *fixed*, then c and $\hbar$ are metrologically stable in this sense; scale dependence (if any) concerns only parameters and couplings *extracted from an effective description* (typically in dimensionless combinations). The scale parameter used in this treatise therefore denotes *resolution/coarse-graining* (RG scale) and *not* a temporal drift along a worldline. The central task of this treatise is therefore:

- to reconstruct the constants *operationally* as derived normalization and conversion quantities from budget calculus and calibration protocols (i.e. as assignments/normalizations, not as a prediction of their SI numerical values),
- to formulate a precise notion of scale flow in the FBA,
- and to derive from this testable, scheme-robust parameter relations across scales.

VII.1.2 Relevance of the constants in the FBA

In standard presentations, c, $\hbar$, and G appear as fixed inputs. In the FBA their conceptual status is different: constants mark *how* we compare and normalize operational quantities, not *what* the world is fundamentally. This is precisely why they are so central to the internal consistency of the series:

- **c as front calibration:** c is the normalization of the fastest admissible signal fronts and thus the bridge between "time calibration" and "space calibration".[4]
- **$\hbar$ as quantum normalization:** $\hbar$ appears wherever the channel and state description requires a fixed conversion between generator/phase normalization and operational

[1] An overview of all parts of the FBA treatise including download links can be found in Section VII.7 of this document.

[2] See FBA Part I: FBA – Foundations, Secs. I.3–I.4 (Calibration, Front, Proper Time).

[3] See FBA Part VI: Gravity & Geometry from Budget Flows, Secs. VI.3–VI.6 (Proxy Metric, Coupling, Tests).

[4] See FBA Part I: FBA – Foundations, Sec. I.3 (Front, Signal Front, Calibration).

cost/budget scales.[5] [6]

- *G* **as gravitational coupling:** In the FBA, G is the effective coupling that connects the geometrization of budget inhomogeneities with observable gravitational phenomena.[7]

This also makes clear why renormalization is not an add-on topic here: once constants are normalizations, the question of stability under coarse-graining and regime changes is unavoidable. Scale flow in the FBA is therefore primarily a *calibration and coupling flow* in effective descriptions (at fixed operational protocol).

VII.1.3 Logical path of the treatise

The structure is chosen such that the operational fixations of normalization come first, and only then do we discuss flows, fixed points, and constraints. Otherwise, renormalization would look like a formal appendage rather than a consequence of the FBA reading of constants and admissible processing:

- **Section VII.1 - Introduction & Target Picture:** We set the target picture and mark the central dividing line of this treatise: metrological calibrations (as protocols) must be stable, whereas effective couplings may be scale-dependent only within admissible coarse-graining. This is the prerequisite for ensuring that "scale dependence" is not later confused with a choice of units or with temporal drift.
- **Section VII.2 - Preliminary foundations & conventions – import:** We fix the minimal FBA building blocks presupposed by any scale question (balance, calibration, proper time, admissible dynamics, locality). This is necessary because a scale flow is physically interpretable only if it is grounded in invariant protocols and admissible operations.
- **Section VII.3 - Constants and normalization parameters in the FBA:** We reconstruct c, $\hbar$, and G as calibrated conversion and coupling factors. This step makes explicit which quantities serve as metrological anchors (and therefore are not renormalized at fixed protocol) and which quantities are couplings tied to regimes and scales.
- **Section VII.4 - Renormalization groups and scale flow in the FBA:** On this basis, renormalization is formulated as admissible CPTP coarse-graining. Monotonicities (DPI/Spohn) and notions of fixed points then become automatic structural consequences, and it becomes precise which statements are scheme-robust. In particular, we justify why c and $\hbar$ do not "run" when the calibration protocol is fixed, whereas effective couplings such as G_{eff} (or dimensionless combinations thereof) can be scale-dependent only within budget and causal constraints.
- **Section VII.5 - Predictions and testability:** From invariants, monotonicities, and proximity to fixed points we obtain concrete, measurable parameter relations. This step is the transition from structure to falsifiability: the theory yields pass/fail criteria and bounded windows.

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[5] See FBA Part III: Quantum Kinematics & CPTP Channels, Secs. III.3–III.6 (CPTP Channels, Kinematics).

[6] See FBA Part IV: Dynamics & Measurement (GKLS), Secs. IV.3–IV.7 (Dynamics, Measurement, GKLS).

[7] See FBA Part VI: Gravity & Geometry from Budget Flows, Secs. VI.4–VI.6.

- **Section VII.6 - Conclusion and outlook:** Finally, we condense the derivation chain into a compact decision logic and mark the next necessary building blocks in the series, so that scale questions do not remain isolated but are embedded consistently into field-theory and gravity tests.

VII.1.4 Outlook and relevance for further work

Part VII is the scale layer of the series: it clarifies how the normalizations used in other parts fit together consistently when switching between micro and macro regimes. This later enables two things:

- In cosmology, a large-scale calibration dynamics can be cleanly separated as an independent degree of freedom from local physics.[8]
- In the bridge and test treatise, pass/fail criteria can be formulated so that they do not depend on arbitrary units, but on stable parameter relations across scales.[9]

VII.1.5 Scope and delimitation

This treatise introduces no new physics "beyond the FBA". In particular:

- We do not postulate a fundamental time variation of c, $\hbar$, or G. Scale dependence here means regime dependence under changes of resolution (RG scale) and, in physically robust terms, primarily concerns dimensionless effective parameters; it must not be confused with a temporal drift along a worldline.
- We do not provide an autonomous theory of quantum gravity. The task is the clean scale handling of the normalizations that are already used operationally in the quantum and gravity parts.[10] [11]
- Cosmological large-scale dynamics is referenced only where it is needed as a consistency check for scale flow; the detailed dynamics belongs in Part IX.[12]

[8] See FBA Part IX: Cosmic Dynamics, Time Dilation & Inflation (TDI), Secs. IX.2–IX.7.
[9] See FBA Part X: Predictions, Falsifiability & Bridge FBA $\rightarrow$ QM $\leftrightarrow$ GR, Secs. X.4–X.8.
[10] See FBA Part III: Quantum Kinematics & CPTP Channels, Secs. III.3–III.6.
[11] See FBA Part VI: Gravity & Geometry from Budget Flows, Secs. VI.3–VI.6.
[12] See FBA Part IX: Cosmic Dynamics, Time Dilation & Inflation (TDI), Secs. IX.3–IX.7.

VII.2 Preliminary Foundations & Conventions (Import from Part I: FBA – Foundations)

Why this import is particularly important here. In Part VII it is not only about using the constants c, $\hbar$, and G, but about their status as *normalizations*. So that later statements such as "c is a calibration result" or "couplings run with scale" do not degenerate into matters of convention, the operational fixed points of the theory must already be in place: balance, sequence, minimal events, front calibration, proper time, and admissible dynamics. Exactly these fixed points are adopted unchanged from Part I.[13]

Since Part VII in principle builds on all of Part I, it is crucial for the reading path to mark the building blocks that appear directly in the following Sections as *calibration anchors* and as *scale invariants*. The list is therefore not a new foundation, but a clarification of which tools later carry the normalization arguments.

Imported building blocks (unchanged)

We adopt the following building blocks *without* redefinition from Part I: FBA – Foundations.

- **Sequence of global states & minimal events:** Part I: FBA – Foundations, Sec. I.2: "Global States, Frame Sequence, and Minimal Event (ME)", as well as "Co-Actuality and Refinement Invariance".
- **Difference function & operational minimal difference:** Part I: FBA – Foundations, Sec. I.2 (box): "Difference Function & Operational Minimal Difference".
- **Budget calculus (internal/external/irreversible) & balance:** Part I: FBA – Foundations, Sec. I.3: "One-Step Budget & Decomposition", (formula box) "Balance Equations", and (lemma) "Refinement Invariance of the Balance".
- **External calibration & front:** Part I: FBA – Foundations, Sec. I.3: (definition) "Calibration and Front Costs", (lemma) "Front Bound", (corollary) "Signal Front".
- **Proper time & aging, Minkowski limit:** Part I: FBA – Foundations, Sec. I.4: (definition) "Proper Time", (formula box) "Properties of Proper Time", (definition) "Aging (irreversible)", (formula box) "Minkowski Limit & Quadric", (lemma) "Time Dilation".
- **Admissible dynamics (CPTP/GKLS), DPI/Spohn:** Part I: FBA – Foundations, Sec. I.5: (definition) "Admissible Channels (CPTP)", (formula) "Kraus/Stinespring", (lemma) "Measurement as CPTP", (definition) "GKLS Generators (open systems)", (formula) "Spohn Monotonicity", (lemma) "Semigroup–Budget", (definition/corollary) "DPI Arrow & No-Recovery".
- **Composition, locality & no-signalling:** Part I: FBA – Foundations, Sec. I.6: (definition) "Symmetric-Monoidal Structure", (formula) "Budget Additivity", (lemma) "No-Wire Inflation & Local Operations", (corollary) "Causal Cones & Local GKLS".

