The Frame–Budget Approach (FBA)

How time, dynamics, and geometry emerge from budget flows

An operational bridge between Quantum Mechanics and General Relativity

Part IV: Dynamics & Measurement (GKLS)

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Part IV

Dynamics & Measurement (GKLS)

IV.1 Introduction & Target Picture

IV.1.1 Motivation

Up to this point, Part III has fixed the kinematic language of the FBA[1]: states as density operators, measurement outcomes as POVMs (effects as elements), probabilities via the Born coupling, and admissible discrete processes as CPTP.[2] Thus the next question is no longer *how* to write quantum objects, but *when* a temporal evolution and a measurement procedure count as physically admissible. This is exactly where this treatise begins: we sharpen dynamics for open systems such that composition, no-signalling, and budget fidelity do not remain merely kinematic guardrails, but are deployed as explicit conditions for process classes and (in the continuum limit) generators.[3]

Here, the order is decisive: CPTP is not merely a "format" for channels, but a minimal, composition-stable closure that keeps positivity and normalization stable even under ancillary registers and composition. Only once this admissibility is secured does it make sense to take the continuum limit and ask for generators. And only once generators are fixed can irreversibility be evaluated operationally as a monotonicity statement (DPI/Spohn, with the respective required assumptions), without silently using post-selection or hidden side assumptions.[4]

IV.1.2 Target Picture

By the end of this part, three levels should be cleanly separated and yet composed:

1. **Admissibility of discrete steps:** channels, (quantum) instruments, and coarse-grainings as CPTP structure with Stinespring/Kraus (instrument = outcome-indexed CP maps whose sum is a CPTP channel).
2. **Continuous flow:** Markov and continuity assumptions lead (under the usual regularity conditions) to CP semigroups and their GKLS generator form.
3. **Irreversibility as monotonicity:** DPI (for CPTP) and Spohn's inequality (in the GKLS setting under suitable stationarity/regularity) provide a protocol-independent direction for unselected processes and thus an operational separation between reversible limiting cases and dissipative contributions.

This separation is not cosmetic: it decides whether later statements about aging, entropy production, and measurement costs must be read as genuine consequences or as conventions.[5]

[1]An overview of all parts of the FBA treatise, including download links, can be found in Section IV.9 of this document.
[2]See FBA Part III: Quantum Kinematics & CPTP Channels, Secs. III.2–III.6.
[3]See FBA Part I: FBA – Foundations, Secs. I.5–I.6.
[4]See FBA Part I: FBA – Foundations, Sec. I.5.
[5]See FBA Part VIII: Classical Limit, Thermodynamics & Aging, Secs. VIII.6–VIII.8.

IV.1.3 Logical Path

1. *Import of the kinematics.* We take over states, effects, and Born coupling from Part III, because dynamical statements only make sense once it is clear which objects they transform.[6]

2. *Admissible process class.* We fix CPTP not as a definition of QM, but as a consequence of the requirement that processes remain admissible under coupling to ancillary registers and under composition. Stinespring/Kraus are then not extra knowledge, but the implementation form that explains why open-system dynamics appears as coupling plus coarse-graining.[7]

3. *Measurement as an instrument.* POVMs describe outcomes, but dynamics requires updates. Therefore measurement is formulated as a (quantum) instrument: an outcome-indexed family of CP maps whose sum is a CPTP channel, and which carries both statistics and state update without violating admissibility.[8]

4. *Continuum limit and GKLS.* Under Markov and continuity assumptions, discrete channel concatenation becomes a semigroup, and (under suitable regularity assumptions) the GKLS generator form follows. This is the precise point at which "differential equation" appears not as an additional postulate, but as the limiting form of admissible composition.[9]

5. *DPI/Spohn as an operational arrow.* Once the dynamics is fixed as CPTP/GKLS, monotonicities are no longer merely interpretive, but become defensible as mathematical statements: DPI and Spohn provide a direction for unselected processes and thus a robust irreversibility structure that is later translated into aging and thermodynamics.[10]

IV.1.4 Scope and Delimitation

We work locally and flat: no curvature, no backreaction. The spacetime and calibration reference (front protocol, c, cone structure) is assumed as already established.[11] Admissible dynamics is treated as CPTP, or as GKLS in the Markov limit, and locality as well as no-signalling are carried along via the composition principles.[12] No $c = 1$ units are used.

IV.1.5 Contribution Relative to Standard QM

Standard texts often begin with Hilbert space, observables, and a measurement rule, and add open systems via additional postulates. Here the direction is reversed: Process admissibility is formulated as a composition-stable guardrail, measurement as budget-faithful coarse-graining is embedded into the same CPTP structure, and GKLS appears as the limiting form of admissible concatenation. This places unitary evolution, measurement, open systems, and irreversibility into one consistent toolkit that can later be translated directly into pass/fail protocols.[13]

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[6] See FBA Part III: Quantum Kinematics & CPTP Channels, Secs. III.2–III.4.
[7] See FBA Part I: FBA – Foundations, Sec. I.5.
[8] See FBA Part I: FBA – Foundations, Sec. I.5.
[9] See FBA Part I: FBA – Foundations, Sec. I.5.
[10] See FBA Part VIII: Classical Limit, Thermodynamics & Aging, Secs. VIII.6–VIII.8.
[11] See FBA Part II: Time, Proper Time & Minkowski Geometry, Secs. II.5–II.9.
[12] See FBA Part I: FBA – Foundations, Sec. I.6.
[13] See FBA Part X: Predictions, Falsifiability & Bridge FBA $\to$ QM $\leftrightarrow$ GR, Secs. X.6–X.8.

IV.1.6 Reading Guide

The Section order is chosen so that we (i) fix the process class and composition rules, (ii) embed measurement as admissible coarse-graining, (iii) only then take the continuum limit (semigroups/GKLS), and (iv) on that basis evaluate irreversibility and the thermodynamic connection as a monotonicity structure of unselected processes.

Section IV.2 - Foundations & Conventions: Import of the building blocks used and fixation of the notation. *Why first?* Because GKLS and DPI statements are only unambiguous once it is clear which process class and which composition rules apply.

Section IV.3 - Dynamics and Admissible Processes: CPTP as the admissibility core and structural closure under ancillary registers and composition. *Why here?* Because the continuum limit only makes sense if the discrete concatenation is well-defined and admissible.

Section IV.4 - Measurement as Coarse-Graining: measurement as an instrument, POVMs as families of effects, unselected measurement as CPTP coarse-graining. *Why now?* Because irreversibility and entropy production can only be formulated cleanly once it is clear what counts as an unselected channel and what is post-selection.

Section IV.5 - Markov Processes and GKLS: continuous dynamics as a semigroup and GKLS as an admissible generator structure. *Why afterwards?* Because GKLS is a statement about limiting forms of admissible composition, not about single steps.

Section IV.6 - Transition to the Semiclassical Regime: controlled limiting assumptions under which GKLS passes over into classical drift–diffusion descriptions. *Why here?* Because the classical connection only makes sense once the admissible continuous flow (GKLS) has already been fixed precisely.

Section IV.7 - FBA and Irreversibility: DPI/Spohn as an operational arrow, entropy production rate, and a budget reading of the irreversible contribution. *Why last in the core?* Because monotonicities are only robust once process class and generator structure are fixed.

Section IV.8 - Conclusions: consolidation of the results, a route map to thermodynamics, scale questions, as well as the field and geometry side, and the transition to testable pass/fail criteria.

