The Frame–Budget Approach (FBA)

How time, dynamics, and geometry emerge from budget flows

An operational bridge between quantum mechanics and general relativity

Part VI: Gravity & Geometry from Budget Flows

Dipl. Wirt.-Inf. Jens Tetzner

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Part VI

Gravity & Geometry from Budget Flows

VI.1 Introduction & Target Picture

VI.1.1 Motivation

Up to this point, the Frame–Budget Approach (FBA)[1] has established a reference stage on which time becomes measurable as an order and proper time as an integrated internal budget flow, and on which the causal structure condenses, via front calibration, into a flat Minkowski limit.[2] [3] The next methodological step is then not to "add gravity as an extra assumption", but to clarify what happens as soon as this reference assumption of homogeneity breaks operationally: If budget densities and budget flows vary spatially or systematically, calibrations become effectively location- and state-dependent, and precisely thereby clock rates, free-fall motion, and light propagation no longer appear as trivial Minkowski transport, but as geometrically distorted quantities.

In the standard picture of GR, this distortion is postulated as curvature of spacetime. In the FBA, we want to obtain the same phenomenological structure from the bookkeeping itself: from the distribution of budget, its gradients, and the resulting effective calibration statements.

The *target picture* of this treatise is therefore a continuous, operational derivation path from budget flows to (i) an effective geometric description (curvature proxy), (ii) geodesics as free fall, (iii) gravitational time dilation, and (iv) a continuum limit in which Einstein-like field equations crystallize as an approximate language.

VI.1.2 Logic path

The order is chosen so that each new object is introduced only once it is clear which operational necessity it serves and which observable it fixes:

1. *From the reference limit to the source:* First we identify which nontrivial but measurable deviations from the flat reference limit can count as "gravitational information" at all. This leads to a curvature proxy constructed from budget density and budget gradients.

2. *From the source to motion:* Only once such a geometric effect is fixed precisely does it make sense to define free motion. Geodesics then appear as the motion that remains compatible with budget-faithful update logic, rather than as a separately postulated "law of inertia".

3. *From motion to clock rate:* With geodesics as reference paths, time dilation becomes a comparison statement between clocks that integrate different budget environments. Thus dilation is not introduced as an extra rule, but as a consequence of the same bookkeeping that defines proper time in the first place.

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[1] An overview of all parts of the FBA treatise, including download links, can be found in Section VI.8 of this document.

[2] See FBA Part II: Time, Proper Time & Minkowski Geometry, Secs. II.5–II.9.

[3] See FBA Part V: Spacetime, Light Cones & Local Field Theory, Secs. V.3–V.6.

4. *Closure in the continuum:* To go beyond individual paths, we need a field description. The continuum limit bundles the budget-induced geometry effects into effective equations that, in suitable regimes, mimic the form of the Einstein field equations.

5. *Limiting cases as diagnostics:* Horizons and redshift occur where the dilation factor or the curvature proxy exhibits extreme behavior. These limiting cases are not mere "folklore", but provide hard, observable diagnostics of the proxies introduced above.

VI.1.3 Scope and delimitations

We work on the flat Minkowski reference limit as a kinematic stage and treat "curvature" first as an *effective* geometry derived from inhomogeneous budget flows and location-dependent calibrations, not as a fundamentally presupposed spacetime metric.[4] The dynamics remains CPTP/GKLS-conform; locality and no-signalling are taken over as already secured consistency requirements.[5] [6] Questions of scale handling, renormalization, and cosmic large-scale dynamics are deliberately outsourced, because they involve additional, regime-dependent calibration flows.[7] [8]

VI.1.4 Contribution relative to standard relativity theory

The FBA does not replace GR by an alternative "geometry postulation", but attempts to reconstruct geometric language as a *condensation* of operational budget and calibration statements: Gravity becomes readable as emergent, measurement-protocol-bound geometry, whose central effects (time dilation, free fall, redshift) are directly coupled to budget proxies. This not only makes GR-proximity in the appropriate regime explainable, but simultaneously yields additional consistency and pass/fail criteria by which deviations can be localized precisely.[9]

VI.1.5 Reading guide

- **Section VI.2 - Foundations & Conventions** bundles the imported foundations so that the gravitational derivation proceeds without circularity: We use only building blocks that are already fixed operationally, thereby marking clearly what is *not* newly assumed here.

- **Section VI.3 - Connections between budget flows and geometry** is the conceptual core: Here the transition from budget flows to a geometric proxy is carried out, because only then can "gravity" be formulated as an unambiguously identifiable deviation from the reference limit.

- **Section VI.4 - Continuum limit and field equations** provides the necessary closure: Individual phenomena are condensed into a field description so that the comparison to GR rests not on analogies but on equation structure in the continuum limit.

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[4]See FBA Part II: Time, Proper Time & Minkowski Geometry, Secs. II.10–II.12.
[5]See FBA Part IV: Dynamics & Measurement (GKLS), Secs. IV.3–IV.7.
[6]See FBA Part V: Spacetime, Light Cones & Local Field Theory, Secs. V.4–V.6.
[7]See FBA Part VII: Constants, Scales & Renormalization, Secs. VII.1–VII.4.
[8]See FBA Part IX: Cosmic Dynamics, Time Dilation & Inflation (TDI), Secs. IX.2–IX.6.
[9]See FBA Part X: Predictions, Falsifiability & Bridge FBA $\rightarrow$ QM $\leftrightarrow$ GR, Secs. X.4–X.8.

- **Section VI.5 - Horizons and redshift** treats horizons and redshift as limiting cases of the same proxy structure, because these extreme regimes deliver the sharpest observable consequences.
- **Section VI.6 - Tests and predictions** translates the relations obtained above into test signatures and comparison protocols, so that the approach does not stop at plausibility but becomes falsifiable.
- **Section VI.7 - Summary and outlook** summarizes the derivation chain compactly and shows which extensions are next required within the series framework (scale handling, cosmology, bridge tests).

VI.2 Preliminary Foundations & Conventions (Import from Part I: FBA – Foundations)

Why an import? In Part VI we do not want to assume gravity as an additional geometric structure, but to reconstruct it as a controlled deviation from the flat reference limit. To ensure that this reconstruction does not contain unnoticed circularities, sequence, balance, calibration, proper time, and admissible dynamics under composition must already be fixed. This is exactly what Part I provides as the foundation.[10]

In this sense we import the foundation from Part I in full.[11] The following box lists the building blocks that appear *explicitly* as tools in Part VI, so that it remains clear later which statements appear here as consequences rather than as concealed redefinitions.