[13] See FBA Part I: FBA – Foundations, Secs. I.2–I.6 (Sequence/ME, Balance, Front, Proper Time, Admissible Dynamics, Locality).

On this basis we can discuss constants as normalizations in the following Sections without silently dragging along unit changes or shifts of convention. What is needed next is a notation that cleanly separates discrete and continuum and simultaneously keeps dimensional issues transparent, because scale flow is otherwise hard to distinguish from mere reparametrization.

Notation & conventions

- **Discrete vs. continuum:** step index $n \in \mathbb{N}$ for successive frames; $\Delta(\cdot)$ for discrete increments, $d(\cdot)$ for differential quantities in the limit.
- **Budget decomposition:** per step δb_{int}, δb_{ext}, δb_{irr} (internal/external/irreversible) with

$$\delta b_{\text{int}} = \delta b_{\text{rev}} + \delta b_{\text{irr}}, \qquad \delta b_{\text{irr}} \geq 0,$$

  and path sums $\sum \delta(\cdot)$ and, respectively, $\int d(\cdot)$. *Note:* δb denotes the same budget account as ΔB in Part I; the lower-case notation here is merely a choice of symbols.[a]
- **Proper time & aging (keep calibration explicit):** proper time arises from the internal budget along a worldline γ, but is read as a *time measure* only after calibration. We therefore write in the continuum limit

$$d\tau_{\text{geo}} := \alpha_\tau \, db_{\text{rev}}, \qquad dA := \alpha_\tau \, db_{\text{irr}} \geq 0, \qquad d\tau_{\text{tot}} = d\tau_{\text{geo}} + dA = \alpha_\tau \, db_{\text{int}}.$$

  The time calibration α_τ (equivalently $1/\kappa_\tau$) is fixed explicitly in the later calibration/dimension boxes of this part; here it serves as a guardrail against "silent units".[b]
- **Calibration (front):** c is the *calibration constant* of the fastest admissible fronts (not postulated, but fixed metrologically). Operationally this is tied to $\kappa_t, \kappa_x > 0$, such that $c := \kappa_t/\kappa_x$ normalizes the front bound.[c]
- **Spacetime language (flat, kinematic):** four-vector $x^\mu = (x^0, x^1, x^2, x^3) = (ct, x, y, z)$; Minkowski signature $\eta = \text{diag}(-1, 1, 1, 1)$, so that

$$\eta_{\mu\nu} dx^\mu dx^\nu = -c^2 dt^2 + dx^2 + dy^2 + dz^2.$$

  Light cone given by $\eta_{\mu\nu} dx^\mu dx^\nu = 0$.[d]
- **Worldlines & paths:** γ denotes a worldline of a system through the frame sequence; concatenation $\Gamma = \Gamma_1 \circ \Gamma_2$; additivity of all integrated budgets along Γ.[e]
- **Composition/locality:** parallel composition $\otimes$; serial composition $\circ$. Local CPTP operations respect no-signalling and budget additivity.[f]
- **Sign conventions:** vector norms $\|\cdot\|$; Euclidean inner products "$\cdot$" in space; c explicit (no c=1 units in this treatise). expectations $\mathbb{E}[\cdot]$; supremum sup.

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[a] See FBA Part I: FBA – Foundations, Sec. I.3 (Budget Accounts, Balance Notation).
[b] See FBA Part I: FBA – Foundations, Sec. I.4 (Proper Time, Aging, Calibration).
[c] See FBA Part I: FBA – Foundations, Sec. I.3 (Front Costs, Calibration, Signal Front).
[d] See FBA Part I: FBA – Foundations, Sec. I.4 (Minkowski Limit, Quadric, Light Cone).
[e] See FBA Part I: FBA – Foundations, Sec. I.4 (Worldlines/Paths in the Limit, Additivity).
[f] See FBA Part I: FBA – Foundations, Sec. I.6 (Composition, Locality, No-Signalling).

These conventions are not mere formalities in Part VII: they ensure that any later "running" of c, $\hbar$, or G is read as a statement about calibration theorems and coarse-graining, and not as an artifact of a silent choice of units.

VII.3 Constants and normalization parameters in the FBA

In the FBA, we clarify the role of fundamental constants as *calibration and coupling parameters* between (i) sequence- and budget-level quantities and (ii) observed measurement scales. Three constants are central: the front calibration c, the quantum of action $\hbar$ as a phase/action calibrator, and the gravitational coupling G as a geometry–budget calibrator.[14] [15] We begin with a dimensional positioning, because otherwise it remains unclear whether an apparent "running of constants" is physical or merely a reparametrization. Only then do we formulate the operational calibration protocols and the structural origin of the constants.

VII.3.1 Dimensions, budgets, and measurement scales

Budgets are primarily bookkeeping quantities of the sequence. To become measurement quantities, protocols are needed that translate budget increments into measures of time, length, and phase. This is precisely where c and $\hbar$ sit: not as "nature postulates", but as fixed conversions that must remain consistent across all admissible representations. The following box makes these translation points explicit so that later statements about scale flow do not accidentally depend on silent unit choices.

[14] See FBA Part I: FBA – Foundations, Secs. I.3–I.4 (Front Calibration, Proper Time).

[15] See FBA Part VI: Gravity & Geometry from Budget Flows, Sec. VI.4 (Effective Gravitational Coupling in the Continuum Limit).

Formula Box VII.3.1.1: Dimensional scheme of FBA calibrations

The sequence supplies *discrete* budget increments $\Delta b_{\rm int}, \Delta b_{\rm ext}$ per step and an irreversible part $\Delta b_{\rm irr} \geq 0$. Measurement scales arise via *calibrations*:

$$\underbrace{d\tau_{\rm geo}}_{\text{time measure (reversible)}} \equiv \alpha_\tau \, db_{\rm rev}, \qquad \underbrace{dA}_{\text{aging}} \equiv \alpha_\tau \, db_{\rm irr} \geq 0, \qquad \underbrace{d\tau_{\rm tot}}_{\text{total proper time}} = d\tau_{\rm geo} + dA = \alpha_\tau \, db_{\rm int},$$

where $db_{\rm rev} \equiv db_{\rm int} - db_{\rm irr}$ denotes the reversible part of the internal budget. For external calibration we write

$$\underbrace{d\ell}_{\text{length measure}} \equiv \alpha_\ell \, db_{\rm ext}, \qquad \underbrace{d\varphi}_{\text{phase measure}} \equiv \alpha_\varphi \, db_{\rm rev}.$$

In the flat, kinematic Minkowski limit, the calibrations are fixed such that the budget quadric can be brought into the usual metric form. In 1+3 we write $d\ell^2 := dx^2 + dy^2 + dz^2$ and obtain along a worldline

$$c^2 \, d\tau_{\rm geo}^2 \;=\; c^2 \, dt^2 - d\ell^2,$$

where t denotes the externally calibrated coordinate time.[a] [b] For the unitary (reversible) quantum sector, the phase calibration is fixed such that

$$d\varphi = \frac{dS}{\hbar}$$

holds, where S denotes the associated action/generator functional of the effective continuum description. The gravitational coupling G finally calibrates the transition from budget inhomogeneity to effective geometry in the continuum limit.[c]

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[a] See FBA Part I: FBA – Foundations, Sec. I.4 (Budget Quadric, Minkowski Limit).

[b] See FBA Part V: Spacetime, Light Cones & Local Field Theory, Secs. V.3–V.4 (Spacetime/Causal Structure).

[c] See FBA Part VI: Gravity & Geometry from Budget Flows, Secs. VI.3–VI.4.

The box is deliberately "technical": it shows that constants in the FBA appear exactly where an operational conversion from budget to measurement scale is required. This simultaneously fixes which quantities can be meaningfully compared under coarse-graining.

Remark VII.3.1.1: Reading

Constants do not appear as postulated quantities, but as conversion factors that (i) map budgets to measurement scales and (ii) normalize couplings between sectors. This reading makes calibration protocols central and ties constants to operational procedures rather than to presupposed units.

With Formula Box VII.3.1.1 and the reading above, the guiding line is set: we define constants exactly where silent normalizations would otherwise creep in, and then ensure that these normalizations remain stable under admissible dynamics and coarse-graining. In what follows, this becomes explicit for c, $\hbar$, and G in the same three-step form each time: operational definition, metrological protocol, invariance or coupling argument.