IV.2 Preliminaries & Conventions (Import from Part I: FBA – Foundations)

Why an import? In Part IV we do not want to add dynamics as extra postulates "on top", but rather as a controlled continuation of the kinematics and admissibility fixed in Part III: states, effects, and Born coupling are given, and admissible steps must remain stable under composition, ancillary registers, and coarse-graining. Exactly for this reason we import the primitives from Part I unchanged. The practical gain is clear: once the process class (CPTP) and the composition rules are fixed, GKLS generators and DPI/Spohn are no longer interpretive add-ons, but statements about limiting forms and monotonicities of that class. The details and full proofs remain where they belong – with the basic building blocks.[14]

Reading-guide note. Part III provides the kinematic operator language; Part IV uses it as a working tool and adds exactly the additional structure needed for continuous flows, measurement instruments, and irreversible behavior.[15] The metrological reference (time, c, causal structure) is taken over from Part II and is not re-derived here.[16]

[14] See FBA Part I: FBA – Foundations, Secs. I.5–I.6 "CPTP/GKLS, DPI/Spohn & Composition".

[15] See FBA Part III: Quantum Kinematics & CPTP Channels, Secs. III.2–III.6 "States, POVMs & CPTP Channels".

[16] See FBA Part II: Time, Proper Time & Minkowski Geometry, Secs. II.5–II.9 "Calibration, Quadric & Cone Structure".

Imported building blocks (unchanged)

- **Sequence of global states & minimal events:** frame sequence, minimal event, as well as co-actuality and refinement invariance.[a]
- **Difference function & operational minimal difference:** operational distance measure as the basis of refinement-stable orders.[b]
- **Budget calculus (internal/external/irreversible) & balance:** one-step budget, balance equations, and refinement invariance.[c]
- **Calibration & front (reference structure):** calibration, front bound, and signal front.[d]
- **Proper time & aging (for the connection to irreversibility):** proper time, aging, Minkowski limit, and time dilation as a reference limit.[e]
- **Admissible dynamics (CPTP/GKLS), DPI/Spohn:** admissible channels (CPTP), Kraus/Stinespring, measurement as CPTP, GKLS generators, Spohn monotonicity, semigroup–budget, DPI arrow, and no-recovery.[f]
- **Composition, locality & no-signalling:** symmetric-monoidal structure, budget additivity, no-wire inflation and local operations, causal cone, and local GKLS.[g]

[a] See FBA Part I: FBA – Foundations, Section I.2.
[b] See FBA Part I: FBA – Foundations, Section I.2.
[c] See FBA Part I: FBA – Foundations, Section I.3.
[d] See FBA Part I: FBA – Foundations, Section I.3.
[e] See FBA Part I: FBA – Foundations, Section I.4.
[f] See FBA Part I: FBA – Foundations, Sec. I.5.
[g] See FBA Part I: FBA – Foundations, Sec. I.6.

What exactly are these imports used for in Part IV? CPTP provides the process class for discrete steps. GKLS is the limiting form of this class under Markov and continuity assumptions. And DPI/Spohn is the monotonicity structure that is only robust if one cleanly separates unselected processes from post-selection. That is exactly why admissibility and composition come before any thermodynamic interpretations here.

Notation & Conventions

- **Discrete vs. continuum:** step index $n \in \mathbb{Z}$ for discrete updates; $\Delta(\cdot)$ for discrete increments, $d(\cdot)$ for differential quantities in the limit.
- **States, effects, POVMs:** states $\rho \geq 0$, $\mathrm{Tr}(\rho) = 1$ (in finite-dimensional systems: ρ a matrix; in general: ρ positive and trace-class). effects $0 \leq E \leq \mathbb{I}$. POVM $\{E_i\}_i$: $E_i \geq 0$, $\sum_i E_i = \mathbb{I}$. Born rule: $p(i) = \mathrm{Tr}(\rho E_i)$.
- **Channels and pictures:** CPTP channels Φ (also $\mathcal{E}$) act in the Schrödinger picture on states: $\rho \mapsto \Phi(\rho)$. The adjoint Heisenberg picture is Φ^* (duality trace-class $\leftrightarrow$ bounded operators) with

$$\mathrm{Tr}(\Phi(\rho)\,A) = \mathrm{Tr}(\rho\,\Phi^*(A)) \quad \text{for all states } \rho \text{ and } A \in \mathcal{B}(\mathcal{H}).$$

  For CPTP Φ, Φ^* is completely positive and unital (CP-unital): $\Phi^*(\mathbb{I}) = \mathbb{I}$.
- **Composition:** serial composition $\circ$, parallel composition $\otimes$. identity id. partial trace Tr_E as coarse-graining.
- **Instruments (measurement with update):** A (quantum) instrument is a family of completely positive maps $\{\mathcal{I}_i\}_i$ that are *trace-non-increasing*, i.e. $\mathrm{Tr}(\mathcal{I}_i(\rho)) \leq \mathrm{Tr}(\rho)$ for all $\rho \geq 0$, such that the unselected sum

$$\Phi \;\equiv\; \sum_i \mathcal{I}_i$$

  is a CPTP channel. The associated POVM is defined via

$$E_i = \mathcal{I}_i^*(\mathbb{I}).$$

  Outcome probabilities and posterior states:

$$p(i) = \mathrm{Tr}(\mathcal{I}_i(\rho)), \qquad \rho_i = \mathcal{I}_i(\rho)/p(i) \quad (p(i) > 0).$$

  [1–3]
- **Semigroups and GKLS:** A (quantum dynamical) semigroup is $\{\mathcal{T}_t\}_{t\geq 0}$ with $\mathcal{T}_0 = \mathrm{id}$, $\mathcal{T}_{t+s} = \mathcal{T}_t \circ \mathcal{T}_s$, and each $\mathcal{T}_t$ CPTP. In the Markov setting we assume (at least) strong continuity in the trace norm; the generator $\mathcal{L}$ is then generally densely defined (bounded in finite-dimensional systems), and $\rho_t = \mathcal{T}_t(\rho_0)$ solves $\frac{d}{dt}\rho_t = \mathcal{L}(\rho_t)$ for ρ_0 in the domain of definition. The GKLS form is derived in Sec. Section IV.5 and then used.[4–6]
- **Entropies, DPI, and Spohn (where needed):** von Neumann entropy $S(\rho) = -\mathrm{Tr}(\rho \log \rho)$. relative entropy $S(\rho\|\sigma) = \mathrm{Tr}(\rho(\log \rho - \log \sigma))$ for $\mathrm{supp}(\rho) \subseteq \mathrm{supp}(\sigma)$, otherwise $S(\rho\|\sigma) = +\infty$. DPI: $S(\Phi(\rho)\|\Phi(\sigma)) \leq S(\rho\|\sigma)$ for CPTP Φ. Spohn monotonicity denotes the GKLS-specific entropy production inequality under the respective required stationarity/regularity assumptions.[7–9]
- **Budget decomposition (reference):** per step δb_{int}, δb_{ext}, δb_{irr} (internal/external/irreversible). We keep the time calibration κ_τ from Part II explicit:[a]

$$d\tau_{\mathrm{geo}} \equiv \frac{db_{\mathrm{int}}^{\mathrm{rev}}}{\kappa_\tau}, \qquad dA \equiv \frac{db_{\mathrm{irr}}}{\kappa_\tau} \geq 0, \qquad d\tau_{\mathrm{tot}} = d\tau_{\mathrm{geo}} + dA.$$

  (Discrete analog: $\Delta\tau_{\mathrm{geo}} = \Delta b_{\mathrm{int}}^{\mathrm{rev}}/\kappa_\tau$, $\Delta A = \Delta b_{\mathrm{irr}}/\kappa_\tau$.)
- **Spacetime and calibration reference:** c remains explicit (no $c = 1$ units). time parameter t is the calibrated scale from the front protocol.[b]
- **Sign conventions:** expectation values $\mathbb{E}[\cdot]$, supremum sup. norms are indicated context-dependently, e.g. trace norm $\|\cdot\|_1$ and operator norm $\|\cdot\|_\infty$.

[a] See FBA Part II: Time, Proper Time & Minkowski Geometry, Secs. II.5–II.9.
[b] See FBA Part II: Time, Proper Time & Minkowski Geometry, Secs. II.5–II.9.

Consequence. With these imports and conventions we can, in Section IV.4, directly embed measurement as an instrument into the CPTP structure, in Section IV.5 sharpen the GKLS limit as a semigroup generator, and in Section IV.7 evaluate DPI/Spohn as a robust, unselected arrow of irreversibility – without hidden shifts of convention.

