The Frame–Budget Approach (FBA)

How time, dynamics, and geometry emerge from budget flows

An operational bridge between quantum mechanics and general relativity

Part II: Time, Proper Time & Minkowski Geometry

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Part II

Time, Proper Time & Minkowski Geometry

II.1 Introduction & Target Picture

II.1.1 Motivation

The Frame–Budget Approach (FBA)[1] replaces a presupposed time parameter by an ordering of global states according to minimal differences: a frame change is not progress along an external axis, but the realization of at least one minimal event. Time therefore becomes measurable only *downstream*: as what accumulates as the integrated *internal* budget flow of a system along its evolution. To make this internal quantity comparable across systems, an external calibration is additionally required; it is fixed via signal fronts and sets the scales, in particular the operational limiting rate c.[2] The key gain of this perspective is that geometry is not introduced as a stage: from budget-faithful bookkeeping and front calibration there emerges an invariance structure whose homogeneous limit becomes a Minkowski quadric.

II.1.2 Logical Path

The exposition is constructed so that each new object appears only once it has been fixed operationally and no circularity arises:

1. **Sequence.** Global frames and minimal events provide the minimal structure required to speak meaningfully of "before" and "after".
2. **Budget.** Each step obeys a balance law. Without it there would be no integrable quantity from which proper time can be extracted as a measurable observable.
3. **Calibration.** A signal front defines the maximal propagation rate c metrologically. Only then do internal rates and external relations become comparable across systems.
4. **Quadric.** Under homogeneity assumptions and budget-faithful composition, the balance condenses into a quadratic invariance, whose continuous limit yields Minkowski geometry with light cones and Lorentz isometries. Time dilation then appears not as an additional postulate, but as a balance-sheet redistribution from the internal to the external share under motion.

Why this order? Without (1) the update order would be undefined; without (2) there would be no measurable accumulation of "proper time"; without (3) the physical scale for comparisons would be missing; and only with (4) does the causal structure become visible as geometry. The necessary primitives and consistency conditions are imported from the FBA – Foundations and are bundled in Section II.2 so that later deductions can proceed without revisiting foundational debates.

[1] An overview of all parts of the FBA series, including download links, can be found in Section II.12 of this document.

[2] See FBA Part I: FBA – Foundations, Secs. I.1–I.3: sequence, budget & calibration.

II.1.3 Scope and Delimitation

In *Part II* we work in the *flat, kinematical limit*: locally inertial, without spacetime curvature (homogeneous budgets, $\nabla b = 0$) and without backreaction. The goal is to fix front calibration and the budget quadric so that the Minkowski limit is unambiguous as a reference case. Once budget densities become inhomogeneous ($\nabla b \neq 0$), an effective curvature emerges and with it gravitational time dilation; this extension is developed in *Part VI: Gravitation & Geometry from Budget Flows*.[3] Questions of scale and possible running of normalizations are treated in *Part VII: Constants, Scales & Renormalization*.[4]

II.1.4 Remark: Flat Limit vs. Emergent Curvature

The Minkowski geometry used in Secs. II.6–II.8 is not a postulate, but the reference limit of homogeneous budgets: budget quadric $\rightarrow$ light cones and Lorentz isometries. Deviations from it measure budget gradients and appear in Part VI as curvature and gravitational time dilation. Empirical relevance is operationalized in Part X via pass/fail criteria.[5]

II.1.5 Contribution Beyond SR

Instead of postulating the speed of light and inertial symmetry, c and the Lorentz structure are *derived* from calibration and the quadric. This shifts the status of time dilation: it is not primarily a coordinate effect, but a balance statement about how a fixed step budget is split between internal progress (proper time) and external relation (motion). At the same time, the FBA strictly separates reversible τ_{geo} from irreversible aging as a share of the internal budget; for unselective processes this separation is supported by DPI/Spohn via a monotonicity structure that provides an additional, measurable signature beyond standard SR.[6][7] The test horizon is formulated in Part X as a compact pass/fail catalogue.[8]

II.1.6 Reading Guide

The Section order is chosen so that we (i) establish the update order, (ii) define an integrable observable, (iii) fix scales, and (iv) only then geometrize.