VII.3.2 Front calibration and the constant c

External accounting generates front costs and a front bound. From this, c is extracted as a metrologically fixed quantity of the maximal signal rate: c is not "put in by hand", but emerges from the demand that a minimal signal, at minimal external cost, defines a maximal propagation relation.[16] [17] The next definition therefore does not fix "a speed", but the saturation limit of a cost and distinguishability protocol.

Definition VII.3.2.1: Front calibration and the signal-front bound

Let $\mathcal{P}_{\text{front}}$ be a front protocol that realizes minimally possible external budget trajectories for transmitting an operationally distinguishable signal. Then there exists a constant $c > 0$ such that for all admissible fronts

$$d\ell \leq c\,dt \quad \text{and} \quad d\ell = c\,dt \;\Leftrightarrow\; \text{front saturation (null-like).}$$

The number c is thus operationally determined by the saturation cases of the fixed protocol $\mathcal{P}_{\text{front}}$.

The definition is intentionally formulated as an existence and protocol statement. To keep it from remaining abstract, a metrological playbook follows immediately: it shows how to isolate the saturation cases in practice and how the calibration is tied to the limiting trajectories.

Formula Box VII.3.2.1: Front protocol (metrological)

1. Choose two localized carriers and a minimal signal (operationally distinguishable).
2. Execute $\mathcal{P}_{\text{front}}$ such that external costs are minimal (no internal relabeling).
3. Fix time and length normalization so that saturation cases (front saturation) satisfy $d\ell = c\,dt$.

For Part VII, the decisive point is not only the existence of c, but the *robustness of the bound*: under admissible coarse-graining, no effective description may realize a larger signal rate than the fundamental saturation limit. This is why this stability is stated as a separate lemma and tied explicitly to DPI/monotonicity of distinguishability.

Lemma VII.3.2.1: Stability of the front bound (no $c_{\text{eff}} > c$)

Under any admissible coarse-graining (CPTP) of the front protocols, the maximal signal rate that can be operationally extracted from $\mathcal{P}_{\text{front}}$ cannot increase. In particular, for any effective description $c_{\text{eff}} \leq c$. If c is calibrated via the same saturation protocol $\mathcal{P}_{\text{front}}$, the value extracted in this way remains unchanged.

[16] See FBA Part I: FBA – Foundations, Sec. I.3 (Front Costs, Front Bound, Signal Front).

[17] See FBA Part II: Time, Proper Time & Minkowski Geometry, Sec. II.5 (Placement of c-Invariance within the Time/Minkowski Structure).

Proof Sketch VII.3.2.1: Stability of the front bound

Coarse-graining cannot increase operational distinguishability (DPI/monotonicity of admissible processing), and therefore cannot "simplify" a signal such that it transports the same distinguishability at smaller external cost. Hence no effective description can realize a larger signal rate than the finest front-saturating protocol. Composition and refinement may change the parametrization of steps, but not the saturation relation, because the latter is defined precisely as a minimal-cost bound.

A more detailed derivation of the front bound and its stability is given in Part I and Part V.[a] [b]

[a]See FBA Part I: FBA – Foundations, Secs. I.3–I.6.

[b]See FBA Part V: Spacetime, Light Cones & Local Field Theory, Sec. V.4 (Null-Cone Interpretation of Front Saturation).

VII.3.3 The quantum of action $\hbar$ as a phase and budget calibrator

Interference makes phases operationally accessible. This forces the question of how "reversible internal budget" is translated into a phase measure. $\hbar$ is the normalization that keeps this translation stable, so that phase comparisons remain independent of the resolution at which the reversible dynamics is described.[18] The core reason to treat $\hbar$ here in parallel with c is the same: without a calibrated, coarse-graining-stable conversion between "budget flow" and "phase", an RG comparison of quantum regimes would not be well-defined.

Definition VII.3.3.1: Phase and action calibration $\hbar$

There exists $\hbar > 0$ and a calibration α_φ such that for every reversible channel evolution (unitary sector of CPTP dynamics) along a path γ

$$d\varphi \;=\; \alpha_\varphi \, db_{\mathrm{rev}} \;=\; \frac{dS}{\hbar},$$

where φ denotes the observable interference phase and S the associated action/generator functional of the effective continuum description. Thus, in the infinitesimal continuum limit, the canonical parametrization of the unitary part holds:

$$U(dt) = \exp\left(-\frac{i}{\hbar} H \, dt\right),$$

with an (effective) generator H.

Here, too, a protocol follows immediately after the definition, because $\hbar$ is a meaningful normalization only if the measurement isolates the reversible part *precisely.* Therefore the front budget is held explicitly fixed in the interference setup, while only $\int db_{\mathrm{rev}}$ is varied.

[18]See FBA Part III: Quantum Kinematics & CPTP Channels, Secs. III.4–III.6 (Unitary Sector from CPTP Structures).

Formula Box VII.3.3.1: Interference protocol (metrological)

1. Prepare two paths γ_1, γ_2 with identical front budget (db_{ext} identical), but different reversible $\int db_{\text{rev}}$.
2. Measure the phase shift $\Delta\varphi$ of the outputs (e.g. Mach–Zehnder, Ramsey).
3. Set $\hbar$ via $\Delta\varphi = \Delta S/\hbar$, where ΔS is reconstructed from the generator of the reversible part.

The final step is again the stability test: in a real implementation, GKLS contributions occur. The FBA claim is not that noise disappears, but that it does not "redefine" the calibration: visibility may drop, but the relation between phase and action remains the same.

Lemma VII.3.3.1: Phase coherence: GKLS damps visibility, not $\hbar$

For a fixed interference protocol, the conversion $d\varphi = dS/\hbar$ remains stable as a calibration relation. Dissipation (GKLS) can reduce visibility/coherence and shift effective generators (e.g. Lamb-shift terms) within S, but it does not replace the conversion constant $\hbar$.

Proof Sketch VII.3.3.1: Proof sketch for $\hbar$ stability

The reversible part of the dynamics is isometric and carries the phase evolution. GKLS terms contract state differences and can damp coherence, but they do not replace the unitary parametrization of the remaining reversible part: they typically change the contrast and (via effective Hamiltonian contributions) the form of S, but not the operational definition of the phase measure via interference. As long as the phase notion is defined operationally through interference protocols and the variation in the protocol is conditioned on the reversible budget component, the protocol fixes the conversion $dS \leftrightarrow d\varphi$ and thus $\hbar$.

For the GKLS structure and Spohn monotonicity, see Part IV.[a]

[a]See FBA Part IV: Dynamics & Measurement (GKLS), Sec. IV.5 (GKLS/Spohn).

VII.3.4 The gravitational coupling G as a geometry and budget calibrator

Unlike c and $\hbar$, G is in the FBA the prototype of a coupling whose *effective* value can become regime-dependent: G calibrates the translation from budget densities and flows to curvature and potential quantities in the continuum limit. In the weak-field regime this calibration is fixed by the Newtonian limit.[19] The reason to include G in this Section is structural: only once c and $\hbar$ are established as stable normalizations can a running of G_{eff} be read as a physical statement rather than as an artifact of inconsistent measure choice.

[19]See FBA Part VI: Gravity & Geometry from Budget Flows, Secs. VI.4–VI.6 (Newton Limit, Tests).

Definition VII.3.4.1: Budget–curvature coupling and G

Let $T^{(B)}_{\mu\nu}$ be the energy–momentum proxy from budget density and budget flows in the continuum limit. Then there exists a coupling κ_B of the form

$$\kappa_B \equiv \frac{8\pi G}{c^4},$$

such that the effective geometric closure takes an Einstein form in the appropriate regime,

$$G^{\text{eff}}_{\mu\nu} = \kappa_B \, T^{(B)}_{\mu\nu}.$$

In the static, weak-field limit this yields the Newtonian limit

$$\nabla^2 \Phi_B = 4\pi G \, \rho^{(B)},$$

where $\rho^{(B)}$ denotes the appropriately calibrated density (budget $\to$ energy density) and Φ_B the corresponding potential.

The definition deliberately ties G to two control points: the continuum closure (Einstein form) and the Newtonian limit (Poisson equation). Thus the normalization is not freely choosable, but anchored in a weak-field, operationally accessible regime. The following protocol makes this anchoring measurable.

Formula Box VII.3.4.1: Gravimetry protocol (metrological)

1. Determine redshift or geodesic deviation in a weak, static field (lab or orbit).
2. Reconstruct Φ_B from clock and front data via α and $\Phi_B = c^2 \ln \alpha$.[a]
3. Calibrate G via $\nabla^2 \Phi_B = 4\pi G \rho^{(B)}$, equivalently via the linearized field closure in the continuum limit.[b]

[a] See FBA Part VI: Gravity & Geometry from Budget Flows, Sec. VI.3.
[b] See FBA Part VI: Gravity & Geometry from Budget Flows, Sec. VI.4.