IV.3 Dynamics and Admissible Processes

Part III fixed the kinematic language (states, POVMs, Born coupling) and already made clear why *process admissibility* cannot be arbitrary: once one allows ancillary registers and composes processes serially as well as in parallel, the class of updates must be closed under exactly these operations.[17] This Section takes the next step: we fix the admissible process class for discrete steps as CPTP and then introduce the Markov and continuity limit, in which generators and the GKLS form are meaningfully defined.[18]

IV.3.1 Admissible Transformations and CPTP Channels

In the FBA, a "step" is at first nothing more than an update along the frame sequence. For such updates to become physics, one must decide which transformations count as *admissible*. Admissibility is not aesthetic, but operational: states must map to states again, probabilities must remain positive, and normalization must not be lost under admissible operations. Exactly these requirements lead to the class of CPTP channels.[19]

Definition IV.3.1.1: CPTP Channel

Let $\mathcal{H}$ be a (here: finite-dimensional) Hilbert space and $\mathcal{B}(\mathcal{H})$ the space of linear operators on $\mathcal{H}$. A *CPTP channel* is a linear map $\mathcal{E} : \mathcal{B}(\mathcal{H}) \to \mathcal{B}(\mathcal{H})$ with:

- **Complete positivity:** For every ancillary register $\mathcal{H}'$, $\mathcal{E} \otimes \mathrm{id}_{\mathcal{H}'}$ is positive on $\mathcal{B}(\mathcal{H} \otimes \mathcal{H}')$.
- **Trace preservation:** $\mathrm{Tr}(\mathcal{E}(X)) = \mathrm{Tr}(X)$ for all $X \in \mathcal{B}(\mathcal{H})$.

In particular, one then has: $\rho \in \mathcal{S}(\mathcal{H}) \Rightarrow \mathcal{E}(\rho) \in \mathcal{S}(\mathcal{H})$. (In infinite-dimensional systems one typically works with a normal CP map on $\mathcal{B}(\mathcal{H})$ or, equivalently, with a trace-preserving CP map on the trace-class operators; the domain/continuity issues are treated explicitly in the GKLS Section.) [3, 10, 11]

That it is precisely *complete* positivity that is needed is the point where ancillary registers and composition really "bite": an update must remain physically consistent not only in isolation, but also in conjunction with arbitrary, unobserved auxiliary registers.

[17] See FBA Part III: Quantum Kinematics & CPTP Channels, Secs. III.2–III.4 "States, Effects & Process Closure".
[18] See FBA Part I: FBA – Foundations, Sec. I.5 "Admissible Channels (CPTP) & GKLS Generators".
[19] See FBA Part I: FBA – Foundations, Sec. I.5 "Admissible Channels (CPTP)".

Why CPTP is the right admissibility class (FBA reading)

- **CP rather than merely positive:** Positivity alone only ensures that $\mathcal{E}$ does not send isolated states to "negative probabilities". In real protocols, however, systems couple to ancillary registers (environment, measurement apparatus, references). The requirement that $\mathcal{E}$ also produces no negativity on ρ_{SE} is exactly complete positivity.
- **Trace preservation as a normalization criterion:** Trace preservation guarantees that an *unselected* step preserves total normalization. Selected subprocesses (e.g. measurement updates) are CP and trace-non-increasing; their sum is again a CPTP channel (see instruments in Section IV.2).
- **Closure under composition:** Serial concatenation and parallel composition of admissible steps must again be admissible. CPTP is closed under $\circ$ and $\otimes$ and thus compatible with the imported composition structure.[a]

[a]See FBA Part I: FBA – Foundations, Sec. I.6 "Symmetric-Monoidal Structure & No-Wire Inflation".

This makes the connection to the frame sequence clean: each step $F_n \to F_{n+1}$ corresponds to an admissible update $\rho \mapsto \mathcal{E}(\rho)$, whose internal, external, and irreversible budget contributions encode the physical costs of this update.[20]

IV.3.2 Markov Semigroups

For many applications, the discrete view of updates is not sufficient: one wants to parametrize processes continuously and specify local rates. For this one needs a continuum limit that respects closure under composition. The operational assumption behind it is called "Markov": if one knows the state $\rho(t)$ at time t, then the information relevant for the future is already completely contained in it. Mathematically, this is formulated as a semigroup structure.

Definition IV.3.2.1: Markov Semigroup (quantum dynamical)

A *Markov semigroup* is a family $\{\mathcal{T}_t\}_{t\geq 0}$ of CPTP channels acting on states via $\rho \mapsto \mathcal{T}_t(\rho)$, with:

- **Semigroup:** $\mathcal{T}_{t+s} = \mathcal{T}_t \circ \mathcal{T}_s$ for all $t, s \geq 0$,
- **Initial value:** $\mathcal{T}_0 = \mathrm{id}$,
- **Strong continuity (on states):** $t \mapsto \mathcal{T}_t(\rho)$ is continuous in the trace norm $\|\cdot\|_1$ for all states ρ.

[5, 6]

The continuity assumption is the technical lever that turns "concatenation of many small CPTP steps" into a generator. Without it, one could define semigroups, but not a well-defined infinitesimal rate.

[20]See FBA Part I: FBA – Foundations, Sec. I.3 "One-Step Budget & Balance".

Formula Box IV.3.2.1: Generator of a CPTP Semigroup

If $\{\mathcal{T}_t\}_{t\geq 0}$ is strongly continuous (in $\|\cdot\|_1$ on states), then the generator $\mathcal{L}$ is defined as a (generally unbounded) operator with domain

$$\mathcal{D}(\mathcal{L}) \;\equiv\; \left\{\rho : \lim_{t\downarrow 0} \frac{\mathcal{T}_t(\rho)-\rho}{t} \text{ exists in } \|\cdot\|_1\right\}$$

given by

$$\mathcal{L}(\rho) \;\equiv\; \lim_{t\downarrow 0} \frac{\mathcal{T}_t(\rho)-\rho}{t} \qquad (\rho \in \mathcal{D}(\mathcal{L})).$$

For initial data $\rho(0) \in \mathcal{D}(\mathcal{L})$ one then has (in the appropriate sense) the master equation

$$\frac{d}{dt}\rho(t) = \mathcal{L}(\rho(t)), \qquad \rho(t) = \mathcal{T}_t(\rho(0)).$$

IV.3.3 GKLS Form and How It Arises

Now comes the decisive point: *Not* every linear differential equation defines a CPTP evolution, and that is precisely why the GKLS form is not an arbitrary ansatz, but the structurally correct characterization of generators that preserve complete positivity and trace preservation for all times. In the FBA, this is the continuum version of the same admissibility logic as for discrete CPTP steps.[21]

Lemma IV.3.3.1: GKLS Equation (Theorem of Gorini–Kossakowski–Lindblad–Sudarshan)

Let $\mathcal{H}$ be finite-dimensional and $\{\mathcal{T}_t\}_{t\geq 0}$ a strongly continuous CPTP Markov semigroup. Then its generator $\mathcal{L}$ can be written in GKLS form:

$$\frac{d}{dt}\rho(t) = \mathcal{L}(\rho(t)),$$

with

$$\mathcal{L}(\rho) = -i[H,\rho] + \sum_{k=1}^{K}\Big(L_k\rho L_k^\dagger - \frac{1}{2}\{L_k^\dagger L_k, \rho\}\Big),$$

where $H = H^\dagger$ is the effective Hamiltonian operator, $\{A,B\} = AB + BA$ denotes the anticommutator, and $\{L_k\}_{k=1}^{K}$ are Lindblad operators. [4–6]

This also makes clear why the GKLS form provides a natural "interface" to the budget side: the Hamiltonian term is the reversible part, the dissipator term describes exchange with unresolved degrees of freedom and is the place where irreversible budget contributions (aging, entropy production) come in. The required monotonicity statements (DPI/Spohn) are evaluated precisely only after a clean separation of unselected channels and post-selection.[22]

[21] See FBA Part I: FBA – Foundations, Sec. I.5 "GKLS Generators (Open Systems)".
[22] See FBA Part I: FBA – Foundations, Sec. I.5 "Spohn Monotonicity & Semigroup Budget".