Section II.2 – Foundations & Conventions: We bundle terminology, notation, and measurement protocols in one place so the subsequent deductions do not rely on tacit additional assumptions.[9] *Why first?* Without a shared language, difference ordering, balance, and calibration are not unique, and later conclusions would be interpretive rather than forced.

Section II.3 – Time from Difference: We construct an ordering of global states *without* presupposed time and show how this ordering becomes visible in protocols as a "step

[3] See FBA Part VI: Gravitation & Geometry from Budget Flows, Secs. VI.3–VI.5: budget geometry, field equations, redshift.

[4] See FBA Part VII: Constants, Scales & Renormalization, Secs. VII.1–VII.4: calibration, scale flow & RG.

[5] See FBA Part X: Predictions, Falsifiability & Bridge FBA $\rightarrow$ QM $\leftrightarrow$ GR, Secs. X.6–X.7: predictions & experiments.

[6] See FBA Part I: FBA – Foundations, Sec. I.4: proper time & aging.

[7] See FBA Part VIII: Classical Limit, Thermodynamics & Aging, Secs. VIII.6–VIII.8: DPI/Spohn & an aging measure.

[8] See FBA Part X: Predictions, Falsifiability & Bridge FBA $\rightarrow$ QM $\leftrightarrow$ GR, Secs. X.6–X.8: tests & comparison.

[9] See FBA Part I: FBA – Foundations, Secs. I.1–I.3: sequence, budget & calibration.

structure". *Why now?* Only once "which step follows which" is precise can a budget flow be defined as a physical, integrable quantity.

Section II.4 – Section II.5 – Proper Time, Aging & Front Calibration: We define proper time/aging as internal quantities and calibrate c via front protocols. *Why before geometry?* Because geometry in the FBA cannot be chosen freely: it must be tied to measurement operations (front, budget balance), otherwise one would effectively presuppose the later quadric.

Section II.6 – Budget Quadric & Minkowski Limit: The balance yields a quadric; its homogeneous limit delivers Minkowski geometry together with a cone structure of operational meaning.[10] *Why here?* Only after calibration is it clear which combinations of internal and external shares must be invariant so that all observers reconstruct the same front bound and thus the same causal structure.

Section II.7 – Relativity & Lorentz Isometries: We show Lorentz symmetries as a consequence of the budget invariance of the quadric. *Why important?* This turns inertial symmetry into a derived consistency criterion of bookkeeping rather than a starting postulate.

Section II.8 – Time Dilation as Budget Redistribution: Time dilation appears as a redistribution between internal and external share under motion. *Added value:* The effect gains a measurement-near interpretation directly tied to protocols (front costs, balance) and thereby provides handles for deviation tests.

Section II.9 – Section II.11 – Context, SR Comparison & Pass/Fail: We compare systematically to SR and formulate a checklist mapping each core claim to observables.[11] *Goal:* No result should merely be "plausible"; each should stand as a protocol decision with explicit failure modes.

[10] See FBA Part V: Spacetime, Light Cones & Local Field Theory, Secs. V.3–V.5: light cones & microcausality.

[11] See FBA Part X: Predictions, Falsifiability & Bridge FBA → QM ↔ GR, Secs. X.6–X.8: tests & comparison.

II.2 Preliminaries & Conventions (Import from Part I: FBA – Foundations)

Why an import? The following Sections are meant to provide a single continuous line of deduction: from the sequence (without presupposed time) via balance and calibration to the quadric and its Minkowski limit. To keep this path from fraying into repeated foundational Sections, we adopt the required primitives from the FBA – Foundations *unchanged.* The practical effect is twofold: (i) in the running text we can work consistently with the same terms and symbols; (ii) the burden of justification and proof sits where it belongs—with the primitives and ground assumptions and their first elaboration in Part I. Here we therefore do not "define again" what the building blocks mean, but rather which building blocks we use as tools in order to proceed from Section II.3 to 8 without circularity.