Why add a proof sketch? Because with G it is easy to fall into the wrong intuition that it is “just a fit parameter”. In the FBA the logic is tighter: once the proxy geometry is fixed, in the weak-field regime only a single normalization remains, and this is fixed by the Newtonian limit.

Proof Sketch VII.3.4.1: Why G is a coupling

The proxy geometry couples inhomogeneities of budget densities and flows to effective curvature quantities. In the continuum limit, this coupling yields—*under the assumptions stated explicitly in Part VI* (local balance, an appropriate continuum description, weak field)—an Einstein-like closure; in the weak field this reduces to a Poisson equation for a potential Φ_B. Thus only the overall normalization remains as a free factor, and it is fixed operationally by the Newtonian limit.

Scale-dependent corrections appear as additional terms in the effective description generated under coarse-graining, and these are precisely the subject of the following RG section.

VII.3.5 Consolidation and outlook towards scales and RG

Before we introduce RG and scale flow formally, we consolidate the three constants once more such that their roles remain clearly separated: $c, \hbar$ as metrological anchors (fixed via protocols), G as a coupling whose effective value may run. This separation is the prerequisite for reading scale flow in the next Section as a physical statement rather than as a change of convention.

Concept balance

- c: front calibrator from minimal external budgets; the front bound is CPTP-stable (no $c_{\text{eff}} > c$).
- $\hbar$: phase calibrator; links reversible budget to S via $d\varphi = dS/\hbar$; GKLS damps visibility, not the conversion.
- G: coupling calibrator between budget curvature and energy-density proxy; the Newtonian limit fixes the normalization, effective corrections can be scale-dependent.

These constants are not inserted, but calibrated conversion and coupling factors of the budget mechanics. Their role in scale flow (fixed points, plateaus, possible running of effective couplings) is formalized in Section VII.4.[a]

[a] See FBA Part V: Spacetime, Light Cones & Local Field Theory, Sec. V.6 (Placement of Local Relativistic EFT Language in the FBA).

The concept balance already fixes what is even allowed to be discussed as a “running” in the RG Section: not the calibration anchors themselves, but effective couplings and parameters that arise from admissible coarse-graining.

Remarks on scale dependence

The FBA distinguishes metrological calibrations (e.g. $c, \hbar$) from effective couplings (e.g. $G_{\text{eff}}(\mu)$ in an EFT reading). While $c, \hbar$ are fixed by protocols and protected against an "artificial" increase by distinguishability monotonicities (DPI) and composition, the effective gravitational coupling (and other field couplings) can carry scale-dependent corrections.

Section VII.4 introduces RG flows as CPTP-compatible coarse-grainings and makes precise when a running is physical and when it is merely a reparametrization of the calibration.

VII.4 Renormalization groups and scale flow in the FBA

Building on Section VII.3, the next step is to make the notion of "scale" in the FBA precise. Once c and $\hbar$ are understood as *metrological* calibrations and G as an *effective* coupling, the question of scale dependence becomes unavoidable: which statements remain invariant under coarse-graining, which quantities are allowed to run, and which monotonicities follow from admissible processing?

In the FBA, renormalization therefore appears not primarily as a computational technique, but as an *admissible dynamical coarse-graining process* on states and (induced) on observables and processes: scale flow is the controlled loss of resolution while preserving (i) budget faithfulness, (ii) locality, and (iii) information monotonicities (DPI/Spohn).[20] [21] In this form, RG becomes a structural consequence of the FBA principles rather than an externally imported tool.[1, 2]

VII.4.1 RG as admissible coarse-graining (CPTP semigroup)

The core is a clean separation between two things: a physically admissible coarse-graining (which respects the allowed operations) and a mere reparametrization (which only renames units or coordinates). The semigroup property encodes the minimal demand that "coarse-grain twice" equals "coarse-grain once more strongly". We state precisely this minimal demand first so that later statements about flows do not depend on a particular formalism.[3, 4]

Definition VII.4.1.1: RG step as admissible coarse-graining

A family $\{R_\ell\}_{\ell\geq 1}$ is called an *RG transformation semigroup* on states if

$$R_{\ell_2} \circ R_{\ell_1} \;=\; R_{\ell_2\ell_1}, \qquad R_1 = \mathrm{id}, \qquad R_\ell \text{ is CPTP and local.}$$

Local means: R_ℓ is built from local block maps (e.g. tensor products/network compositions) and respects no-signalling under composition. *Budget faithfulness* here means: the underlying protocols preserve additive budget relations under composition; trace preservation of R_ℓ is the state normalization (probability balance), not the budget balance.[5, 6]

On observables, R_ℓ acts in the Heisenberg picture via the dual map R_ℓ^* (CP-unital), and on processes/channels via an induced admissible process map (a superchannel) constructed from admissible pre- and post-processing.[4]

An RG step increases the coarse-graining factor ℓ (blocking, binning, convolution) and defines an *effective theory*

$$\mathcal{T}_\ell := R_\ell[\mathcal{T}_1]\,.$$

What matters is what ℓ means here: ℓ is a coarse-graining factor (dimensionless) describing the mapping from fine to coarse. Physical length or energy scales arise only once ℓ is tied to a concrete discretization or a spectral cutoff. This is exactly why the connection to Section VII.3 is crucial: without stable calibrations, a comparison of "the same" scale between two descriptions is not well-defined.

[20] See FBA Part I: FBA – Foundations, Sec. I.5 (DPI, No-Recovery, Admissible Processing).

[21] See FBA Part IV: Dynamics & Measurement (GKLS), Sec. IV.5 (GKLS Semigroup Structure, Spohn Monotonicity).

Next we need a language in which the semigroup becomes a *flow*. This is not extra structure for the sake of mathematics, but the prerequisite for distinguishing parameter running from mere renaming: a generator makes precise what actually changes under coarse-graining. (Here $\ln \ell$ is a *scale parameter*, not a physical time variable; GKLS structures from Part IV are to be read only as an analogy for admissible positivity/continuity requirements.)[7–9]

Formula Box VII.4.1.1: Flow equation and generator

Under a suitable continuity assumption in $\ln \ell$ (e.g. strong continuity), the semigroup can be described by a (possibly non-unique) generator $\mathcal{G}$:

$$\partial_{\ln \ell} R_\ell = \mathcal{G} \circ R_\ell.$$

Working assumption: In finite-dimensional models the technical implementation is straightforward. In infinite-dimensional regimes (algebras/distributions) we assume normality- and continuity-compatible constructions so that the dual action on observables is well-defined and $\mathcal{G}$ can be interpreted on an appropriate domain.

For effective parameters $g_i(\ell)$, extracted via a fixed procedure $\mathrm{Param}(\cdot)$, this yields a *scale flow*

$$\frac{dg_i}{d \ln \ell} = \beta_i(\mathbf{g}), \qquad \mathbf{g}(\ell) = \mathrm{Param}\big(R_\ell[\mathcal{T}_1]\big).$$

The β-functions depend on the microscopic model, on the choice of admissible coarse-graining R_ℓ, and on the parameter extraction $\mathrm{Param}(\cdot)$ (scheme/projection dependence).

Thus RG in the FBA is not tied to a special representation: different admissible R_ℓ yield different flows. Universal statements are precisely those that remain stable for whole classes of admissible coarse-grainings. The next step is therefore to isolate such a universal structure that does not depend on the chosen parametrization. This is provided by DPI.[1, 4]

VII.4.2 DPI ⇒ monotonicities and a scale function $C(\ell)$

A scale flow is physical only if it captures irreversible information loss. This is exactly where DPI enters: coarse-graining must not increase distinguishability. Hence an entire class of monotonicities becomes available automatically, without additional structure.

Lemma VII.4.2.1: DPI monotonicity under RG

Let D be a divergence on states that is contractive under CPTP maps. Then for all $\ell \geq 1$,

$$D\big(R_\ell(\rho) \,\|\, R_\ell(\sigma)\big) \;\leq\; D(\rho \| \sigma)\,.$$

Proof Sketch VII.4.2.1: DPI monotonicity under RG

R_ℓ is CPTP. Contractivity of D means by definition non-increase under any CPTP processing. Applying this to R_ℓ yields the claim.

This fixes the direction, but not yet comparability across ℓ: if one looks at D directly, information loss mixes with trivial size effects (more degrees of freedom per block). We therefore introduce a normalized scale function that enforces a "per block" statement and makes plateaus visible in the first place.