Proof Sketch IV.3.3.1: Why GKLS follows from CPTP + semigroup

1. **Small-step dilation:** For small times t, $\mathcal{T}_t$ is CPTP and thus admits a Stinespring dilation as a unitary coupling to an ancillary register plus partial trace.

2. **Semigroup structure:** The relation $\mathcal{T}_{t+s} = \mathcal{T}_t \circ \mathcal{T}_s$ ensures that the small-step structure can be concatenated consistently, without losing admissibility.

3. **Infinitesimal form:** Taking the continuous limit $t \downarrow 0$, the first order splits into a commutator (reversible) contribution and a dissipative contribution that must be structured so as to preserve CP and TP for all times.

Full treatments and regularity details can be found at the foundational location.[a] [4–6]

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[a]See FBA Part I: FBA – Foundations, Sec. I.5 "GKLS Generators (Open Systems)".

Transition. In the following Section IV.4 we use the CPTP structure to cast measurement as admissible coarse-graining (instrument). Only afterwards do we evaluate irreversibility as a monotonicity statement: DPI and Spohn provide the operational arrow for *unselected* processes and thus separate reversible limiting cases from dissipative contributions.

IV.4 Measurement as Coarse-Graining

After Section IV.3 fixed admissible dynamics as CPTP (discrete) and, in the Markov limit, as GKLS (continuous), measurements must fit into the same framework. In the FBA this is not a matter of taste, but a consistency requirement: if measurement is a physical operation, it must not lie outside the admissibility class that we obtain from composition, ancillary registers, and budget fidelity.[23] The correct formulation is therefore: measurement is *budget-faithful coarse-graining* — either with a classical outcome register (selective, i.e. conditioned on information), or as an unselected channel if the outcome is discarded.

IV.4.1 Measurements as special coarse-grainings

A POVM alone describes only the statistics of the outcomes. For dynamics we additionally need the state update. Exactly this combination of statistics and update is an *instrument*: a family of CP maps whose sum is a CPTP channel. This turns measurement into a special case of admissible dynamics, rather than leaving it as an exception.

Definition IV.4.1.1: Measurement instrument (outcome + update) as a CP decomposition of a CPTP channel

Let $\mathcal{H}$ be finite-dimensional and $\mathcal{B}(\mathcal{H})$ the operator space. A *(quantum) instrument* is a family $\{\mathcal{I}_i\}_i$ of linear maps $\mathcal{I}_i : \mathcal{B}(\mathcal{H}) \to \mathcal{B}(\mathcal{H})$ with:

1. **CP property:** Each $\mathcal{I}_i$ is completely positive (CP).
2. **Selectively trace-non-increasing:** For all $\rho \geq 0$, $\mathrm{Tr}(\mathcal{I}_i(\rho)) \leq \mathrm{Tr}(\rho)$.
3. **Unselected admissibility:** The sum $\Phi \equiv \sum_i \mathcal{I}_i$ is a CPTP channel.

For an input state ρ,

$$p(i) = \mathrm{Tr}(\mathcal{I}_i(\rho)), \qquad \rho_i = \frac{\mathcal{I}_i(\rho)}{p(i)} \quad (p(i) > 0)$$

are the outcome probability and the a-posteriori state. [1–3]

In this view, the POVM is not an additional axiom, but the “Heisenberg shadow side” of the instrument: it encodes exactly the outcome statistics, independent of which update one implements afterwards.

[23] See FBA Part I: FBA – Foundations, Secs. I.5–I.6 “Measurement as CPTP, Composition & No-Signalling”.

Lemma IV.4.1.1: POVM as an effect family induced by the instrument

Let $\{\mathcal{I}_i\}_i$ be an instrument and $\mathcal{I}_i^*$ the adjoint map in the Heisenberg picture. Then the effects

$$E_i \;\equiv\; \mathcal{I}_i^*(\mathbb{I})$$

define a POVM, i.e. $E_i \geq 0$ and $\sum_i E_i = \mathbb{I}$. Moreover, for all states ρ,

$$p(i) = \mathrm{Tr}(\mathcal{I}_i(\rho)) = \mathrm{Tr}(\rho\, E_i).$$

[2, 3]

Operationally, the statement is: *an instrument already carries the statistics in its Heisenberg side.* The effects $E_i = \mathcal{I}_i^*(\mathbb{I})$ are exactly the objects that reproduce the observed frequencies for *all* inputs ρ.

Proof Sketch IV.4.1.1: Why an instrument automatically induces a POVM

For $\rho \geq 0$, $\mathcal{I}_i(\rho) \geq 0$ (CP), hence $p(i) = \mathrm{Tr}(\mathcal{I}_i(\rho)) \geq 0$. With the definition of the adjoint it follows that

$$p(i) = \mathrm{Tr}(\mathcal{I}_i(\rho)) = \mathrm{Tr}\big(\rho\, \mathcal{I}_i^*(\mathbb{I})\big) = \mathrm{Tr}(\rho\, E_i).$$

Since $\mathrm{Tr}(\rho E_i) \geq 0$ for all $\rho \geq 0$, we have $E_i \geq 0$. Furthermore, $\Phi = \sum_i \mathcal{I}_i$ (unselected) is CPTP, in particular trace-preserving, so

$$\sum_i p(i) = \sum_i \mathrm{Tr}(\mathcal{I}_i(\rho)) = \mathrm{Tr}(\Phi(\rho)) = \mathrm{Tr}(\rho).$$

On the other hand, $\sum_i p(i) = \sum_i \mathrm{Tr}(\rho E_i) = \mathrm{Tr}\big(\rho \sum_i E_i\big)$. Thus $\mathrm{Tr}\big(\rho \sum_i E_i\big) = \mathrm{Tr}(\rho)$ for all states ρ, hence $\sum_i E_i = \mathbb{I}$.

Remark (notation in the QM literature). Often, instead of the effects E_i, so-called measurement operators (Kraus operators) M_i are given, with $E_i = M_i^\dagger M_i$. Then the selective update is $\mathcal{I}_i(\rho) = M_i \rho M_i^\dagger$, and the unselected map reads $\Phi(\rho) = \sum_i M_i \rho M_i^\dagger$. In the FBA this notation is merely a representation of the same instrument.[24] [3, 11, 12]

[24] See FBA Part I: FBA – Foundations, Sec. I.5 "Kraus/Stinespring & Measurement as CPTP".

Measurement as coarse-graining: selective vs. unselected

- **Selective (conditioned on outcome):** One conditions on an outcome i and obtains ρ_i. As a map on the system alone this is a CP, trace-non-increasing operation $\mathcal{I}_i$ (thus *not* CPTP). Physically, selectivity corresponds to carrying along a classical outcome register; on $\mathcal{H} \otimes \mathcal{H}_{\text{out}}$ the overall description is CPTP again.
- **Unselected (outcome discarded):** One discards the outcome and describes only the resulting system dynamics $\Phi = \sum_i \mathcal{I}_i$. This is exactly coarse-graining: finer information (which i occurred) is no longer tracked.

In the FBA it is important to keep both levels strictly separate: admissibility and monotonicities (DPI/Spohn) refer to unselected CPTP dynamics, not to post-selection.

IV.4.2 Dilation principle: why measurement must be embedded into CPTP

That measurement can be formulated as an instrument is no accident: it is the operational form of the same idea as Stinespring dilation. Measurement arises from coupling to an ancillary register plus subsequent coarse-graining (partial trace or classical readout). This makes transparent where irreversibility comes from: not from a "mystical collapse", but from discarding information about ancillary degrees of freedom.