Imported building blocks (unchanged)

Source: [a]

- **Sequence of global states & minimal events:** frames/sequence and minimal event (ME), as well as co-actuality and refinement invariance (Sec. I.2).
- **Difference function & operational minimal difference:** a difference measure as the operational basis for building an order (Sec. I.2).
- **Budget calculus (internal/external/irreversible) & balance:** one-step budget, balance equations, and refinement invariance of the balance (Sec. I.3).
- **External calibration & front:** calibration and front costs, front bound, and signal front (Sec. I.3).
- **Proper time & aging, Minkowski limit:** proper time, aging (irreversible), Minkowski limit and time dilation (Sec. I.4).
- **Admissible dynamics (CPTP/GKLS), DPI/Spohn:** CPTP as admissible channels, measurement as CPTP, GKLS generators, Spohn monotonicity, the DPI arrow, and no-recovery (Sec. I.5).
- **Composition, locality & no-signalling:** symmetric monoidal structure, budget additivity, local operations, causal cones, and local GKLS (Sec. I.6).

[a] See FBA Part I: FBA – Foundations, Secs. I.2–I.6: sequence, difference function, budget & calibration, proper time/aging, CPTP/GKLS, composition/locality.

What is the role of this import in the reading path? The points above are exactly the "load-bearing beams" we do not want to re-argue later, but to use: Section II.3 uses the sequence and the difference function to construct a time order without an external clock; Section II.4 then sharpens proper time/aging as integrated budget contributions; Section II.5 fixes c via front protocols; Sections II.6 to II.8 drive the balance to the quadric and read off Minkowski structure, Lorentz isometries, and time dilation. Where new steps are needed later, we give the proof idea in the running text; full proofs and axiomatic details remain in Part I so that the argument line here stays tight.

So that the following Sections can be read without silent convention changes, we now fix the notation. Important: Minkowski language appears here only as *notation*; its physical justification comes only once the budget quadric has been derived in Section II.6.

Notation & Conventions

- **Discrete vs. continuum:** step index $n \in \mathbb{Z}$ for successive update steps $U_n \to U_{n+1}$ (frames). A single update step may bundle one or more minimal events (ME). $\Delta(\cdot)$ for discrete increments, $d(\cdot)$ for differential quantities in the limit.
- **Budget decomposition:** per step we distinguish *external* budget δb_{ext} as well as the *internal* share, which splits into a reversible and an irreversible contribution:

  $$\delta b_{\text{int}} = \delta b_{\text{int}}^{\text{rev}} + \delta b_{\text{irr}}, \qquad \delta b_{\text{irr}} \geq 0.$$

  Path integrals $\sum \delta(\cdot)$ resp. $\int d(\cdot)$. (Cumulative budgets are correspondingly $\Delta B_* = \sum \delta b_*$ in the discrete setting.)
  Aging is the calibrated irreversible accumulation

  $$dA := \frac{db_{\text{irr}}}{\kappa_\tau} \geq 0.$$

  Geometric proper time is the calibrated reversible internal accumulation

  $$d\tau_{\text{geo}} := \frac{db_{\text{int}}^{\text{rev}}}{\kappa_\tau},$$

  and coincides in the (nearly) reversible limit with the reversible proper time τ_{rev} used in Part I. The *total proper time* along a path is

  $$d\tau_{\text{tot}} := \frac{db_{\text{int}}}{\kappa_\tau} = d\tau_{\text{geo}} + dA.$$

- **Calibration:** external calibration uses κ_t, κ_x (and κ_τ for internal time calibration). The front constant of the fastest admissible signal fronts is thus

  $$c := \frac{\kappa_t}{\kappa_x},$$

  i.e., c is not postulated but metrologically fixed by the defined front protocol.
- **Spacetime language (flat, kinematic):** four-vector $x^\mu = (ct, x, y, z)$; Minkowski signature $\eta = \text{diag}(-1, 1, 1, 1)$. *Light cones* given by $\eta_{\mu\nu} dx^\mu dx^\nu = 0$.
- **Worldlines & paths:** γ denotes a worldline of a system through the frame sequence; concatenation $\Gamma = \Gamma_1 \circ \Gamma_2$; additivity of all integrated budgets along Γ.
- **Composition/locality:** parallel composition $\otimes$; serial composition $\circ$. Local CPTP operations respect no-signalling and budget additivity.
- **Sign conventions:** vector norms $\|\cdot\|$; Euclidean dot products "$\cdot$" in space; c explicit (no c=1 units in this part). expectations $\mathbb{E}[\cdot]$; supremum sup.

Consequence: With these imports and conventions, the following Sections can build directly on the balance and calibration structure—without renewed definitional work and without hidden notation changes.