[10] See FBA Part I: FBA – Foundations, Secs. I.2–I.6.
[11] See FBA Part I: FBA – Foundations, Secs. I.2–I.6.

Imported building blocks (unchanged)

We adopt the following building blocks *without* redefinition from the FBA – Foundations and refer *in the text* to section/box and heading/title:

- **Sequence of global states & minimal events:** *FBA – Foundations*, Sec. I.2 "Global states, frame sequence, and minimal event (ME)"; *FBA – Foundations*, Section/Box I.2 "Co-actuality and refinement invariance".
- **Difference function & operational minimal difference:** *FBA – Foundations*, Box I.2 "Difference function & operational minimal difference".
- **Budget calculus (internal/external/irreversible) & balance:** *FBA – Foundations*, Section/Box I.3 "One-step budget & decomposition"; *FBA – Foundations*, formula box I.3 "Balance equations"; *FBA – Foundations*, Lemma I.3 "Refinement invariance of the balance".
- **External calibration & front:** *FBA – Foundations*, Definition I.3 "Calibration and front costs"; *FBA – Foundations*, Lemma I.3 "Front bound"; *FBA – Foundations*, Corollary I.3 "Signal front".
- **Proper time & aging, Minkowski limit:** *FBA – Foundations*, Definition I.4 "Proper time (proper time)"; *FBA – Foundations*, formula box I.4 "Properties of proper time"; *FBA – Foundations*, Definition I.4 "Aging (irreversible)"; *FBA – Foundations*, formula box I.4 "Minkowski limit & quadric"; *FBA – Foundations*, Lemma I.4 "Time dilation".
- **Admissible dynamics (CPTP/GKLS), DPI/Spohn:** *FBA – Foundations*, Definition I.5 "Admissible channels (CPTP)"; *FBA – Foundations*, Formula I.5 "Kraus/Stinespring"; *FBA – Foundations*, Lemma I.5 "Measurement as CPTP"; *FBA – Foundations*, Definition I.5 "GKLS generators (open systems)"; *FBA – Foundations*, Formula I.5 "Spohn monotonicity"; *FBA – Foundations*, Lemma I.5 "Semigroup budget"; *FBA – Foundations*, Definition/Corollary I.5 "DPI arrow & no-recovery".
- **Composition, locality & no-signalling:** *FBA – Foundations*, Definition I.6 "Symmetric-monoidal structure"; *FBA – Foundations*, Formula I.6 "Budget additivity"; *FBA – Foundations*, Lemma I.6 "No-wire inflation & local operations"; *FBA – Foundations*, Corollary I.6 "Light cones & local GKLS".

This fixes what is operationally admissible in Part VI and how reference causality is calibrated.[12] Next we fix the notation so that the later constructions (curvature proxy, geodesics, time dilation) do not hinge on symbol choices or mixtures of discrete and limit statements, but on unambiguously identifiable budget quantities.

[12] See FBA Part I: FBA – Foundations, Secs. I.2–I.6.

Notation & Conventions

- **Discrete vs. continuum:** Step index $n \in \mathbb{Z}$ for successive frames. $\Delta(\cdot)$ denotes discrete step increments (e.g. Δb_n), $d(\cdot)$ differential quantities in the continuum limit.

- **Budget decomposition (step form → path form):** Per step Δb_n^{int}, Δb_n^{ext}, Δb_n^{irr} (internal/external/irreversible). Along a worldline Γ:

$$B^{(\cdot)}(\Gamma) = \sum_n \Delta b_n^{(\cdot)} \qquad \text{or in the limit} \qquad B^{(\cdot)}(\Gamma) = \int_\Gamma db^{(\cdot)}.$$

  Irreversibility: $db^{\text{irr}} \geq 0$.

- **Proper-time/aging calibration (import from Part I):** κ_τ is the (metrologically fixed) time calibration for internal/irreversible budget contributions. In the limit we define

$$d\tau_{\text{geo}} \equiv \frac{db^{\text{int}}}{\kappa_\tau}, \qquad dA \equiv \frac{db^{\text{irr}}}{\kappa_\tau} \geq 0, \qquad d\tau_{\text{tot}} = d\tau_{\text{geo}} + dA.$$

  In the reversible limit $db^{\text{irr}} = 0$ and hence $\tau_{\text{tot}} = \tau_{\text{geo}}$.[a]

- **Calibration of the front speed:** κ_t and κ_x are the external calibrations of the front protocol; the fastest admissible front defines

$$c := \frac{\kappa_t}{\kappa_x},$$

  i.e. c is not postulated but metrologically fixed via the defined front protocol. (*No* c=1 units in this treatise.)[b]

- **Spacetime language (flat, kinematic):** Four-vector $x^\mu = (ct, x, y, z)$; Minkowski signature $\eta = \text{diag}(-1, 1, 1, 1)$, so that $\eta_{\mu\nu} dx^\mu dx^\nu = -c^2 dt^2 + dx^2 + dy^2 + dz^2$. *Light cone* given by $\eta_{\mu\nu} dx^\mu dx^\nu = 0$.[c]

- **Worldlines & paths:** γ denotes a system's worldline through the frame sequence; concatenation $\Gamma = \Gamma_1 \circ \Gamma_2$. All integrated budgets (and thus $\tau_{\text{geo}}, A, \tau_{\text{tot}}$) are additive under concatenation.[d]

- **Composition/locality:** Parallel composition $\otimes$; serial composition $\circ$. Local CPTP operations respect no-signalling and budget additivity.[e]

- **Sign conventions:** Vector norms $\|\cdot\|$; Euclidean inner products "$\cdot$" in space; expectations $\mathbb{E}[\cdot]$; supremum sup.

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[a] See FBA Part I: FBA – Foundations, Sec. I.4 ("Proper time, aging, Minkowski limit").
[b] See FBA Part I: FBA – Foundations, Sec. I.3 ("Calibration/front costs, signal front").
[c] See FBA Part I: FBA – Foundations, Sec. I.4 ("Minkowski limit & quadric").
[d] See FBA Part I: FBA – Foundations, Secs. I.2–I.4 ("Sequence, balance, proper time").
[e] See FBA Part I: FBA – Foundations, Secs. I.5–I.6 ("CPTP/GKLS, composition, no-signalling").