Formula Box IV.4.2.1: Dilation representation of a measurement (unitary coupling + readout)

For every POVM $\{E_i\}_i$ there exist (in the finite-dimensional setting, with a finite outcome set) an ancillary register $\mathcal{H}_A$, a state σ_A, a unitary coupling U on $\mathcal{H} \otimes \mathcal{H}_A$ and projectors $\{\Pi_i\}_i$ on $\mathcal{H}_A$, such that

$$p(i) = \mathrm{Tr}\Big(U(\rho \otimes \sigma_A)U^\dagger\,(\mathbb{I} \otimes \Pi_i)\Big).$$

A *canonical* instrument that realizes this POVM is obtained by coarse-graining the ancillary register after the readout Π_i:

$$\mathcal{I}_i(\rho) = \mathrm{Tr}_A\Big((\mathbb{I} \otimes \Pi_i)\,U(\rho \otimes \sigma_A)U^\dagger\,(\mathbb{I} \otimes \Pi_i)\Big), \qquad \Phi(\rho) = \sum_i \mathcal{I}_i(\rho).$$

[2, 3, 10]

IV.4.3 Coarse-graining and entropy: information loss as a DPI statement

The key point is: *coarse-graining is irreversible* because it destroys distinguishability. This can be formulated directly as a data-processing inequality, without interpretation. Thus entropy production becomes not a metaphor, but an operational monotonicity statement about admissible channels.[25]

[25] See FBA Part I: FBA – Foundations, Sec. I.5 "DPI Arrow & Spohn Monotonicity".

Formula Box IV.4.3.1: DPI for measurement: distinguishability can only decrease under coarse-graining

Let Φ be a CPTP channel (in particular, an unselected measurement map) and let ρ, σ be two states. Then the relative entropy satisfies

$$S(\Phi(\rho)\|\Phi(\sigma)) \ \leq \ S(\rho\|\sigma), \qquad S(\rho\|\sigma) \equiv \mathrm{Tr}\big(\rho(\log\rho - \log\sigma)\big),$$

where $S(\rho\|\sigma) = +\infty$ if $\mathrm{supp}(\rho) \nsubseteq \mathrm{supp}(\sigma)$. The difference

$$\Delta_\Phi(\rho,\sigma) \ \equiv \ S(\rho\|\sigma) - S(\Phi(\rho)\|\Phi(\sigma)) \ \geq \ 0$$

quantifies the information loss due to coarse-graining: after applying Φ, the two hypotheses ρ and σ are operationally less distinguishable. [3, 7, 8]

This formulation is deliberately pairwise: it makes clear what "information loss" means in experiments, namely a loss of distinguishability between possible preparations. For continuous GKLS flows, the same idea is translated into an entropy production rate via Spohn monotonicity; that is the next step in Section IV.7.

IV.5 Markov Processes and the GKLS Equation

In Section IV.3 we fixed admissible discrete updates as CPTP channels and thus set the process class that remains stable under composition and ancillary registers. For many applications, however, this discrete view is not sufficient: one wants to specify *rates*, formulate master equations, and sharpen the separation between reversible and dissipative contributions in a continuum limit. This is exactly where the Markov limit enters: it is the controlled modeling assumption that the effective description of an open system (after the chosen coarse-graining) is carried by a semigroup-like, memoryless evolution. The GKLS form is then not an additional ansatz, but the characterization of those generators that preserve this admissibility in the continuum.[26]

IV.5.1 Markov Semigroups

In the FBA, the Markov assumption is not a metaphysical claim about nature, but a modeling decision about what we no longer resolve *explicitly*: the environment and correlation memory are coarse-grained so that the future dynamics depends only on the current system state. In this way, the concatenation of many small admissible steps becomes a semigroup.[6]

Definition IV.5.1.1: Markov semigroup (strongly continuous)

Let $\mathcal{H}$ be (here) finite-dimensional and $\mathcal{B}(\mathcal{H})$ the operator space. A *Markov semigroup* is a family $\{\mathcal{T}_t\}_{t\geq 0}$ of CPTP channels $\mathcal{T}_t : \mathcal{B}(\mathcal{H}) \to \mathcal{B}(\mathcal{H})$, acting on states, with:

- **Semigroup:** $\mathcal{T}_{t+s} = \mathcal{T}_t \circ \mathcal{T}_s$ for all $t, s \geq 0$,
- **Initial value:** $\mathcal{T}_0 = \mathrm{id}$,
- **Strong continuity (on states):** For all states ρ, $t \mapsto \mathcal{T}_t(\rho)$ is continuous in the trace norm $\|\cdot\|_1$.

(In infinite-dimensional systems one typically formulates Schrödinger dynamics on the trace-class operators and Heisenberg dynamics as a normal, unital CP semigroup on $\mathcal{B}(\mathcal{H})$; domain/topology issues then become essential.) [5, 6]

The continuity requirement is the operational lever that turns the mere concatenation $\mathcal{T}_{t+s} = \mathcal{T}_t \circ \mathcal{T}_s$ into a well-defined infinitesimal description. Without it, there would be a "time parametrization", but no controlled passage to rates and generators.

[26] See FBA Part I: FBA – Foundations, Sec. I.5 "GKLS Generators (Open Systems)".

Formula Box IV.5.1.1: Generator and master equation

If $\{\mathcal{T}_t\}_{t\geq 0}$ is strongly continuous, then the generator $\mathcal{L}$ is defined as a (generally unbounded) operator on the domain

$$\mathcal{D}(\mathcal{L}) \;\equiv\; \left\{\rho : \lim_{t\downarrow 0} \frac{\mathcal{T}_t(\rho) - \rho}{t} \text{ exists in } \|\cdot\|_1\right\}$$

given by

$$\mathcal{L}(\rho) \;\equiv\; \lim_{t\downarrow 0} \frac{\mathcal{T}_t(\rho) - \rho}{t} \qquad (\rho \in \mathcal{D}(\mathcal{L})).$$

Then for $\rho(0) \in \mathcal{D}(\mathcal{L})$ one has (in the appropriate sense) the master equation

$$\frac{d}{dt}\rho(t) = \mathcal{L}(\rho(t)), \qquad \rho(t) = \mathcal{T}_t(\rho(0)).$$

[6]

In the FBA, the "rate description" formulated in Formula Box IV.5.1.1 is exactly what one needs in order to interpret dissipative budget contributions as flows: discrete budget sums become integrals, and irreversible behavior becomes visible as a structural monotonicity of admissible flows. This is evaluated operationally later in Section IV.7 using DPI/Spohn.

IV.5.2 GKLS form and how it arises

Not every linear differential equation generates a CPTP evolution for all $t \geq 0$. The actual statement of the GKLS structure is therefore an admissibility characterization: *if* a Markov semigroup exists in this sense, then its generator must have a very specific form so that complete positivity and trace preservation remain intact for all times.[27]

Lemma IV.5.2.1: GKLS form (characterization of admissible Markov generators)

Let $\mathcal{H}$ be finite-dimensional and $\{\mathcal{T}_t\}_{t\geq 0}$ a strongly continuous CPTP Markov semigroup. Then its generator $\mathcal{L}$ can be written in GKLS form:

$$\frac{d}{dt}\rho(t) = \mathcal{L}(\rho(t)),$$

with

$$\mathcal{L}(\rho) = -i[H,\rho] + \sum_{k=1}^{K}\Big(L_k\rho L_k^\dagger - \frac{1}{2}\{L_k^\dagger L_k, \rho\}\Big),$$

where $H = H^\dagger$ is the effective Hamiltonian operator, $\{A, B\} = AB + BA$ denotes the anticommutator, and $\{L_k\}_{k=1}^K$ are Lindblad operators. [4–6]

The form in Lemma IV.5.2.1 already separates structurally what in the FBA is read as the "reversible contribution" and as "dissipative exchange": the commutator is the reversible part, the dissipator describes precisely the non-unitary contributions that nevertheless remain

[27] See FBA Part I: FBA – Foundations, Sec. I.5 "GKLS Generators (Open Systems)".