II.3 Time from an Order of Minimal Differences

In this Section we take the first, decisive step: we separate *order* from *scale*. The succession of global frames provides only a "earlier/later" relation—no clock yet. We close exactly this gap operationally, without presupposing an external time: we define a time scale as a *strictly increasing embedding* of the (ordinally given) sequence into the real numbers. Thus, "time" is initially an admissible parametrization of the ordering, not yet a metrologically fixed quantity. The required building blocks (global frames and minimal events, co-actuality/refinement invariance, as well as the difference function and the operational minimal difference) were introduced in Part I.[12]

The central idea is: from the operational minimal difference we obtain a *countable* structure that partitions the sequence into "steps". What matters is the logic fixed in the FBA – Foundations: an update step $U_n \to U_{n+1}$ may bundle one or more minimal events (MEs); MEs are the smallest realized changes, and the step index counts updates. A "time" is then a monotone reparametrization of this step index. The physical scale (i.e., the unit of t) is fixed only later via calibration using signal fronts.[13]

[12]See FBA Part I: FBA – Foundations, Section I.2: Global states, frame sequence, minimal event (ME), co-actuality/refinement invariance, difference function & operational minimal difference.

[13]See FBA Part I: FBA – Foundations, Section I.3: Calibration & front costs.

Definition II.3.1: Time as a strictly increasing embedding

Source: [a]

Let $(F, \prec)$ be the strict order on global frames generated by minimal events. We consider a representation of the sequence as a chain

$$F_0 \prec F_1 \prec F_2 \prec \dots$$

(frames in temporal order), where adjacent frames in this representation differ operationally only minimally. The associated step index is then the numbering $n(F_k) = k$, i.e. $n : F \supset \{F_k\} \to \mathbb{Z}$. Under refinements, additional intermediate frames may be inserted without changing the order content (co-actuality/refinement invariance).
A *time* is a map $t : F \to \mathbb{R}$ with:

1. **Monotonicity:** $F \prec F' \Rightarrow t(F) < t(F')$.
2. **Step-compatibility:** For adjacent frames of the chosen chain,

   $$n(F') = n(F) + 1 \;\Rightarrow\; t(F') > t(F),$$

   and equivalently there exists a strictly increasing function $\phi : \mathbb{Z} \to \mathbb{R}$ with $t = \phi \circ n$ *on the chain* (see Formula Box II.3.1).
3. **Refinement invariance:** Under a refinement (inserting additional intermediate frames), the order content is preserved; different step indices of different representations are related by a strictly increasing renumbering. Accordingly, t is determined only up to a strictly increasing reparametrization ϕ.
4. **Freedom of calibration:** The unit and scale of t are *not* part of the definition and are fixed metrologically only via external calibration (signal front).

[a] See FBA Part I: FBA – Foundations, Sections I.2 – I.3: sequence/frames/MEs, co-actuality/refinement, difference function, calibration.

Why this definition? Monotonicity ensures that t truly reflects only the ordering. Step-compatibility is the point where a mere order becomes a “countable” structure: if a representation is chosen in which adjacent frames differ only minimally in an operational sense, then an admissible time scale must not introduce additional, physically unmotivated substructure “between” two neighboring steps. Refinement invariance prevents spurious information by ensuring that any finer representation carries the same order content and merely induces a strictly increasing renumbering. Freedom of calibration keeps the definition clean: without a front protocol there is no reason why Δt should take a particular numerical value.

Formula Box II.3.1: Properties of the time embedding

On a chosen chain $F_0 \prec F_1 \prec \cdots$ with step index $n(F_k) = k$, every time t that is monotone and step-compatible satisfies

$$t = \phi \circ n \quad \text{with} \quad \phi : \mathbb{Z} \to \mathbb{R} \text{ strictly increasing.}$$

A frequently used *uniform* choice is the affine parametrization

$$\phi(k) = t_0 + \alpha_t\, k, \qquad \alpha_t > 0,$$

i.e. discretely $\Delta t = \alpha_t$ per step. In the continuum picture of a strongly refined representation one may use a dimensionless parameter λ that locally approximates the (renumbered) steps; then locally

$$dt = \alpha_t\, d\lambda.$$

Here α_t is a pure convention and unit choice on the *uncalibrated* order; the metrological fixation of the time scale occurs only via front calibration (cf. Section II.5).