VI.3 Connections between Budget Flows and Geometry

Up to this point, the spacetime language in the FBA has been fixed as a *reference limit*: Minkowski geometry serves as a calibrated stage on which proper time is operationalized as an integrated internal budget contribution and fronts provide the metrological fixation of c.[13] Exactly here, gravity becomes a necessary follow-up question in the FBA: as soon as budget densities and budget flows are *not* homogeneous, the calibrations fixed in Part I can no longer be treated as globally constant. Then clock comparisons, free motion, and signal propagation must depend on location, even though the reference stage remains flat.

This Section is therefore deliberately kinematic: we construct an *effective geometric language* that bundles precisely those location-dependent calibration effects that follow from budget inhomogeneities. The dynamical closure in the form of field-equation-like relations is carried out only in Section VI.4.

VI.3.1 Budget Flows as Curvature Sources

The central step is to identify a quantity that (i) comes from the budget calculus and (ii) is directly coupled to observable effects. For this purpose, not just any tensor ansatz is suitable, but rather the quantity that brings together clock rates and front calibration: the *position-dependent lapse* as the ratio of geometric proper time to coordinate time for an observer at rest. This quantity is measurable via local clock comparisons, without presupposing a field equation.

From this lapse one then obtains an effective metric, and from that in turn an effective connection. Only then does it make sense to state in what precise sense budget gradients act like curvature, without introducing curvature as a fundamental assumption.[1, 2]

[13] See FBA Part I: FBA – Foundations, Secs. I.3–I.4.

Formula Box VI.3.1.1: Curvature proxy from the budget lapse

In a region with $\alpha(x) > 0$, for observers at rest (no spatial drift relative to the chosen coordinate time t, i.e. $d\mathbf{x} = 0$), we define the position-dependent *budget lapse*

$$\alpha(x) \;\equiv\; \frac{d\tau_{\rm geo}}{dt}\bigg|_{\rm rest},$$

where t is the coordinate time calibrated via fronts and $\tau_{\rm geo}$ denotes geometric proper time (reversible-internal). (For the expressions below we assume α to be locally sufficiently smooth, at least so that $\partial_i \alpha$ exists.)

To describe the kinematic effects compactly, we encode the position-dependent calibration in an *effective* (not fundamentally postulated) metric. In the minimal model, in which a *single* scalar proxy $\alpha(x)$ organizes both clock and length calibration isotropically, we choose the conformal isotropic form

$$ds^2_{\rm eff} = g^{\rm eff}_{\mu\nu}\, dx^\mu dx^\nu = -\alpha(x)^2\, (dx^0)^2 \;+\; \alpha(x)^{-2}\, (dx^2 + dy^2 + dz^2), \qquad (x^0 \equiv ct),$$

equivalently written as

$$ds^2_{\rm eff} = -\alpha(x)^2\, c^2\, dt^2 \;+\; \alpha(x)^{-2}\, (dx^2 + dy^2 + dz^2).$$

We define the associated Christoffel symbols $\Gamma^\mu_{\alpha\beta}[g^{\rm eff}]$. (For purely kinematic statements about clock rates and slow motion, the lapse-driven time component suffices; the spatial conformal factorization is the minimal assumption needed to treat light propagation consistently within the same proxy framework.)

For slow motion ($\|\dot{\mathbf{x}}\| \ll c$, with $\dot{\mathbf{x}} = d\mathbf{x}/dt$), this yields at leading order the effective free acceleration

$$\frac{d^2 x^i}{dt^2} \;\approx\; -\,c^2\, \partial_i \ln \alpha(x) \;=\; -\,\partial_i \Phi_B(x), \qquad \Phi_B(x) \equiv c^2 \ln \alpha(x),$$

so that Φ_B takes the role of a budget-induced gravitational potential.

A *curvature proxy* is then provided by the effective curvature of $g^{\rm eff}$ generated by α (or Φ_B). The *dynamical* identification of a source quantity (Einstein/Poisson structure and normalization of the coupling) is fixed only in the continuum limit; see Section VI.4 and in particular Formula Boxes VI.4.1.1 and VI.4.1.2.

What matters here is the direction of the logic: we introduce α because it is the quantity directly delivered by clock comparisons. The effective metric is then a compressed language that organizes this clock physics so that free motion and redshift can be described as unified consequences.

VI.3.2 Geodesics and Free Fall

Once $g^{\rm eff}$ is fixed as the encoding of position-dependent calibration, “free fall” in the FBA is no longer an additional postulation, but the precise requirement that a system without external intervention realizes the worldline compatible with budget-faithful proper-time integration. Operationally: among all nearby paths between two events, the free path is characterized by making $\tau_{\rm geo}$ extremal, because $\tau_{\rm geo}$ is exactly the reversible internal budget contribution that is maximally realized without intervention.[14] [1, 2]

Definition VI.3.2.1: Geodesics in the FBA

An *FBA geodesic* is a timelike worldline γ that (in the kinematic proxy regime) satisfies the geodesic equation of the effective metric:

$$\frac{d^2x^\mu}{d\tau_{\text{geo}}^2} + \Gamma^\mu_{\alpha\beta}\left[g^{\text{eff}}\right] \frac{dx^\alpha}{d\tau_{\text{geo}}} \frac{dx^\beta}{d\tau_{\text{geo}}} = 0,$$

where $\Gamma^\mu_{\alpha\beta}[g^{\text{eff}}]$ is determined from the effective metric and thus ultimately depends on the budget lapse $\alpha(x)$.

Proof Sketch VI.3.2.1: Extremal principle for τ_{geo} (idea chain)

The geodesic equation is the Euler–Lagrange equation of the functional

$$\int_\gamma d\tau_{\text{geo}} \;=\; \int \frac{1}{c}\sqrt{-\,g^{\text{eff}}_{\mu\nu}(x)\,\frac{dx^\mu}{d\lambda}\frac{dx^\nu}{d\lambda}}\;d\lambda,$$

where λ is an arbitrary parameter along the worldline. Choosing $\lambda = \tau_{\text{geo}}$ (affine parametrization) yields directly the geodesic equation with the Christoffel symbols of g^{eff}.