CPTP. Thus GKLS is not optional, but (under the stated assumptions) the appropriate characterization of admissible Markov generators.

Why these assumptions, and what they mean

1. **CPTP as admissibility:** Without complete positivity, stability under ancillary registers would not be secured, and without trace preservation, the unselected bookkeeping would not be budget-faithful.
2. **Semigroup as Markov limit:** $\mathcal{T}_{t+s} = \mathcal{T}_t \circ \mathcal{T}_s$ is the formal expression of "no effective memory at the system level" after the chosen coarse-graining (not: absence of microscopic correlations).
3. **Continuity as existence of rates:** Only (strong) continuity legitimizes the generator as a well-defined infinitesimal rate.
4. **Finite dimension as a technical framework:** In infinite dimensions, domain questions, normality, and additional regularity conditions enter; the operational idea remains the same, the mathematical development is more subtle.[a]

[a]See FBA Part I: FBA – Foundations, Sec. I.5 "GKLS Generators (Open Systems)".

Transition. With Markov semigroups and GKLS generators, the dynamical infrastructure is in place. In the following Section IV.6 we formulate the controlled transition to semiclassical limiting descriptions. Building on that, we use DPI and Spohn in Section IV.7 to cast irreversibility as a robust monotonicity statement for *unselected* processes, and to keep measurement, entropy production, and aging precisely distinct within the FBA.

IV.6 Transition to the Semiclassical Regime

Sections IV.3 and IV.5 fixed the dynamical infrastructure: discrete updates as CPTP, continuous Markov flows as GKLS. If the FBA is to provide a bridge between quantum description and classical thermodynamics, it must now become clear how classical transport equations and classical entropy balances emerge from GKLS under controlled limiting assumptions. The point is not to invoke $\hbar \to 0$ as a slogan, but to say *which* properties of states and rates remain stable in this limit and which information is deliberately coarse-grained.[28] Scale and renormalization issues that inevitably arise in such transitions are bundled separately in the FBA.[29]

IV.6.1 Continuity assumptions and semiclassical approximations

A GKLS equation is an equation for operators. The semiclassical transition becomes operational only once one translates the operator description into a (quasi-)classical state description, for instance via phase-space representations. Under smoothness and scale assumptions, the operator dynamics can become an effective drift/diffusion dynamics for probability densities. The central roles are played by continuity (for the generator) and the assumption that, on the scale considered, relevant states no longer carry strongly oscillatory quantum fine structure.

Definition IV.6.1.1: Semiclassical transition as a controlled coarse-graining limit

A *semiclassical transition* in the FBA is a limiting description in which

1. states ρ are described by a phase-space representation $W_\hbar[\rho]$ (e. g. Wigner or Husimi representation) that, for the state families under consideration, converges in the limit $\hbar \to 0$ in a suitable sense (e. g. weakly or after smoothing) to a genuine probability density f,
2. the GKLS evolution $\dot{\rho} = \mathcal{L}(\rho)$ is approximated on this descriptive level by an effective equation $\partial_t f = \mathcal{G}(f)$,
3. in the diffusion-dominated regime, $\mathcal{G}$ has a drift–diffusion form (Fokker–Planck type).

The limit is thus not a postulate, but a statement about which information is resolved in the considered scale window and which is not. [6, 13, 14]

[28] See FBA Part VIII: Classical Limit, Thermodynamics & Aging, Secs. VIII.1–VIII.5 "Classical Limit & Thermodynamic Interpretation".

[29] See FBA Part VII: Constants, Scales & Renormalization, Secs. VII.1–VII.4 "Scales, Constants & Renormalization".

Which assumptions support the Fokker–Planck limit

The statement "GKLS $\rightsquigarrow$ Fokker–Planck" is not an automatism claim: it is a controlled approximation that requires additional regularity and a suitable scale regime. Typical sufficient conditions are:

- **Scale separation:** macroscopic observables change slowly compared to the microscopic correlations of the environment (Markov limit).
- **Smoothness:** relevant state families are sufficiently smooth in phase space on the scale considered so that a gradient or Kramers–Moyal expansion makes sense.
- **Diffusive limit:** many small random contributions add up to effective diffusion (rather than to higher-order jump processes); otherwise, one obtains master equations with jump terms rather than pure Fokker–Planck forms.

Full developments and the thermodynamic reading of the resulting entropy balances are given at the interface location.[a] [15, 16]

[a] See FBA Part VIII: Classical Limit, Thermodynamics & Aging, Secs. VIII.1–VIII.5 "Classical Limit & Thermodynamic Interpretation".

Formula Box IV.6.1.1: GKLS $\rightsquigarrow$ drift–diffusion on phase space (schematic)

Under these assumptions, the effective dynamics for a phase-space density $f(x,t)$ can often be written in conservative form as

$$\partial_t f(x,t) \;=\; -\nabla\cdot J(x,t), \qquad J(x,t) \;=\; a(x)\, f(x,t) \;-\; \frac{1}{2}\, D(x)\, \nabla f(x,t),$$

(Fokker–Planck type; conventions for factors and the exact placement of f are model-dependent). Here the drift a carries the reversible contribution (classical limit of the Hamiltonian term), and the diffusion matrix $D(x) \succeq 0$ encodes the dissipative contribution (classical limit of suitable Lindblad terms). [15, 17]

What is structurally preserved in the semiclassical limit

This makes the connection to the FBA logic clear: the distinction "reversible vs. dissipative" is preserved; only the language changes from operators to densities. Monotonicities also have classical counterparts: DPI corresponds to contraction of relative entropy under stochastic (Markov) maps, and Spohn-type statements become (under suitable stationary references) classical entropy production rates in the Markov flow.[a] [15, 16]

[a] See FBA Part VIII: Classical Limit, Thermodynamics & Aging, Secs. VIII.6–VIII.8 "DPI/Spohn & Thermodynamic Production".

IV.6.2 Examples of GKLS processes

The following examples serve as orientation for which physical situations are described by GKLS generators and how their semiclassical limit is typically read. They are deliberately kept close to protocols: the goal is not to exhaust a special model, but to make the structure "admissible dynamics ⇒ cleanly separated reversible and dissipative contributions" visible.

IV.6.2.1 Decoherence in a two-level system

Decoherence is the prototypical situation in which a system continues to evolve dynamically, but loses phase information. In the GKLS picture this appears as a dissipative term that damps certain coherences without changing the trace.

Phase decoherence of a qubit (dephasing)

For a qubit with Hamiltonian operator H and dephasing along the Pauli-z matrix $\sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$, a GKLS generator is

$$\mathcal{L}(\rho) = -i[H, \rho] + \frac{\gamma}{2}(\sigma_z \rho \sigma_z - \rho), \qquad \gamma \geq 0.$$

The Hamiltonian term is reversible; the dissipative term damps off-diagonal entries in the σ_z basis and models the loss of phase information due to an unobserved environment. [5, 6]

From a semiclassical point of view, the dissipative contribution corresponds to effective noise that smears fine structure in the conjugate variables. Which classical drift–diffusion description emerges depends on the chosen observable family and the scaling regime, not on the formula for $\mathcal{L}$ alone.

IV.6.2.2 Thermodynamic processes in open systems

Thermodynamic situations typically correspond, within the GKLS framework, to a system interacting with a bath over long times. Here the semiclassical transition is particularly important because it shows how energy flows, relaxation, and entropy production emerge as macroscopic quantities from admissible microscopic dynamics.