Definition VII.4.2.1: Scale function $C(\ell)$

For a class $\mathcal{F}$ of test state pairs and a volume normalization $V(\ell)$ (number of blocks, i.e. effective degrees of freedom at scale ℓ) we define

$$C(\ell) \;:=\; \frac{1}{V(\ell)} \sup_{(\rho,\sigma)\in\mathcal{F}} D\big(R_\ell(\rho)\,\|\,R_\ell(\sigma)\big)\,.$$

We choose V as an extensive normalization compatible with the block structure (in particular such that "trivial" increases in degrees of freedom are compensated by the division). Then for all $\ell_2, \ell_1 \geq 1$,

$$C(\ell_2\ell_1) \leq C(\ell_1),$$

i.e. $C(\ell)$ is non-increasing in ℓ. If C is differentiable in $\ln \ell$, equivalently $\frac{d}{d\ln\ell}C(\ell) \leq 0$ follows.

Proof Sketch VII.4.2.2: Monotonicity of $C(\ell)$

From Lemma VII.4.2.1 and the semigroup property we obtain $D(R_{\ell_2\ell_1}(\rho)\|R_{\ell_2\ell_1}(\sigma)) \leq D(R_{\ell_1}(\rho)\|R_{\ell_1}(\sigma))$. Choosing $V(\ell)$ as an extensive normalization that correctly tracks the block count, the division compensates the trivial size change of the description. Taking the supremum over $\mathcal{F}$ preserves the direction.

This also makes clear why plateaus can be meaningful: if $C(\ell)$ remains practically constant over a window, then the RG step in that window (relative to $\mathcal{F}$ and V) has not deleted further "distinguishability per block".

Remark VII.4.2.1: Interpretation

$C(\ell)$ measures "effective distinguishability per block" relative to a chosen test class $\mathcal{F}$. Plateaus mark regimes in which the considered diagnostics exhibit hardly any further information loss per block under additional coarse-graining. In this sense, $C(\ell)$ is an FBA-analogue of a C/c scale function, motivated from information monotonicities rather than from a specific path-integral representation.

VII.4.3 Fixed points, linearization, and relevance classes

Once one has a flow, the next step is to classify what remains stable under scale flow. Fixed points are not merely mathematically convenient, but operational: they are regimes in which further coarse-graining does not change new, measurable predictions (up to pure renormalizations/calibrations).

Definition VII.4.3.1: RG fixed point and stability matrix

A vector $\mathbf{g}^\star$ is a fixed point if $\boldsymbol{\beta}(\mathbf{g}^\star) = 0$. The linearization around $\mathbf{g}^\star$ reads

$$\frac{d}{d\ln\ell}\,\delta\mathbf{g} = \mathbf{J}\,\delta\mathbf{g}, \qquad \mathbf{J}_{ij} := \left.\frac{\partial\beta_i}{\partial g_j}\right|_{\mathbf{g}=\mathbf{g}^\star}.$$

Eigen-directions are classified as relevant ($\Re\lambda > 0$), irrelevant ($\Re\lambda < 0$), or marginal ($\Re\lambda = 0$).

Lemma VII.4.3.1: Idempotence ⇒ fixed point

If $R_\ell[\mathcal{T}_{\ell^\star}] = \mathcal{T}_{\ell^\star}$ for all $\ell \geq 1$, then the associated parametrization is constant, i.e. $\mathbf{g}(\ell) \equiv \mathbf{g}^\star$ and $\boldsymbol{\beta}(\mathbf{g}^\star) = 0$.

Proof Sketch VII.4.3.1: Idempotence ⇒ fixed point

Idempotence means: further coarse-graining does not change the effective theory. Hence (for a fixed parameter extraction $\mathrm{Param}(\cdot)$) no parameter drift can occur; therefore $\mathbf{g}(\ell)$ is constant and $\boldsymbol{\beta} = 0$.

The converse ($\boldsymbol{\beta} = 0 \Rightarrow$ "theory unchanged") is a question of completeness of the parametrization: if Param captures all physically relevant directions, fixed-point behavior follows up to pure calibration/rescaling freedoms; otherwise $\boldsymbol{\beta} = 0$ describes only the stagnation of the chosen parameter coordinates.

Corollary VII.4.3.1: Plateau diagnosis via $C(\ell)$

If $C(\ell)$ remains practically constant over a scale window, this is an indication of a universal regime: either the flow is very small in the directions tested by $\mathcal{F}$ ("slow flow"), or the effective description lies close to a fixed-point window.

VII.4.4 Parameter relations from budget and causal constraints

So far RG has been a statement about information loss. For physical theories one additionally needs hard side constraints: budget faithfulness and causal structure. These conditions carve out the space of admissible effective theories and produce parameter relations that cannot be dismissed by "renormalizing differently". In the FBA it is particularly important that fronts, as boundary objects of information propagation, remain protected, because otherwise the metrological role of c would be undermined.

Formula Box VII.4.4.1: Null-flux positivity as an admissibility constraint

As a compact constraint formulation (in the proxy regime) we require: for every null front $\mathcal{N}$ and the budget-side stress-energy proxy $T^{(B)}_{\mu\nu}$,

$$\int_{\mathcal{N}} T^{(B)}_{\mu\nu}\, k^\mu k^\nu \, d\lambda \ \geq 0,$$

where k^μ is the null tangent. This positivity forbids "cost-free" amplification of distinguishability along fronts and thereby constrains admissible effective couplings and flows.

This condition is not introduced here as a new dynamics, but as an explicit *admissibility constraint* in the proxy regime: it summarizes the intuition that front calibration (limiting propagation) and information monotonicities must not be turned into an amplifier channel by effective description tricks.[22] In analogy to GR language, this is a null-flux/null-energy-like positivity requirement on suitable front hypersurfaces.[10, 11] Which concrete form of the constraint is appropriate in a given regime is controlled in the test Sections via observable front/clock and redshift data.

Corollary VII.4.4.1: Metrological invariance vs. coupling running

The calibration constants c and $\hbar$ extracted via fixed protocols are stable under admissible R_ℓ (in particular: no effective description realizes $c_{\mathrm{eff}} > c$, and the phase/action calibration remains protocol-fixed). See Lemmas VII.3.2.1 and VII.3.3.1.[a] [b] By contrast, G, as an effective coupling $G_{\mathrm{eff}}(\mu)$, may exhibit scale running as long as budget and causal constraints (in particular Formula Box VII.4.4.1) are respected.

[a]See FBA Part I: FBA – Foundations, Sec. I.5 (DPI/Monotonicities of Admissible Processing).
[b]See FBA Part IV: Dynamics & Measurement (GKLS), Sec. IV.5 (GKLS/Spohn).

VII.4.5 Practical construction of RG steps

The definitions above are deliberately abstract to keep them representation-independent. For applications, however, one needs a constructive scheme that produces local CPTP coarse-grainings and then extracts parameters. What matters is that each step actually respects the monotonicities; otherwise it is not an admissible RG in the FBA sense.

[22]See FBA Part VI: Gravity & Geometry from Budget Flows, Secs. VI.3–VI.4 (Budget Proxies, $T^{(B)}_{\mu\nu}$).

Algorithm VII.4.5.1: CPTP RG playbook (local, causal, budget-faithful)

1. **Partition and blocking:** Choose a local-causal decomposition into blocks of size ℓ.
2. **Coarse-graining map:** Define a local CPTP map $R_\ell = \bigotimes_{\text{blocks}} \mathcal{R}_\ell$ (trace-preserving on states) and ensure that the underlying protocols respect budget additivity and no-signalling.
3. **Reparametrization:** Determine $\mathbf{g}(\ell)$ by matching effective correlations, response functions, or process-tomography data.
4. **Monotonicity checks:** Verify $C(\ell_2\ell_1) \le C(\ell_1)$ (or $dC/d\ln\ell \le 0$ if differentiable) and stability tests for metrological calibrations (front, phase); see Corollary VII.4.4.1.

VII.4.6 Positioning relative to the standard picture

Comparison with standard RG

Standard RG typically formulates β-functions from rescalings in path-integral or operator language (Wilsonian RG, effective Lagrangians). The FBA starts one level earlier: RG is any admissible CPTP coarse-graining on states, supplemented by the dual action on observables and an induced admissible process map that respects budgets, locality, and DPI.[12–14] This yields the same structural elements (semigroup, fixed points, relevance classes), augmented by two FBA-specific points:

- a scale function $C(\ell)$ from information monotonicities,
- explicit metrological stability (protocol protection of $c, \hbar$) and thus a clean separation between calibration versus coupling running.