This makes the bridge transparent: budget inhomogeneity $\to$ position-dependent lapse $\alpha(x)$ $\to$ effective connection $\Gamma[g^{\text{eff}}]$ $\to$ free motion. In this order it is clear that the geodesic equation introduces no new assumption about gravity; it formulates the minimal requirement for "no intervention" in an environment with position-dependent calibration.

VI.3.3 Gravitational Time Dilation

In the FBA, time dilation is initially not a field phenomenon but a comparison statement about integrated budgets: two identical clocks operated in different budget environments accumulate different τ_{geo}, even though they measure the same coordinate time t according to the same front protocol. This is precisely why $\alpha(x) = d\tau_{\text{geo}}/dt$ is the right primary quantity: it translates budget distribution directly into clock rates.

The separation of geometric proper time and aging is conceptually mandatory: gravitational dilation concerns the reversible component τ_{geo}. Irreversible contributions A may occur additionally, but they are not the gravitational core mechanism.

[14] See FBA Part I: FBA – Foundations, Secs. I.3 ("Balance") and I.4 ("Proper time, aging").

Definition VI.3.3.1: Time dilation in the FBA

For an observer at rest at position x,

$$d\tau_{\text{geo}} \;=\; \alpha(x)\,dt, \qquad d\tau_{\text{tot}} \;=\; d\tau_{\text{geo}} + dA,$$

where $dA \geq 0$ is irreversible aging.

For two clocks at rest at positions x_1, x_2, this yields the gravitational clock-rate comparison

$$\frac{d\tau_{\text{geo}}(x_1)}{d\tau_{\text{geo}}(x_2)} \;=\; \frac{\alpha(x_1)}{\alpha(x_2)}.$$

In the weak-field regime with $\alpha = 1 + \Phi_B/c^2 + o(c^{-2})$, one obtains the familiar linear form

$$\frac{d\tau_{\text{geo}}(x_1)}{dt} - \frac{d\tau_{\text{geo}}(x_2)}{dt} \;\approx\; \frac{\Phi_B(x_1) - \Phi_B(x_2)}{c^2}.$$

Thus the conceptual loop is closed without anticipating a dynamical field equation: $\alpha(x)$ is metrologically accessible, encodes via the effective metric both free motion and time dilation (and, in the minimal model, also the null-geodesic structure), and thereby provides the operative entry point for the dynamical closure in Section VI.4.

VI.4 Continuum Limit and Field Equations

Section VI.3 introduced an effective geometric language that constructs an effective metric g^{eff} from the metrologically accessible budget lapse $\alpha(x)$. This makes time dilation and free fall formulable as *kinematic* consequences of position-dependent calibration. What is still missing, however, is *closure*: without a relation that couples $\alpha(x)$ or g^{eff} to the budget distribution, the effective geometry remains a freely choosable proxy.

The continuum limit is precisely the step that enables this closure. It does not replace the discrete bookkeeping; rather, it compresses it into fields that can carry local statements: budget density, budget current, and their sources or sinks. This compression is necessary because only then can the comparison with GR take place at the level of differential equations, instead of stopping at mere analogies.[15] [16]

VI.4.1 Field Equations in the FBA Limit

In the discrete FBA, the per-step balance is the hard consistency condition.[17] In the continuum limit, this becomes a local balance in the form of a continuity equation: a budget density $\rho_B(x)$ can change only through the divergence of a budget current and through local sources or sinks. This structure is not an additional postulate; it is the minimal form in which refinement invariance and bookkeeping can be carried over as a local statement in the limit.

For the effective geometry, this implies an additional consistency requirement: if a field equation couples g^{eff} to budget fields and the budget side, in the regime under consideration, carries a local balance, then the geometric side must be chosen so that this balance is not violated by the coupling. In what follows, ∇ is always the Levi–Civita connection of g^{eff}.

Formula Box VI.4.1.1: Effective field equation in the FBA limit – Part 1

We combine the coarse-grained budget quantities in the continuum limit into an effective energy–momentum proxy $T^{(B)}_{\mu\nu}$, which, in a locally inertial rest frame (orthonormal basis), carries the component identifications

$$T^{(B)}_{\hat{0}\hat{0}} \equiv \rho_B c^2, \qquad T^{(B)}_{\hat{0}\hat{\imath}} \equiv c\, j^{(B)}_{\hat{\imath}},$$

where ρ_B is the budget-operational density and $j^{(B)}$ the associated budget current.

For the regimes below we restrict to effective descriptions in which the sector under consideration is *closed* in the proxy sense, so that the local balance can be formulated as covariant conservation,

$$\nabla^{\mu} T^{(B)}_{\mu\nu} = 0,$$

where ∇ is the Levi–Civita connection of g^{eff}.

[15] See FBA Part X: Predictions, Falsifiability & Bridge FBA → QM ↔ GR, Secs. X.4–X.6.
[16] See FBA Part VII: Constants, Scales & Renormalization, Secs. VII.1–VII.3.
[17] See FBA Part I: FBA – Foundations, Sec. I.3.

Formula Box VI.4.1.2: Effective field equation in the FBA limit – Part 2

If, in a concrete application, known exchange/source terms occur ($\nabla^\mu T^{(B)}_{\mu\nu} = J_\nu \neq 0$), this is an extension case: then either a correspondingly extended effective $T^{(B)}_{\mu\nu}$ (including exchange contributions) must be considered, or the coupling structure must be extended beyond the minimal model so that Bianchi compatibility is preserved.

Under the following (GR-typical) minimal assumptions, an Einstein-like coupling form is a particularly natural choice: (i) the left-hand side is local and built purely from g^{eff}, (ii) in 4D the equation is second order in derivatives of g^{eff}, (iii) no additional dynamical fields or new length scales are introduced, (iv) the left-hand side is divergence-free (with ∇ associated with g^{eff}). Within this framework, $G^{\text{eff}}_{\mu\nu}$ (possibly plus $\Lambda g^{\text{eff}}_{\mu\nu}$) is the standard choice; for the structural narrowing under (i)–(iv) see, e.g., Lovelock.[3] We choose $\Lambda = 0$ in the regime considered. As the left-hand side we therefore use

$$G^{\text{eff}}_{\mu\nu}[g^{\text{eff}}] \;\equiv\; R^{\text{eff}}_{\mu\nu} - \frac{1}{2}\, R^{\text{eff}}\, g^{\text{eff}}_{\mu\nu},$$

which satisfies the Bianchi identity $\nabla^\mu G^{\text{eff}}_{\mu\nu} = 0$. The effective field equation thus reads