Ornstein–Uhlenbeck limit as a classical limiting case (schematic)

A common semiclassical end product is a drift–diffusion dynamics for a macroscopic state variable x of the form

$$\partial_t f(x,t) = \partial_x\big(\lambda\, x\, f(x,t)\big) + \frac{D}{2}\,\partial_x^2 f(x,t), \qquad \lambda > 0,\ D > 0,$$

which combines relaxation (drift) and fluctuations (diffusion). In the FBA reading, the drift carries the effective restoring flow, while D encodes the strength of the dissipative, irreversible influence. [17, 18]

The thermodynamic evaluation (entropy production, flows, stationary states and their stability) then takes place at the level of the classical density f and is directly tied to DPI/Spohn. This is exactly where the advantage of the FBA becomes visible: irreversibility is not introduced as an additional interpretation, but read as a robust monotonicity structure of admissible processes.[30]

Transition. After the semiclassical connection has been sharpened as a controlled coarse-graining limit, we formulate irreversibility again purely operationally in the following Section IV.7: DPI and Spohn provide the arrow for *unselected* processes.

[30]See FBA Part VIII: Classical Limit, Thermodynamics & Aging, Secs. VIII.6–VIII.8 "DPI/Spohn & Irreversible Thermodynamics".

IV.7 FBA and Irreversibility

Sections IV.3 to IV.5 fixed admissible dynamics in such a way that we can formulate irreversibility no longer as an interpretation, but as a *structural statement*: coarse-graining (discarding information about ancillary registers or outcomes) is a CPTP step, and CPTP steps contract distinguishability. In the FBA, it is precisely this loss of distinguishability that is booked as an irreversible budget contribution. Thus the "arrow of time" is not an additional assumption, but becomes operationally visible within this separation between unselected channels and post-selection.[31]

IV.7.1 Irreversibility from budget flows

In the budget calculus there is a contribution that, by definition, cannot be recovered as reversible internal clock power. This quantity is the operational interface between "bookkeeping" and "irreversibility": as long as a step is fully reversible, one can invert it in the description without shifting additional bookkeeping into the irreversible contribution. As soon as information is discarded (environment not resolved, outcome not tracked), a positive irreversible contribution arises.[32]

Definition IV.7.1.1: Irreversible budget flow and aging

The *irreversible budget flow* δb_{irr} is the part of a step that cannot be realized as a reversible internal flow $\delta b_{\mathrm{int}}^{\mathrm{rev}}$.

For the translation into the language of time we use the time calibration κ_τ from Part II; the notation conventions are collected in Section IV.2.[a] We define in the continuum:

$$d\tau_{\mathrm{geo}} \equiv \frac{db_{\mathrm{int}}^{\mathrm{rev}}}{\kappa_\tau}, \qquad dA \equiv \frac{db_{\mathrm{irr}}}{\kappa_\tau} \geq 0, \qquad d\tau_{\mathrm{tot}} = d\tau_{\mathrm{geo}} + dA.$$

We call A the *aging functional. (Discrete analog:* $\Delta\tau_{\mathrm{geo}} = \Delta b_{\mathrm{int}}^{\mathrm{rev}}/\kappa_\tau$, $\Delta A = \Delta b_{\mathrm{irr}}/\kappa_\tau \geq 0$, $\Delta\tau_{\mathrm{tot}} = \Delta\tau_{\mathrm{geo}} + \Delta A$.*)*

[a] See FBA Part II: Time, Proper Time & Minkowski Geometry, Secs. II.5–II.9 "Calibration, Quadric & Cone Structure".

This makes the logic clear: to make irreversibility robust, one needs a criterion that says, without additional interpretation, when coarse-graining is truly "not undoable". In the FBA, this criterion comes from data processing: unselected CPTP channels can only reduce distinguishability between states.

[31] See FBA Part I: FBA – Foundations, Sec. I.5 "DPI Arrow, Spohn Monotonicity & No-Recovery".

[32] See FBA Part I: FBA – Foundations, Secs. I.3–I.5 "Budget Balance, CPTP & Measurement as Coarse-Graining".

Formula Box IV.7.1.1: DPI as an operational statement of irreversibility

Let Φ be a CPTP channel and ρ, σ two states. For the relative entropy

$$S(\rho\|\sigma) \equiv \mathrm{Tr}\big(\rho(\log\rho - \log\sigma)\big)$$

(valid for $\mathrm{supp}(\rho) \subseteq \mathrm{supp}(\sigma)$, otherwise $S(\rho\|\sigma) = +\infty$) the data-processing inequality (DPI) holds:

$$S(\Phi(\rho)\|\Phi(\sigma)) \;\le\; S(\rho\|\sigma).$$

The difference

$$\Delta_\Phi(\rho,\sigma) \;\equiv\; S(\rho\|\sigma) - S(\Phi(\rho)\|\Phi(\sigma)) \;\ge\; 0$$

is the (pair-dependent) *contraction deficit* of the relative entropy under Φ and quantifies operationally the loss of distinguishability between the hypotheses ρ and σ. [3, 7, 8]

What DPI means here (without presupposing thermodynamics)

The DPI statement is deliberately not thermodynamically loaded: it first only says what is testable in protocols – namely whether two preparations become less distinguishable after an admissible, *unselected* step. A thermodynamic reading arises only once one chooses σ as a suitable reference (e. g. a stationary state or a Gibbs reference in a model). Then the information loss can be interpreted as production. Exactly at this point, Spohn provides a rate form of this monotonicity for GKLS flows (under the respective required regularity/stationarity assumptions).[6, 9]

IV.7.2 Thermodynamic consequences

Thermodynamics does not begin here with heat and work, but with a robust monotonicity: for Markov flows, the information loss is not only global (DPI), but has a local rate form once the time derivative is well-defined. This is exactly the structure needed to capture "entropy production" in the sense of a second law, without building in post-selection.

Formula Box IV.7.2.1: Spohn monotonicity and entropy production rate

Let $\{\mathcal{T}_t\}_{t\ge 0}$ be a CPTP Markov semigroup (in the finite-dimensional or norm-continuous GKLS setting) and let σ be a stationary state, i. e. $\mathcal{T}_t(\sigma) = \sigma$ for all $t \ge 0$. Then $t \mapsto S(\rho_t\|\sigma)$ is monotonically decreasing, where $\rho_t = \mathcal{T}_t(\rho_0)$. If $t \mapsto S(\rho_t\|\sigma)$ is differentiable, then (Spohn)

$$\frac{d}{dt} S(\rho_t\|\sigma) \;\le\; 0.$$

Thus the entropy production rate

$$\dot{\Sigma}(t) \;\equiv\; -\frac{d}{dt} S(\rho_t\|\sigma) \;\ge\; 0$$

is nonnegative. In particular, $\dot{\Sigma}(t) = 0$ when $\rho_t = \sigma$ (stationary trajectory); further equality conditions are model- and generator-dependent. [6, 9]

Why Spohn is the bridge to bookkeeping in the FBA

In the FBA, the practical value of this statement is twofold:

- **Unselected vs. selective:** Spohn (like DPI) refers to *unselected* dynamics. Selective updates are CP and trace-non-increasing; by conditioning they can seemingly “lower entropy”, but they always use an outcome register and thus do not belong to the same unselected balance logic (cf. Subsection IV.4.1).
- **Connection to the irreversible budget contribution:** If the process destroys distinguishability at the system level, then in the FBA this is exactly the part that is booked as the irreversible budget flow dA. The thermodynamic calibration of this assignment (scales, k_B, concrete flows) is made precise in Part VIII.[a]

[a]See FBA Part VIII: Classical Limit, Thermodynamics & Aging, Secs. VIII.6–VIII.8 “DPI/Spohn & Thermodynamic Production”.

What already follows here, and what is deliberately deferred

- **Follows here:** irreversibility for unselected dynamics is a monotonicity statement (DPI/Spohn) and thus protocol-robust. coarse-graining is the structural source of this monotonicity.
- **Not spelled out here:** identifying $\dot{\Sigma}$ with classical quantities such as heat flow and entropy balance requires additional model assumptions (baths, stationary references, scale limits). Exactly this translation is developed in the classical limit and in the thermodynamics treatise.[a]
- **Consequence for Part IV:** once we cleanly separate measurement, environment, and unselected updates in concrete models, “no anti-irreversibility trick” is possible anymore: a genuine violation of DPI/Spohn would be a direct fail of the admissibility assumptions, not merely an interpretive dispute.