The mapping to the usual local QFT language is a regime question and is placed via the local field-theory perspective.[a]

[a]See FBA Part V: Spacetime, Light Cones & Local Field Theory, Sec. V.6 (Local Field-Theory/EFT Perspective in the FBA).

For the remainder of the treatise, only one condensation is still needed: which building blocks are the robust outcomes of this Section that must reappear in test protocols, independent of which concrete RG scheme is used later?

Takeaways on RG in the FBA

- RG = admissible CPTP coarse-graining (on states) with the semigroup property, local and compatible with budget and causal structure; dual/induced action on observables/processes.
- Scale flow via β-functions from a generator $\mathcal{G}$; fixed points via $\boldsymbol{\beta} = 0$; relevance classes from the linearization $\mathbf{J}$.
- The scale function $C(\ell)$ is non-increasing and measures distinguishability per block relative to $\mathcal{F}$; plateaus diagnose universal or "slow-flow" regimes.
- Metrological calibrations $(c, \hbar)$ are protocol-fixed; effective couplings such as $G_{\text{eff}}(\mu)$ may run, but only under budget and causal constraints.

With this RG reading, the transition to the test part is prepared: in Section VII.5 we formulate parameter relations and concrete signatures that follow from fixed-point proximity, monotonicities, and admissible runnings.

VII.5 Predictions and testability

This Section bundles *falsifiable* FBA statements about constants, scales, and RG flows and deliberately provides lean protocols. The guiding idea is simple: if constants in the FBA are read as calibration and coupling results, then (i) the metrological calibrations must remain stable under admissible processing and (ii) any genuine scale dependence must be describable as an effect of admissible coarse-graining. Both are structurally driven by budget faithfulness, composition, and DPI/Spohn.[23] [24] The metrological fixations themselves are the front and interference calibrations from Subsections VII.3.2 and VII.3.3.[25] [26] [27] Gravitational calibrations are developed in Part VI and are referenced here only as a test channel.[28]

VII.5.1 Prediction classes and invariants

The most important pass/fail dividing line is between *metrological stability (protocol protection)* and *effective coupling running.* The former must not break under admissible coarse-graining, because it is what makes measurements comparable across scales in the first place. The latter is exactly what RG describes, but it must remain within budget and causal constraints. The following box summarizes this separation so that it can be read directly as a test plan: what *must* remain protocol-fixed, and what *may* run only in a strictly controlled way?

[23] See FBA Part I: FBA – Foundations, Sec. I.5 "DPI Arrow & No-Recovery" and Sec. I.6 "Composition, Locality & No-Signalling".

[24] See FBA Part IV: Dynamics & Measurement (GKLS), Sec. IV.5 "Spohn Monotonicity".

[25] See FBA Part II: Time, Proper Time & Minkowski Geometry, Sec. II.5 "Front/Time Calibration in the Minkowski Regime".

[26] See FBA Part III: Quantum Kinematics & CPTP Channels, Secs. III.4–III.5 "Interference/Phase Calibration".

[27] See FBA Part IV: Dynamics & Measurement (GKLS), Secs. IV.4–IV.5 "Measurement/Coarse-Graining, GKLS".

[28] See FBA Part VI: Gravity & Geometry from Budget Flows, Secs. VI.3–VI.4 "Calibration, Proxy Regime".

Definition VII.5.1.1: Prediction classes (operational)

We distinguish four classes:

1. **Metrological invariants (protocol protection):** Re-extracted calibrations $c_{\text{eff}}(\ell)$, $\hbar_{\text{eff}}(\ell)$ from fixed front and interference protocols must not be "artificially" increased or redefined by admissible CPTP coarse-graining (in particular no $c_{\text{eff}}(\ell) > c$); deviations beyond controlled systematics are a fail.[a] [b] [c]
2. **RG monotonicities:** For every admissible divergence D, $D(R_\ell(\rho)\|R_\ell(\sigma))$ is non-increasing; this implies a scale function $C(\ell)$ which (with a suitable extensive normalization) is monotonically non-increasing (see Section VII.4).[1, 4][d] [e]
3. **Gravitational calibration:** G is calibrated via weak-field limits (redshift, geodesic deviation) and may be scale-dependent as an effective coupling, but *only* under budget and causal constraints.[f]
4. **Causal and null-flux constraints:** Null-flux positivity along fronts constrains admissible couplings and flows and prevents "cost-free" information gain via effective descriptions; see Formula Box VII.4.4.1 as well as the front/causal anchors in Part II and Part V.[g] [h]

[a] See FBA Part II: Time, Proper Time & Minkowski Geometry, Sec. II.5.
[b] See FBA Part III: Quantum Kinematics & CPTP Channels, Secs. III.4–III.5.
[c] See FBA Part IV: Dynamics & Measurement (GKLS), Secs. IV.4–IV.5.
[d] See FBA Part I: FBA – Foundations, Sec. I.5.
[e] See FBA Part IV: Dynamics & Measurement (GKLS), Sec. IV.5.
[f] See FBA Part VI: Gravity & Geometry from Budget Flows, Secs. VI.4–VI.6.
[g] See FBA Part II: Time, Proper Time & Minkowski Geometry, Sec. II.5.
[h] See FBA Part V: Spacetime, Light Cones & Local Field Theory, Sec. V.4.

From this classification it follows how falsifiability is sensibly operationalized here: not via a single "signature experiment", but via a set of invariants and monotonicities that must hold simultaneously across very different implementations. This makes the real risk points visible: as soon as an experiment indicates an apparent running in c or $\hbar$, one must first clarify whether an actually admissible protocol (local, CPTP-modelable, symmetrically controlled) was implemented—or whether an uncontrolled reparametrization was mistaken for a "physical effect".

The following inequalities are chosen so that they can be read directly as measurement criteria: any clear violation is either a protocol error (not admissible, not symmetric, not local) or—if admissibility and systematics are truly controlled—a hard contradiction to the FBA core.

Formula Box VII.5.1.1: Falsifiable invariants & inequalities

With R_ℓ an admissible RG step on states (cf. Definition VII.4.1.1) and volume normalization $V(\ell)$, we have:

$$\begin{aligned}
\text{(I) Front protection:}\quad & \frac{d\ell}{dt} \le c, \qquad d\ell = c\,dt \;\Leftrightarrow\; \text{saturation}, \\
& c_{\mathrm{eff}}(\ell) \le c. \\
\text{(II) Phase calibration:}\quad & d\varphi = \frac{dS}{\hbar}, \\
& \hbar_{\mathrm{eff}}(\ell) \approx \hbar. \\
\text{(III) DPI monotonicity:}\quad & D\big(R_\ell(\rho)\,\|\,R_\ell(\sigma)\big) \le D(\rho\|\sigma). \\
\text{(IV) Scale function:}\quad & C(\ell) = \frac{1}{V(\ell)} \sup_{(\rho,\sigma)\in\mathcal{F}} D\big(R_\ell(\rho)\,\|\,R_\ell(\sigma)\big), \\
& C(\ell_2\ell_1) \le C(\ell_1). \\
\text{(V) Null-flux constraint:}\quad & \int_{\mathcal{N}} T^{(B)}_{\mu\nu} k^\mu k^\nu \, d\lambda \ge 0.
\end{aligned}$$

Here k^μ is a null tangent (front generator). Under admissible coarse-graining, in particular nothing "faster than the front" may occur (front protection), and GKLS may damp visibility but must not redefine the phase/action conversion. If C is differentiable in $\ln\ell$, then (IV) is equivalent to $\frac{d}{d\ln\ell}C(\ell) \le 0$.

Thus Formula Box VII.5.1.1 provides a "minimal contract" between theory and measurement: (I) and (II) fix what must not physically "run" as calibration, (III) and (IV) fix what RG must look like as information loss, and (V) ties possible runnings of effective couplings to hard causal conditions. This also clarifies why we do not jump to gravimetry immediately: without passing the metrological checks, any claimed coupling running would be inherently ambiguous.

VII.5.2 Test protocols (lab, simulation, astrophysics)

The test protocols are deliberately redundant: metrological invariants are tested under controlled admissible perturbations, RG monotonicities are checked as a direct consequence of CPTP coarse-graining, and gravitational calibrations serve as a coupling test, not as a replacement for the metrological checks. Redundancy is not a luxury here but the method for separating protocol artifacts from genuine physics.