$$G^{\text{eff}}_{\mu\nu}[g^{\text{eff}}] = \kappa_B\, T^{(B)}_{\mu\nu}.$$

In the static weak-field regime (with the convention $x^0 = ct$ used in this treatise), the minimal model from Formula Box VI.3.1.1 gives

$$g^{\text{eff}}_{00} \approx -(1 + 2\Phi_B/c^2), \qquad g^{\text{eff}}_{ij} \approx (1 - 2\Phi_B/c^2)\delta_{ij}, \qquad \Phi_B \equiv c^2 \ln \alpha,$$

and $\|\dot{\mathbf{x}}\| \ll c$. To leading order, the locally inertial identifications for $T^{(B)}_{\hat{\mu}\hat{\nu}}$ can then be used with the corresponding coordinate components in the Newton/PPN limit. The 00 component (with ∇^2 the Euclidean spatial Laplace operator) then reduces to

$$G^{\text{eff}}_{00} \;\approx\; \frac{2}{c^2}\, \nabla^2 \Phi_B,$$

so that

$$\nabla^2 \Phi_B \;\approx\; \frac{\kappa_B c^4}{2}\, \rho_B.$$

Defining in the Newton/PPN limit

$$\kappa_B \;\equiv\; \frac{8\pi G_B}{c^4} \qquad \text{(equivalently: } G_B := \frac{\kappa_B c^4}{8\pi}\text{)},$$

one obtains the Poisson form

$$\nabla^2 \Phi_B \;=\; 4\pi G_B\, \rho_B,$$

where G_B is fixed metrologically in the weak-field regime by Newton-type experiments.

The decisive statement here is not that GR is simply being copied, but why an Einstein-like structure is plausible in the minimal model: as soon as (i) the effective geometry is described via g^{eff} and (ii) the budget side, in the regime considered, carries a local balance in the form of covariant conservation, a divergence-free geometric side is a consistent choice so that the coupling does not violate the balance structure. The continuum limit thus provides not only a convenient language, but a closure that remains compatible with budget bookkeeping in the limit.

VI.4.2 Einstein-like Structure

With Formula Boxes VI.4.1.1 and VI.4.1.2, the comparison with GR is precise: the difference lies not in the form of the equation, but in the origin of the right-hand side and in the interpretation of the left-hand side. In the standard picture, $g_{\mu\nu}$ is fundamentally dynamical. In the FBA, $g^{\text{eff}}_{\mu\nu}$ is a proxy constructed from calibration- and clock-based budget quantities, and the "matter side" $T^{(B)}_{\mu\nu}$ is a coarse-grained expression of budget flows and their distribution.

Definition VI.4.2.1: Einstein-like structure in the FBA

For the formal comparison with the Einstein structure we set (as a comparison identification)

$$T_{\mu\nu} \;\equiv\; T^{(B)}_{\mu\nu}, \qquad \kappa \;\equiv\; \kappa_B = \frac{8\pi G_B}{c^4},$$

so that the effective field equation takes the Einstein form

$$R^{\text{eff}}_{\mu\nu} - \frac{1}{2}\,R^{\text{eff}}\,g^{\text{eff}}_{\mu\nu} = \kappa\,T_{\mu\nu}$$

.

The central FBA statement is that $T_{\mu\nu}$ is not introduced as an independent matter postulate, but as a proxy of budget density and budget currents in the continuum limit, while $g^{\text{eff}}_{\mu\nu}$ is the geometric language that consistently brings together clock rates, redshift, and free motion.

Thus GR-proximity becomes tangible as a regime statement: if the coarse-grained budget quantities carry the usual energy–momentum symmetries and conservation structures, and if $\alpha(x)$ or g^{eff} can be parametrized by Φ_B in a weak field, the effective description in this regime coincides with the Einstein structure. Where these prerequisites fail, the FBA expects controlled deviations, formulated as test signatures in Section VI.6.

VI.5 Horizons and Redshift

With the effective geometric language from Section VI.3 and the dynamical closure from Section VI.4, two things are now possible at once: first, we can state precisely when the metrologically fixed front structure *tips* at a surface in the proxy sense and thereby yields an effective horizon surface; and second, we can express the observable frequency shift as a pure comparison of clocks and fronts. Both are limiting cases of the same proxy structure: $\alpha(x)$ controls both the local clock calibration relative to the front-calibrated coordinate time t and (in the minimal model) the null-cone structure of the effective metric.[1, 2]

VI.5.1 Horizons in the FBA

In the FBA, a horizon in the sense used here is not an externally imposed entity, but the operational statement that beyond a certain surface (within the proxy regime) there is no admissible signal chain that can carry information from inside to outside. This statement must be assessed against the front calibration, because c is precisely the maximal speed for information fronts fixed by the front protocol.

In the kinematic setup of this treatise, the budget lapse $\alpha(x)$ encodes the relation between coordinate time t (front-calibrated) and geometric proper time τ_{geo} (reversible-internal), see Formula Box VI.3.1.1 and Definition VI.3.3.1. This allows a proxy horizon condition to be derived directly from the null cone of the effective metric, without additional dynamical assumptions.

Definition VI.5.1.1: Horizons in the FBA (kinematic proxy)

We consider (in the kinematic proxy regime without shift/drift terms) the effective metric from Formula Box VI.3.1.1

$$ds^2_{\text{eff}} = -\alpha(x)^2\, c^2\, dt^2 + \alpha(x)^{-2}\, d\ell^2, \qquad d\ell^2 \equiv dx^2 + dy^2 + dz^2,$$

where t and the spatial coordinates are fixed as a reference chart by the same front-metrological protocol. For a signal front, $ds^2_{\text{eff}} = 0$, hence

$$0 = -\alpha(x)^2\, c^2\, dt^2 + \alpha(x)^{-2}\, d\ell^2 \quad \Longrightarrow \quad \frac{d\ell}{dt} = \alpha(x)^2\, c.$$

We call a surface H an *effective horizon surface* in the kinematic proxy sense if, for outward-directed null paths, the front-calibrated outward propagation collapses, i.e.

$$\alpha(x) \to 0 \quad \text{for } x \to H,$$

so that $\frac{d\ell}{dt} \to 0$. Operationally this means: outward-directed signal fronts become asymptotically "frozen" in the coordinate time t near H (the signal coupling relevant to the exterior region effectively breaks down).