[a]See FBA Part VIII: Classical Limit, Thermodynamics & Aging, Secs. VIII.1–VIII.5 “Classical Limit & Thermodynamic Structure”.

This sharpens the role of irreversibility in the FBA for open systems: it is a controlled effect of the admissible process class under coarse-graining, and it appears at the same time as the irreversible budget contribution dA in bookkeeping. In the conclusion Section we will cast this structure into a compact pass/fail form and mark the interfaces to thermodynamics and gravity.

IV.8 Conclusions

This treatise had a clear goal: in the FBA, dynamics, measurement, and irreversibility are not to appear as separate Sections of an interpretation, but as consequences of the same admissibility class that remains stable under composition, ancillary registers, and coarse-graining. This is exactly what makes it intelligible why CPTP, GKLS, and DPI/Spohn are not interchangeable "models" here, but the structurally appropriate objects once one takes open systems operationally seriously.

IV.8.1 Summary of the main results

What is fixed here once one accepts CPTP as admissibility

- **Measurement is not a special case.** A measurement is a CP decomposition of a CPTP step, i. e. an instrument with an outcome register and a state update (Definition IV.4.1.1 and Lemma IV.4.1.1). The POVM is the effect family induced by the instrument, and the Born coupling remains the consistent statistical formula.
- **Coarse-graining is the operational origin of irreversibility.** Unselected measurement is CPTP coarse-graining; it reduces distinguishability between preparations. Exactly this statement is formulated as DPI (Formula Boxes IV.4.3.1 and IV.7.1.1).
- **The Markov limit is a controlled limiting description.** If the effective dynamics is memoryless and continuous, it carries a semigroup structure and has a generator (Definition IV.5.1.1 and Formula Box IV.5.1.1).
- **GKLS is the admissible generator form in the continuum.** The GKLS form characterizes generators that preserve CPTP in the semigroup flow (Lemma IV.5.2.1).
- **Spohn turns DPI into a rate form.** For GKLS flows, the decrease of relative entropy with respect to a stationary reference is a local monotonicity, and the entropy production rate is nonnegative (Formula Box IV.7.2.1).

Section IV.6 additionally situates this structure: it shows under which smoothness and scale assumptions GKLS dynamics can pass over into drift–diffusion equations (Fokker–Planck type), and why this step is controlled coarse-graining rather than a purely formal replacement $\hbar \to 0$.

IV.8.2 Outlook to other parts of the FBA series

The next progress arises not from more formalism, but from two targeted interfaces: first, the *local* structure (field/causality language), second, the *thermodynamic* translation (scales, flows, production). Both are deliberately outsourced in the series so that the deductive line here does not fray.

Route map: where the results are developed further

- **Causality, locality, field structure.** The light-cone/front structure calibrated in Part II provides the reference causality.[a] Part V builds local generators, microcausality, and continuity equations on top of this.[b]
- **Gravity as deviation from the flat limit.** When budgets become inhomogeneous, backreaction appears as an effective geometry; this is worked out in the continuum in Part VI.[c]
- **Thermodynamics, aging, entropy production.** The DPI/Spohn structure used here is translated into classical quantities (flows, stationary states, production) in Part VIII, including the role of irreversible contributions as aging.[d]
- **Scales and renormalization.** As soon as one compares concrete models, one must clarify which scales are convention-dependent and which remain invariant; in the FBA this is bundled independently in Part VII.[e]
- **Tests and pass/fail criteria.** The series does not end with "explanations", but with protocols. A consolidated catalog of which observations CPTP/GKLS/DPI support or falsify within the FBA is given in Part X.[f]

[a] See FBA Part II: Time, Proper Time & Minkowski Geometry, Secs. II.5–II.9 "Calibration, Quadric & Cone Structure".

[b] See FBA Part V: Spacetime, Light Cones & Local Field Theory, Secs. V.3–V.6 "Light Cones, Microcausality & Local Dynamics".

[c] See FBA Part VI: Gravity & Geometry from Budget Flows, Secs. VI.1–VI.5 "Geometry/Gravity from Budget Flows".

[d] See FBA Part VIII: Classical Limit, Thermodynamics & Aging, Secs. VIII.6–VIII.8 "DPI/Spohn & Aging Measure".

[e] See FBA Part VII: Constants, Scales & Renormalization, Secs. VII.1–VII.4 "Scales, Constants & Renormalization".

[f] See FBA Part X: Predictions, Falsifiability & Bridge FBA $\rightarrow$ QM $\leftrightarrow$ GR, Secs. X.6–X.8 "Predictions, Tests & Comparison".

This makes the role of this treatise within the overall arc unambiguous: we have fixed the dynamical and measurement-theoretic infrastructure in such a way that irreversibility can be formulated as a robust monotonicity of unselected processes. Everything that comes afterwards – local field densities, thermodynamic flows, or gravitational backreaction – builds on exactly this stability under composition and coarse-graining.

IV.9 Appendix: Overview of the FBA Series (Parts I–X)

Click the title to download the PDF

1. ***Part I*: FBA-Foundations: Ordering, Budget, Proper Time & Arrows.** *Goal:* Provide the base layer: ordering, budget, proper time/aging, front and the operational arrow of time (DPI); Minkowski limit from the budget quadric; admissible dynamics and locality/no-signalling. *Import:* – (reference for all subsequent parts). *Extension:* interface contracts, pass/fail checklists, reading guide.
2. ***Part II*: Time, Proper Time & Minkowski Geometry.** *Goal:* Capture proper time/quadric operationally and derive geodesics. *Import:* foundations (ordering, budget, proper time, front/DPI). *Extension:* smooth limit, variational principle on worldlines, calibration κ_τ.
3. ***Part III*: Quantum Kinematics & CPTP Channels.** *Goal:* State spaces and channels (CPTP) including composition. *Import:* foundations (budget, channel viewpoint, composition). *Extension:* concrete divergences/cost functionals $\mathcal{C}$, measurements, and classical registers.
4. ***Part IV*: Dynamics, Measurement & GKLS (Open Systems).** *Goal:* Continuous open dynamics (GKLS) and the operational arrow of time. *Import:* channels/DPI. *Extension:* Spohn monotonicity, stationary/NESS references, flows $b^{\mathrm{rev}}, b^{\mathrm{irr}}, b^{\mathrm{ext}}$.
5. ***Part V*: Spacetime, Light Cones & Local Field Theory.** *Goal:* Local field equations under front/locality. *Import:* front, composition, no-signalling. *Extension:* local GKLS generators, Lieb–Robinson-type bounds, effective light cones.
6. ***Part VI*: Gravity & Geometry from Budget Flows.** *Goal:* Geometrization of budget flows. *Import:* budget quadric/proper time. *Extension:* effective metrics from calibrations (κ_t, κ_x) and internal stresses; coupling to curvature.
7. ***Part VII*: Constants, Scales & Renormalization.** *Goal:* Scale running of the calibration theorems. *Import:* $c = \kappa_t/\kappa_x$, κ_τ. *Extension:* flow equations for $\kappa_t, \kappa_x, \kappa_\tau$; stability of c.
8. ***Part VIII*: Classical Limit, Thermodynamics & Aging.** *Goal:* Macroscopic behavior from $A[\gamma]$ (aging) and DPI. *Import:* proper time/aging, Spohn. *Extension:* entropy production, Euler–Lagrange forms for irreversible flows, effective transport equations.
9. ***Part IX*: Cosmic Dynamics, Time Dilation & Inflation (TDI).** *Goal:* Cosmic ordering & calibration flow. *Import:* budget, proper time/front. *Extension:* budget equations on large-scale slices; time-dilation inflation as calibration dynamics.
10. ***Part X*: Predictions, Falsifiability & Bridge FBA → QM ↔ GR.** *Goal:* Testable differences and bridges FBA ↔ QM/GR. *Import:* all foundational building blocks. *Extension:* protocols, limiting-case tests, overdetermined consistency relations (pass/fail).

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