Algorithm VII.5.2.1: CPTP test suite for constants & RG

1. **Front-c (lab, metrological):** Realize a minimal signal via the front protocol and vary the environment by symmetric, local GKLS dephasing on subsegments. *Expectation:* no increase of the extracted front bound ($c_{\text{eff}}(\ell) \leq c$); saturation cases remain null-like.[a] [b]
2. $\hbar$ **phase stability (interference):** Mach–Zehnder or Ramsey with controlled, path-symmetric noise (CPTP). *Expectation:* visibility $\downarrow$, but the conversion remains protocol-fixed ($\hbar_{\text{eff}}(\ell) \approx \hbar$) and phases are normalized by $\Delta\varphi = \Delta S/\hbar$.[c] [d]
3. **RG monotonicity $C(\ell)$ (simulation or quantum processor):** Prepare ρ, σ, implement block-local R_ℓ (tomography or shadowing), and compute a contractive divergence D (e.g. relative entropy). *Expectation:* $C(\ell_2\ell_1) \leq C(\ell_1)$; plateaus diagnose "slow-flow" or fixed-point windows relative to the test class.[1, 3, 4]
4. **Gravimetry G (lab or orbit):** Compare gravitational redshift or geodesic deviation with the Newtonian limit $\nabla^2\Phi = 4\pi G\rho$. *Expectation:* calibrated G; possible runnings only within null-flux and causal bounds.[e]
5. **Plateau detection (EFT windows):** Measure response functions or correlations across ℓ and identify scale windows with practically constant $C(\ell)$ (or $dC/d\ln\ell \approx 0$ if differentiable). *Expectation:* universal exponents and indicators stable across the plateau.[f]

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[a] See FBA Part II: Time, Proper Time & Minkowski Geometry, Sec. II.5 "Front Protocol/Front Protection".
[b] See FBA Part IV: Dynamics & Measurement (GKLS), Sec. IV.5 "Admissible GKLS Perturbations".
[c] See FBA Part III: Quantum Kinematics & CPTP Channels, Sec. III.5 "Interference/Phase Calibration".
[d] See FBA Part IV: Dynamics & Measurement (GKLS), Secs. IV.4–IV.5 "Measurement as Coarse-Graining, GKLS Effects".
[e] See FBA Part VI: Gravity & Geometry from Budget Flows, Secs. VI.3–VI.5 "Calibration, Continuum Limit, Tests".
[f] See FBA Part V: Spacetime, Light Cones & Local Field Theory, Sec. V.6 "Local Field-Theory/EFT Placement".

The test suite is ordered to enable unambiguous diagnosis: first the calibration anchors are tested, then the RG monotonicities, and only then coupling questions (gravimetry, EFT windows). This creates a hierarchy in which later findings do not rest on an earlier unresolved convention problem.

VII.5.3 Parameter relations and measurement windows

The protocols above provide not only individual checks but also parameter relations: metrological invariants must remain stable across all admissible implementations. For effective couplings, a running is acceptable only if it remains compatible with the causal and budget constraints. Practically, this means one obtains not only "yes/no" statements, but quantitative bound windows that can be used directly as constraints in model comparisons.

Formula Box VII.5.3.1: Bounds from budget & causality

The invariants from Formula Box VII.5.1.1 can be formulated operationally as bounds:

$$c_{\text{eff}}(\ell) \le c + \varepsilon_c, \qquad |\hbar_{\text{eff}}(\ell) - \hbar| \le \varepsilon_\hbar, \qquad C(\ell_2\ell_1) \le C(\ell_1), \qquad \int_{\mathcal{N}} T^{(B)}_{\mu\nu} k^\mu k^\nu \, d\lambda \ge 0.$$

An observed $C(\ell_2\ell_1) > C(\ell_1)$ or a robust violation of front/phase protection (beyond controlled ε-systematics) contradicts the FBA core. For $G_{\text{eff}}(\mu)$, runnings are admissible only if Formula Box VII.4.4.1 and the front-side causal constraints are respected.[a]

[a] See FBA Part VI: Gravity & Geometry from Budget Flows, Sec. VI.4 "Gravitational Closure, Interpretation in the Continuum Limit".

The role of the ε parameters is purely operational: they encode how well a concrete setup actually controls admissibility (symmetry, locality, CPTP modeling). In the pass/fail sense, the point is not that every lab immediately makes ε extremely small, but that a claimed effect is no longer compatible with admissible systematics.

Corollary VII.5.3.1: Minimal pass/fail catalogue

1. Front and phase protection pass ⇒ metrological invariants are protocol-compatible.
2. $C(\ell)$ monotonically non-increasing (relative to $\mathcal{F}, V$) ⇒ RG structure compatible.
3. Gravimetry consistent with calibration ⇒ G normalized; admissible runnings are bounded by constraints.
4. Any clear violation of (i)–(iii) ⇒ FBA rejected *or* the employed protocol was not admissible (systematics/admissibility breakdown).

The catalogue is deliberately minimal: it contains only points that cannot be talked away by "better modeling". Everything beyond that is fine structure—important for precision, but not necessary to decide the core as consistent or inconsistent.

VII.5.4 Positioning and data cross-checks

Comparison, cross-checks & bridge

The tests connect to the formal core statements from Sections VII.2 and VII.4 and form the operational bridge to the two regime languages already developed in the series: local QFT/RG and effective gravitation.[a] [b] A document-spanning review of the measurement programs and bridge theorems is given in Part X.[c]

[a] See FBA Part V: Spacetime, Light Cones & Local Field Theory, Sec. V.6 "Placement within Local Relativistic QFT/EFT".

[b] See FBA Part VI: Gravity & Geometry from Budget Flows, Secs. VI.4–VI.5 "Gravitational Closure, Weak Limits".

[c] See FBA Part X: Predictions, Falsifiability & Bridge FBA → QM ↔ GR, Secs. X.1–X.3.

This makes the status of this Section clear: it provides no additional model building, but a test logic that follows from the previous Sections. The following summary therefore does not condense "content" but marks the points that must reappear in every implementation that appeals to the FBA.

Takeaways: tests in the FBA

- **Invariants:** front and phase protection (protocol-fixed extraction of $c, \hbar$); any robust violation is a hard fail.
- **Monotonicities:** $C(\ell)$ monotonically non-increasing; plateaus diagnose "slow-flow" or fixed-point windows relative to the test class.
- **Gravitation:** G is calibrated in the weak field; possible runnings only under strict causal and budget bounds.
- **Playbook:** front, interference, RG, and gravimetry protocols provide a minimal but complete pass/fail suite.

VII.6 Conclusion and outlook

This treatise has anchored constants, scales, and renormalization in the FBA operationally: c and $\hbar$ appear as *metrological calibration constants* (front calibration and, respectively, phase/action calibration), while G appears as a *geometry and budget coupling* in the continuum limit. Renormalization is understood as *admissible CPTP coarse-graining.* Under mild regularity and parametrization assumptions (generator/extraction), one can formulate β-flows, fixed points, and relevance classes. An information-driven scale function $C(\ell)$ is *monotone* along coarse-graining (semigroup order); the frequently used differential form $\frac{d}{d\ln\ell}C(\ell) \le 0$ is to be read as shorthand only in a smooth limit. Causal and budget constraints (in particular null-flux along fronts) provide hard bounds for admissible effective couplings.[29] [30] [31] [32]

The point of this conclusion is not merely to repeat results, but to mark the derivation chain as a *stable core logic*: which statements depend only on admissibility and calibration and are therefore particularly robust, and which statements depend on the choice of a concrete RG scheme and are therefore to be read as a work program. Exactly this dividing line is made explicit in the following boxes.

Core statements (condensed)

1. **Calibration rather than postulate:** $c, \hbar$ are *protocol-fixed* (front and, respectively, interference calibration); G normalizes budget curvature $\leftrightarrow$ stress/energy in the appropriate continuum limit (Section VII.3).
2. **RG = admissible coarse-graining:** RG steps are local and CPTP (hence DPI-contractive); under additional assumptions (generator/extraction) this yields the usual β-language, fixed points, and relevance classes (Section VII.4).
3. **Information monotonicity:** The scale function $C(\ell)$ is *non-increasing* along coarse-graining (semigroup order). Plateaus are *diagnostic indicators* for universal windows (proximity to fixed points) and depend on the test family/normalization (Section VII.4).
4. **Protection vs. running:** $c, \hbar$ are metrologically *protected*: no admissible coarse-graining may produce an effective violation of the front bound ($c_{\text{eff}} > c$) or a re-calibration of the phase/action relation; realistically one expects only contrast/-precision loss. $G_{\text{eff}}(\mu)$ may run as a coupling, provided null-flux and front bounds are respected (Section VII.5).
5. **Pass/fail tests:** Front, interference, RG, and gravimetry protocols yield a minimally sufficient suite for falsification (Section VII.5).[a]

[a] See FBA Part X: Predictions, Falsifiability & Bridge FBA $\to$ QM $\leftrightarrow$ GR, Secs. X.1–X.3 (Bridge Theorems, Measurement Programs, Pass/Fail Framework).