This formulation also makes the connection to a "limitation of budget flows" clean: it is not a dimensionally unclear gradient that is compared to c; rather, the physically relevant, metrologically calibrated quantity is the coordinate propagation of a null front. Horizons therefore mark precisely those locations where the proxy geometry cuts off the outward information flux operationally. In more general situations (e.g. with effective shift terms or a non-isotropic proxy structure), the horizon condition is modified by the corresponding null-cone structure; but the logic remains identical: an effective horizon surface is where outward-directed null fronts no longer carry outward. [18]

VI.5.2 Redshift

In the FBA, redshift is the direct observation of the same budget lapse: if $\alpha(x)$ is position- or time-dependent, then frequencies referenced to local clocks are not globally invariant. The key point is that, in the stationary proxy regime, the effect requires no additional dynamical assumptions: it follows from comparing (i) phase transported along a null front and (ii) local clock calibration via τ_{geo}.[1, 2]

In the stationary case, the shift can be read off directly from the time calibration of the effective metric: a clock at rest measures $d\tau_{\mathrm{geo}} = \alpha(x)\,dt$, so the locally measured frequency (for the same coordinate phase) is proportional to $1/d\tau_{\mathrm{geo}}$. Thus the shift becomes the ratio of lapse factors.

Definition VI.5.2.1: Redshift in the FBA

For a stationary situation with the effective metric from Formula Box VI.3.1.1 $ds^2_{\mathrm{eff}} = -\alpha(x)^2 c^2 dt^2 + \alpha(x)^{-2} d\ell^2$, the gravitational redshift for two observers at rest at emission x_{em} and observation x_{obs} is

$$\frac{\nu_{\mathrm{obs}}}{\nu_{\mathrm{em}}} = \frac{\alpha(x_{\mathrm{obs}})}{\alpha(x_{\mathrm{em}})}, \qquad 1 + z \equiv \frac{\nu_{\mathrm{em}}}{\nu_{\mathrm{obs}}} = \frac{\alpha(x_{\mathrm{em}})}{\alpha(x_{\mathrm{obs}})}.$$

In particular: if x_{em} approaches an effective horizon surface H with $\alpha(x) \to 0$, then

$$1 + z \to \infty.$$

Thus H is characterized as the limiting case of infinite redshift within the same proxy structure.

For cosmological redshift, a genuine time dependence of the large-scale calibration enters. This is treated separately within the series framework, because it requires scale handling and cosmic dynamics that are deliberately excluded from this treatise.[19] Here the stationary statement suffices: in the FBA, redshift is primarily a lapse effect, and effective horizons are precisely the locations where this effect turns into an operational decoupling.

[18] See FBA Part V: Spacetime, Light Cones & Local Field Theory, Secs. V.3–V.6.
[19] See FBA Part IX: Cosmic Dynamics, Time Dilation & Inflation (TDI), Secs. IX.2–IX.6.

VI.6 Tests and Predictions

Sections Sections VI.3 to VI.5 have delivered two things that are decisive for tests: first, a metrologically accessible proxy quantity $\alpha(x)$ (clock-rate factor) and, derived from it, $g_{\mu\nu}^{\text{eff}}$; second, a closure in the continuum limit via $T_{\mu\nu}^{(B)}$ and the coupling κ_B. For comparisons in the weak-field regime we additionally use a *Newton/PPN calibration* G_B, i.e. the effective parameter extracted from κ_B (and the specific proxy assignment) that parameterizes the standard observables in the Newton/PPN regime (cf. Formula Boxes VI.4.1.1 and VI.4.1.2). This makes the FBA not merely interpretive but operationally testable: one measures α and the resulting light and particle kinematics and checks whether a single coupling κ_B (or its Newton limit G_B) explains these data consistently.

The test paths sketched here are deliberately formulated so as to avoid additional layers of model building. Detailed test architectures, including bridge criteria and falsifiability logic, are consolidated systematically in Part X.[20] Questions of a possible scale dependence of κ_B (and thus also of the derived G_B) as well as effective couplings belong in Part VII: Constants, Scales & Renormalization.[21]

VI.6.1 Comparison with standard models

A comparison with GR is meaningful in the FBA only if it does not stop at formal similarity of equations, but maps to identical observables. The natural comparison level is therefore clock and signal experiments: time dilation, redshift, travel times, and deflection angles. In the FBA these quantities depend directly on $\alpha(x)$ and $g_{\mu\nu}^{\text{eff}}$, see Formula Box VI.3.1.1 and Definitions VI.3.3.1 and VI.5.2.1.

In the weak-field regime, GR-proximity means concretely: there must exist an effective parameter G_B (derived from κ_B) such that $\Phi_B = c^2 \ln \alpha$ reproduces the same post-Newtonian observables that in GR are generated by the corresponding metric potential. Deviations are then not vague, but appear as systematic residual terms in the usual tests (clock comparison, lensing profiles, Shapiro delay, orbital precession). For the standard parameterization in the PPN regime see Will.[4]

[20] See FBA Part X: Predictions, Falsifiability & Bridge FBA $\rightarrow$ QM $\leftrightarrow$ GR, Secs. X.4–X.8.
[21] See FBA Part VII: Constants, Scales & Renormalization, Secs. VII.1–VII.4.

Definition VI.6.1.1: Comparison of curvature and proxy predictions

A direct comparison FBA vs. GR proceeds via the same observable chain:

$$\alpha(x) \;\Rightarrow\; g^{\mathrm{eff}}_{\mu\nu}(x) \;\Rightarrow\; (\text{null cones/null geodesics, geodesics, redshift, lensing}).$$

In the minimal model with the isotropic proxy metric chosen in Formula Box VI.3.1.1, in the weak field we have $\alpha = 1 + \Phi_B/c^2 + o(c^{-2})$ and thus (in the coordinates used)

$$g^{\mathrm{eff}}_{00} = -(1 + 2\Phi_B/c^2) + o(c^{-2}), \qquad g^{\mathrm{eff}}_{ij} = (1 - 2\Phi_B/c^2)\delta_{ij} + o(c^{-2}).$$

Thus, in the leading post-Newtonian regime, both clock rates/redshift and null-geodesic effects (light deflection, Shapiro travel time) are controlled by the same potential $\Phi_B = c^2 \ln \alpha$. In particular, this matches in the leading term the standard PPN structure with $\gamma = 1$ (deviations appear as residuals).[4]

Practically, this means:

- **Clock comparison:** Measure $\alpha(x)$ via local rate comparisons of clocks at rest and check whether a single $\Phi_B = c^2 \ln \alpha$ describes the observed dilations consistently.
- **Light propagation:** Determine the null geodesics from g^{eff} and compare deflection angles and travel times (Shapiro delay) with the standard predictions.
- **Matter motion:** Determine the (timelike) geodesics from g^{eff} and compare orbital parameters (precession, perihelion advance) in a regime in which non-gravitational forces are controllable.