Proof Sketch II.3.1: Properties of the time embedding

On the chosen chain, define $\phi(k) := t(F_k)$. Monotonicity of t along $F_0 \prec F_1 \prec \cdots$ implies $\phi(k+1) > \phi(k)$, i.e. strict increase. Step-compatibility ensures that t varies on the chain only through the step count, so indeed $t(F_k) = \phi(n(F_k))$.

Consequence for the further construction: Up to this point, "time" is an admissible parametrization of the sequence—nothing more. This allows us to cleanly separate two things in the next Sections: (1) *which* evolution steps a system undergoes (order, representation as a chain, step index), and (2) *how much* internal versus external resource is realized per step (budget). Only from (2) do we later obtain proper time as the integrated internal contribution, and only front calibration makes the scale of t compatible with lengths.

Operational reading and the "film" analogy

The sequence $F_0 \prec F_1 \prec \ldots$ can be read as a sequence of global states. The difference function ensures that adjacent frames in a suitable representation differ only minimally, so the step index n provides a physically motivated "count" of this sequence.

The direction of the logic matters: it is not that "time numbers the frames"; rather, a time scale $t : F \to \mathbb{R}$ is, by definition, a strictly increasing embedding that assigns numerical values to an already given order. Before calibration, this assignment is conventional up to strictly increasing reparametrizations ϕ. Physical statements in this section therefore depend only on the order (and the choice of a representation as a chain), not on a particular choice of ϕ.

II.4 Proper Time & Aging as Integrated Internal Budget

Section II.3 established a time order as a strictly increasing embedding of the update sequence—yet still without a physical scale. The next step therefore has to provide an *intrinsic* quantity that a system accumulates along its worldline and that remains stable under concatenation as well as under refinements.

This is exactly what the budget calculus delivers: per step there is an external share and an internal share; the internal share further splits into a reversible and an irreversible contribution, and this decomposition is refinement-safe.[14] This yields a natural definition of *geometric proper time* as the integrated reversible internal flow (after calibration via κ_τ). The irreversible share provides a second, conceptually distinct quantity, which we interpret as *aging*. This separation is not cosmetic: it is necessary in order to later discuss and test geometric time dilation (reversible) separately from dissipative effects (irreversible).

We keep to the guiding idea: first define the quantities so that they do not depend on any coordinate choice and only depend on the worldline and the stepwise balance; only afterwards is the external comparison scale (and thus c) fixed operationally via front calibration (see Section II.5). The conversion "budget → time" for the internal accounts is carried out via the proper-time calibration κ_τ (unit matching), as introduced in the FBA – Foundations.[15]

Definition II.4.1: Geometric proper time

Standard reference: Minkowski/SR proper time [1, 2].[a]

Let S be a system with worldline γ through the sequence of global frames. Let $\delta b_{\text{int}}(n)$ be the internal budget account of S at step $n \to n+1$, and $\delta b_{\text{int}}^{\text{rev}}(n)$ its reversible share. Let $\kappa_\tau > 0$ be the (global) proper-time calibration.

The *geometric proper time* of S on a path segment $\Gamma \subset \gamma$ is the calibrated integral of the reversible internal share:

$$\tau_{\text{geo}}[\Gamma] \equiv \frac{1}{\kappa_\tau} \sum_{n\to n+1\in\Gamma} \delta b_{\text{int}}^{\text{rev}}(n), \qquad \text{in the continuum:} \qquad d\tau_{\text{geo}} \equiv \frac{db_{\text{int}}^{\text{rev}}}{\kappa_\tau}.$$

Here $\delta b_{\text{int}}^{\text{rev}}$ denotes the reversible share of the internal flow. The associated decomposition

$$\delta b_{\text{int}} = \delta b_{\text{int}}^{\text{rev}} + \delta b_{\text{irr}}, \qquad \delta b_{\text{irr}} \geq 0,$$

is fixed by the criteria given in the FBA – Foundations. In the (nearly) reversible limit, τ_{geo} coincides with the reversible proper time τ_{rev} used in Part I.

[a] See FBA Part I: FBA – Foundations, Sections I.3 - I.4: One-step budget/decomposition and proper time & aging.