[29] See FBA Part II: Time, Proper Time & Minkowski Geometry, Secs. II.5 and II.8 (Basic Calibrations, Front Protection).

[30] See FBA Part IV: Dynamics & Measurement (GKLS), Sec. IV.5 (Dynamics, Spohn Monotonicity).

[31] See FBA Part V: Spacetime, Light Cones & Local Field Theory, Sec. V.6 (Field-Theoretic Placement).

[32] See FBA Part VI: Gravity & Geometry from Budget Flows, Secs. VI.3–VI.4 (Geometry from Budget Flows).

From these core statements it follows immediately what a sensible research program must look like: it should start where freedom of choice remains (RG schemes, parameter extraction), yet be formulated so that the metrological anchors remain hard consistency conditions. The next package therefore collects precisely the tasks that become *necessary* once one wants to move from plausibility to quantitative parameter relations.

Research program & next steps

1. **Micro → β:** Derive explicit β-functions from channel-specific CPTP RG steps (blocking, convolution) and compare with known QFT flows.[a]
2. **Measurable $C(\ell)$:** Protocols for $C(\ell)$ on quantum processors (shadowing, robust divergence estimators) and in classical lattice simulations (Section VII.4).
3. **Front and phase metrology:** Precision checks of *front/phase protection* of $c, \hbar$ under deliberately introduced, local GKLS noise (admissible, symmetric, local).[b]
4. **Gravity windows:** Determine possible $G_{\text{eff}}(\mu)$ runnings in weak fields (lab, orbit) under null-flux bounds and tie them to the geometry construction.[c]
5. **Bridge components:** Systematic mapping FBA ↔ EFT (operator schemes, scheme-robust indicators) and cross-checks against universality.[d]

[a] See FBA Part V: Spacetime, Light Cones & Local Field Theory, Sec. V.6 (Mapping to Local Relativistic QFT/Standard RG).
[b] See FBA Part IV: Dynamics & Measurement (GKLS), Sec. IV.5 (GKLS Semigroup Structure, Spohn, Admissible Perturbation Classes).
[c] See FBA Part VI: Gravity & Geometry from Budget Flows, Secs. VI.3–VI.4 (Proxy Geometry, Continuum Limit, Calibration).
[d] See FBA Part V: Spacetime, Light Cones & Local Field Theory, Sec. V.6 (Field-Theoretic Placement, Universality/Fixed Points).

To prevent this program from quietly turning into a mere collection of model-building exercises, the central vulnerability points at which spurious effects can arise must remain visible. In particular, Part VII is prone to two typical confusions: scale dependence as physical running versus scale dependence as a consequence of poorly controlled calibration, and scheme-dependent β-curves versus scheme-robust statements. The next box serves precisely this protective function.

Limits, assumptions & risks

- **Protocol dependence:** Calibrations are operationally defined; sloppy protocols can generate apparent runnings in $c, \hbar$ (spurious effects).
- **RG scheme:** β-flows depend on the chosen CPTP coarse-graining and parameter extraction; scheme-robust statements rest on monotonicities/endpoints (e.g. $C(\ell)$-based diagnostics).
- **Continuum limit:** The map budget $\mapsto$ curvature proxy is a limit statement; corrections (e.g. $\mathcal{O}(\ell^2)$ in a concrete discretization) must be quantified.[a]

[a]See FBA Part VI: Gravity & Geometry from Budget Flows, Sec. VI.4 (Continuum Closure, Correction Terms).

Precisely because these risks are real, it is helpful to phrase the steps toward a "clean result" as a controllable playbook. The following roadmap is therefore structured so that each step either tests an invariant, checks a monotonicity, or calibrates a coupling in a limiting regime.

Algorithm VII.6.1: Roadmap (playbook, brief)

1. **Define** local R_ℓ (CPTP) and verify in the concrete implementation contractivity of chosen divergences (DPI check) as well as Spohn monotonicity in the GKLS regime.[a]
2. **Determine** $\mathbf{g}(\ell)$, $\boldsymbol{\beta}(\mathbf{g})$, $C(\ell)$; test plateaus as fixed-point diagnostics (Section VII.4).
3. **Calibrate** $c, \hbar$ via front and interference protocols; test front/phase protection under admissible perturbations (Section VII.3).
4. **Calibrate** G in the Newtonian limit and evaluate possible runnings under null-flux bounds.[b]
5. **Decide** pass/fail based on invariants and monotonicities (Section VII.5).[c]

[a]See FBA Part IV: Dynamics & Measurement (GKLS), Sec. IV.5.
[b]See FBA Part VI: Gravity & Geometry from Budget Flows, Sec. VI.4.
[c]See FBA Part X: Predictions, Falsifiability & Bridge FBA $\to$ QM $\leftrightarrow$ GR, Secs. X.1–X.3.

The outlook can thus be stated succinctly: Part VII introduces no additional dynamical assumption, but a robust scale architecture. This architecture is the link that later brings cosmological regime changes, local field-theory effects, and gravitational calibrations together under a common pass/fail framework.

Outlook (compact)

The building blocks developed here enable (i) a metrologically clean separation of invariants and effective runnings, (ii) an information-driven RG framework with a monotone scale function, and (iii) a controlled coupling of budget flows to effective geometry. Next milestones are channel-sharp β-analyses, experimental $C(\ell)$ measurements, and precise gravimetry bounds.[a]

[a] See FBA Part X: Predictions, Falsifiability & Bridge FBA $\rightarrow$ QM $\leftrightarrow$ GR, Secs. X.1–X.3 (Synthesis, Data Cross-Checks, Measurement Programs).

VII.7 Appendix: Overview of the FBA Series (Parts I–X)

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1. ***Part I*: FBA-Foundations: Ordering, Budget, Proper Time & Arrows.** *Goal:* Provide the base layer: ordering, budget, proper time/aging, front and the operational arrow of time (DPI); Minkowski limit from the budget quadric; admissible dynamics and locality/no-signalling. *Import:* – (reference for all subsequent parts). *Extension:* interface contracts, pass/fail checklists, reading guide.
2. ***Part II*: Time, Proper Time & Minkowski Geometry.** *Goal:* Capture proper time/quadric operationally and derive geodesics. *Import:* foundations (ordering, budget, proper time, front/DPI). *Extension:* smooth limit, variational principle on worldlines, calibration κ_τ.
3. ***Part III*: Quantum Kinematics & CPTP Channels.** *Goal:* State spaces and channels (CPTP) including composition. *Import:* foundations (budget, channel viewpoint, composition). *Extension:* concrete divergences/cost functionals $\mathcal{C}$, measurements, and classical registers.
4. ***Part IV*: Dynamics, Measurement & GKLS (Open Systems).** *Goal:* Continuous open dynamics (GKLS) and the operational arrow of time. *Import:* channels/DPI. *Extension:* Spohn monotonicity, stationary/NESS references, flows $b^{\mathrm{rev}}, b^{\mathrm{irr}}, b^{\mathrm{ext}}$.
5. ***Part V*: Spacetime, Light Cones & Local Field Theory.** *Goal:* Local field equations under front/locality. *Import:* front, composition, no-signalling. *Extension:* local GKLS generators, Lieb–Robinson-type bounds, effective light cones.
6. ***Part VI*: Gravity & Geometry from Budget Flows.** *Goal:* Geometrization of budget flows. *Import:* budget quadric/proper time. *Extension:* effective metrics from calibrations (κ_t, κ_x) and internal stresses; coupling to curvature.
7. ***Part VII*: Constants, Scales & Renormalization.** *Goal:* Scale running of the calibration theorems. *Import:* $c = \kappa_t/\kappa_x$, κ_τ. *Extension:* flow equations for $\kappa_t, \kappa_x, \kappa_\tau$; stability of c.
8. ***Part VIII*: Classical Limit, Thermodynamics & Aging.** *Goal:* Macroscopic behavior from $A[\gamma]$ (aging) and DPI. *Import:* proper time/aging, Spohn. *Extension:* entropy production, Euler–Lagrange forms for irreversible flows, effective transport equations.
9. ***Part IX*: Cosmic Dynamics, Time Dilation & Inflation (TDI).** *Goal:* Cosmic ordering & calibration flow. *Import:* budget, proper time/front. *Extension:* budget equations on large-scale slices; time-dilation inflation as calibration dynamics.
10. ***Part X*: Predictions, Falsifiability & Bridge FBA → QM ↔ GR.** *Goal:* Testable differences and bridges FBA ↔ QM/GR. *Import:* all foundational building blocks. *Extension:* protocols, limiting-case tests, overdetermined consistency relations (pass/fail).

All parts of the FBA series are available in both English and German at

https://www.frame-budget-approach.eu

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