Agreement in these channels is the *necessary* but not sufficient regime check for GR-proximity. Systematic residuals localize whether (i) the proxy assignment $T^{(B)}_{\mu\nu} \mapsto$ matter/stress-energy proxy, (ii) the extraction of the Newton limit G_B from κ_B, or (iii) a scale-dependent running of the effective couplings (Part VII) must be refined.

VI.6.2 Predictions for experimental tests

The strongest FBA-specific tests are those that simultaneously (i) involve the clock-rate factor α and (ii) challenge the field closure via $T^{(B)}_{\mu\nu}$. Two natural classes are gravitational waves (dynamic, propagating perturbations in an appropriate extension regime) and cosmological parameters (slow, large-scale time dependences). Both go beyond static potential tests and probe whether the proxy character of geometry remains consistent also in dynamical regimes.

Definition VI.6.2.1: Gravitational waves as a dynamical test channel

In a dynamical extension of the proxy framework used here, gravitational waves can be described as propagating perturbations of the effective geometry $g_{\mu\nu}^{\text{eff}}$, i.e. as modes of $\alpha(x,t)$ (and possibly further proxy fields), whose dynamics must be compatible with the closure Formula Boxes VI.4.1.1 and VI.4.1.2.

This yields three robust comparison questions that are already testable without a detailed model:

- **Propagation speed:** Do the dominant modes propagate along the cone structure calibrated by the front protocol, i.e. effectively with c as the limiting speed?
- **Dispersion and damping:** Do frequency-dependent travel times or additional damping occur that cannot be explained as a detector effect or by the matter environment?
- **Polarization/mode content:** In the vacuum/weak-field regime, is the GR-like tensor structure (two tensor polarization modes) sufficient, or does the proxy assignment force additional effective modes in certain coupling regimes?

An FBA regime that reproduces GR in the vacuum limit must deliver GR-compatible answers here. Any stable deviation acts as a direct constraint on the admissible proxy assignment and the effective couplings.[4]

Definition VI.6.2.2: Cosmological parameterization as a large-scale test

Cosmological tests probe whether the large-scale time dependence of the proxy geometry arises consistently from budget flows. In a homogeneous-isotropic ansatz, the observed redshift is typically parameterized via a scale factor $a(t)$:

$$1 + z = \frac{a(t_{\mathrm{obs}})}{a(t_{\mathrm{em}})}.$$

Here t denotes the cosmic time of comoving observers in the *effective* description; the FBA claim is not that $a(t)$ is postulated, but that an effective description of this form must be derivable from the time-dependent budget lapse and the large-scale budget density.

Operationally, this means:

- **Consistency of z and clock rates:** Redshift and cosmic time dilation must share the same underlying proxy structure, rather than being parameterized independently.
- **Horizon diagnostics:** Large-scale horizon phenomena appear as limiting behavior of the effective lapse, analogous to Definition VI.5.1.1.
- **Parameter tying:** A once-calibrated coupling must not be allowed to vary arbitrarily between local gravity tests and cosmological fits without being justified by scale running.

The detailed development of this dynamics, including the connection to the standard formula and to Time-Dilation Inflation (TDI), is the subject of Part IX.[a]

[a] See FBA Part IX: Cosmic Dynamics, Time Dilation & Inflation (TDI), Secs. IX.2–IX.6.

With this, the test logic in Part VI is fully anchored in the reading path: α provides the directly measurable entry point, g^{eff} organizes the kinematics (including null geodesics), and the field closure via $T^{(B)}_{\mu\nu}$ decides whether a single coupling carries the data consistently across regimes.

VI.7 Summary and Outlook

This treatise had a clear goal: not to introduce gravity in the FBA as an additional structure, but to reconstruct it as a robust consequence of inhomogeneous budget flows on an otherwise flat reference stage. The reading path was deliberately chosen so that each new object closes a concrete operational gap: first the measurable clock quantity α, then the effective geometric language derived from it, followed by the dynamical closure in the continuum limit, and finally the limiting cases in which the proxy structure becomes especially sharp.

VI.7.1 Summary of the main points

The central conceptual gain is the decoupling of *language* and *origin*: in the appropriate regime, GR geometry serves as an efficient language, but its core effects are reconstructed in the FBA from budget bookkeeping and calibration. This becomes particularly clear at three nodes: (i) clock comparison as the primary observation, (ii) free fall as an extremal principle for τ_{geo}, (iii) field equations as closure once a local balance is demanded in the continuum limit.

Summary of the FBA results

The derivation chain of this treatise can be read as a compact proxy path:

- **Clock rate as the primary quantity:** The budget lapse $\alpha(x) = d\tau_{\text{geo}}/dt$ is the metrological entry point and organizes time dilation and local calibration; see Formula Box VI.3.1.1 and Definition VI.3.3.1.
- **Effective geometric language:** From α one obtains an effective metric $g^{\text{eff}}_{\mu\nu}$ that bundles null cones, redshift, and free motion in a consistent language; see Formula Box VI.3.1.1 and Definition VI.5.2.1.
- **Free fall as a consequence:** Geodesics appear as extremal paths of τ_{geo} rather than as an additional postulate; see Definition VI.3.2.1.
- **Closure in the continuum limit:** Local balance (proxy conservation) and a divergence-free geometric side make, in the minimal model, an Einstein-like coupling structure for the proxy $T^{(B)}_{\mu\nu}$ a consistent choice; see Formula Boxes VI.4.1.1 and VI.4.1.2 and Definition VI.4.2.1.
- **Limiting cases as diagnostics:** Horizons and infinite redshift are limiting behavior of the same lapse structure; see Definitions VI.5.1.1 and VI.5.2.1.

This makes GR-proximity as a regime statement precisely formulable: if the coarse-grained budget proxies carry the usual conservation and symmetry properties, g^{eff} reproduces the standard tests. Where these prerequisites fail, deviations are not arbitrary, but appear as concrete, localizable residuals in clock, signal, and orbital observations; see Section VI.6.

VI.7.2 Outlook for further development of the FBA

The next step is not to attach additional phenomena to the proxy geometry, but to stabilize the couplings already introduced across regimes. Three extensions are particularly natural:

(1) Scale running and regime changes. In this treatise, κ_B (or G_B) was calibrated as an effective coupling. As soon as one bridges laboratory, astrophysical, and cosmological scales, the question of scale-dependent calibration laws and parameter tying becomes central. This scale running is the subject of Part VII.[22]

(2) Cosmology as time dependence of calibration. Cosmological redshift, horizon phenomena, and accelerated expansion involve a large-scale, time-dependent proxy structure that was only touched upon in its stationary form here. The consistent derivation of $a(t)$, $H(t)$, and a dilation factor $\chi(t)$ from budget flows is worked out in Part IX.[23]

(3) Bridges, pass/fail, and replication protocols. The strongest falsifiability arises where multiple derivation paths overdetermine the same observable, e.g. clock rates, travel times, lensing, and dynamical modes. A systematic test architecture, including H-gates, proxy families, and replication blueprints, is the subject of Part X.[24]

Future applications of the FBA

The next development stage of the FBA lies in consolidating the proxy chains across disciplines:

- **Cosmology:** Large-scale dynamics and observable drifts as consequences of time-dependent budget lapse and budget balances.[a]
- **Scales and renormalization:** Calibration laws as running quantities and parameter tying between experiments of different resolution.[b]
- **Quantum-near bridges:** Consistency of the geometry proxies with CPTP/GKLS structures and local field descriptions.[c] [d] [e] [f]

[a]See FBA Part IX: Cosmic Dynamics, Time Dilation & Inflation (TDI), Secs. IX.3–IX.7.
[b]See FBA Part VII: Constants, Scales & Renormalization, Secs. VII.1–VII.4.
[c]See FBA Part III: Quantum Kinematics & CPTP Channels, Secs. III.4–III.5 (CPTP channels, Hilbert-space formalism).
[d]See FBA Part IV: Dynamics & Measurement (GKLS), Secs. IV.3–IV.5 (admissible dynamics, measurement, GKLS).
[e]See FBA Part V: Spacetime, Light Cones & Local Field Theory, Secs. V.4–V.6 (light cones, microcausality, local QFT).
[f]See FBA Part X: Predictions, Falsifiability & Bridge FBA $\rightarrow$ QM $\leftrightarrow$ GR, Secs. X.3–X.8.

In sum, Part VI provides exactly the layer missing in the overall toolkit between reference geometry and large-scale dynamics: an operational, testable geometrization of budget flows. Everything beyond that is not an additional interpretive step, but the controlled extension of the same proxy chain into regimes where scale running, time dependence, and bridge conditions become unavoidable.

[22]See FBA Part VII: Constants, Scales & Renormalization, Secs. VII.1–VII.4.
[23]See FBA Part IX: Cosmic Dynamics, Time Dilation & Inflation (TDI), Secs. IX.2–IX.7.
[24]See FBA Part X: Predictions, Falsifiability & Bridge FBA $\rightarrow$ QM $\leftrightarrow$ GR, Secs. X.4–X.8.

VI.8 Appendix: Overview of the FBA Series (Parts I–X)

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1. ***Part I*: FBA-Foundations: Ordering, Budget, Proper Time & Arrows.** *Goal:* Provide the base layer: ordering, budget, proper time/aging, front and the operational arrow of time (DPI); Minkowski limit from the budget quadric; admissible dynamics and locality/no-signalling. *Import:* – (reference for all subsequent parts). *Extension:* interface contracts, pass/fail checklists, reading guide.
2. ***Part II*: Time, Proper Time & Minkowski Geometry.** *Goal:* Capture proper time/quadric operationally and derive geodesics. *Import:* foundations (ordering, budget, proper time, front/DPI). *Extension:* smooth limit, variational principle on worldlines, calibration κ_τ.
3. ***Part III*: Quantum Kinematics & CPTP Channels.** *Goal:* State spaces and channels (CPTP) including composition. *Import:* foundations (budget, channel viewpoint, composition). *Extension:* concrete divergences/cost functionals $\mathcal{C}$, measurements, and classical registers.
4. ***Part IV*: Dynamics, Measurement & GKLS (Open Systems).** *Goal:* Continuous open dynamics (GKLS) and the operational arrow of time. *Import:* channels/DPI. *Extension:* Spohn monotonicity, stationary/NESS references, flows $b^{\text{rev}}, b^{\text{irr}}, b^{\text{ext}}$.
5. ***Part V*: Spacetime, Light Cones & Local Field Theory.** *Goal:* Local field equations under front/locality. *Import:* front, composition, no-signalling. *Extension:* local GKLS generators, Lieb–Robinson-type bounds, effective light cones.
6. ***Part VI*: Gravity & Geometry from Budget Flows.** *Goal:* Geometrization of budget flows. *Import:* budget quadric/proper time. *Extension:* effective metrics from calibrations (κ_t, κ_x) and internal stresses; coupling to curvature.
7. ***Part VII*: Constants, Scales & Renormalization.** *Goal:* Scale running of the calibration theorems. *Import:* $c = \kappa_t/\kappa_x$, κ_τ. *Extension:* flow equations for $\kappa_t, \kappa_x, \kappa_\tau$; stability of c.
8. ***Part VIII*: Classical Limit, Thermodynamics & Aging.** *Goal:* Macroscopic behavior from $A[\gamma]$ (aging) and DPI. *Import:* proper time/aging, Spohn. *Extension:* entropy production, Euler–Lagrange forms for irreversible flows, effective transport equations.
9. ***Part IX*: Cosmic Dynamics, Time Dilation & Inflation (TDI).** *Goal:* Cosmic ordering & calibration flow. *Import:* budget, proper time/front. *Extension:* budget equations on large-scale slices; time-dilation inflation as calibration dynamics.
10. ***Part X*: Predictions, Falsifiability & Bridge FBA → QM ↔ GR.** *Goal:* Testable differences and bridges FBA ↔ QM/GR. *Import:* all foundational building blocks. *Extension:* protocols, limiting-case tests, overdetermined consistency relations (pass/fail).

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