Why must geometric proper time be tied to the reversible share? Because we want to use τ_{geo} as a geometric quantity that, in the kinematic limit, appears as the line element of a quadric. Irreversible contributions do add to the "experienced" total duration, but they are not exchangeable by a mere change of coordinates or motion. This distinction later underlies the identification of time dilation as a balance redistribution (reversible) without folding in

[14] See FBA Part I: FBA – Foundations, Section I.3: Balance equations & invariances.
[15] See FBA Part I: FBA – Foundations, Section I.4.3: External calibration and the choice of κ_τ.

dissipative aging effects.

Formula Box II.4.1: Properties of geometric proper time

Standard reference: additivity/proper time as line length in the SR limit [2].[a]

For every path segment $\Gamma \subset \gamma$ the following hold:

1. **Additivity:** $\Gamma = \Gamma_1 \circ \Gamma_2 \Rightarrow \tau_{\text{geo}}[\Gamma] = \tau_{\text{geo}}[\Gamma_1] + \tau_{\text{geo}}[\Gamma_2]$.
2. **Refinement invariance:** Refinements of the step decomposition do not change τ_{geo}.
3. **Locality in composition:** Local operations on disjoint subsystems add; parallel composition respects τ_{geo}.
4. **Coordinate independence:** τ_{geo} depends only on the worldline γ (and the global calibration κ_τ), not on a choice of $(t, \vec{x})$.
5. **Minkowski limit (preview):** In the continuous kinematic limit, $d\tau_{\text{geo}}$ becomes the line element of the budget quadric introduced later (see Section II.6).

[a] See FBA Part I: FBA – Foundations, Sections I.3 - I.6: Refinement invariance, composition/locality.

For the reading path it suffices to read (1) and (2) as direct consequences of the definition and of refinement invariance; an optional short sketch is given in the next box.

Proof Sketch II.4.1: on (1) and (2) in Formula Box II.4.1

(1) is a direct consequence of the definition as a sum / integral along the path.

(2) follows because the balance and the decomposition into reversible and irreversible contributions are refinement-safe: a refinement replaces one step by several sub-steps whose reversible shares (after calibration by κ_τ) add up exactly to the original reversible share.[a]

[a] See FBA Part I: FBA – Foundations, Section I.3: Refinement invariance of the balance.

As a second intrinsic quantity we now define the irreversible share as aging; it adds to the total proper time without changing the geometric line element.

Definition II.4.2: Aging (irreversible share)

The *aging functional* on $\Gamma \subset \gamma$ is the calibrated sum of the irreversible internal share:

$$A[\Gamma] \equiv \frac{1}{\kappa_\tau} \sum_{n\to n+1\in\Gamma} \delta b_{\text{irr}}(n), \qquad \text{in the continuum:} \qquad dA \equiv \frac{db_{\text{irr}}}{\kappa_\tau} \geq 0.$$

This yields the *total* proper time as

$$\tau_{\text{tot}}[\Gamma] \equiv \tau_{\text{geo}}[\Gamma] + A[\Gamma] = \frac{1}{\kappa_\tau} \sum_{n\to n+1\in\Gamma} \delta b_{\text{int}}(n),$$

where τ_{geo} denotes the reversible (geometric) share.

This definition makes a second, test-relevant statement explicit: even if two worldlines have the same geometric share τ_{geo}, their total proper time τ_{tot} can diverge if the irreversible contributions differ. Thus aging becomes an independent quantity that cannot be "transformed away" by a mere change of motion or coordinates.

Formula Box II.4.2: Monotonicity & inequalities (DPI/Spohn)

Standard reference: DPI/monotonicity [3, 4] and Spohn monotonicity [5, 6].[a]

Let the unselective dynamics between steps be CPTP/GKLS-admissible and satisfy Spohn monotonicity. Then, in expectation value:

$$\mathbb{E}[\delta A] \geq 0, \qquad \Rightarrow \qquad \mathbb{E}[d\tau_{\text{tot}}] = \mathbb{E}[d\tau_{\text{geo}}] + \mathbb{E}[dA] \geq \mathbb{E}[d\tau_{\text{geo}}].$$

In particular, no local CPTP operation (without post-selection) can *decrease* the expected value of total proper time (no anti-aging). Equality holds exactly for (effectively) reversible steps.

[a] See FBA Part I: FBA – Foundations, Section I.5: CPTP/GKLS, DPI/Spohn.

Context. Up to this point we have cleanly separated three layers: (1) the order of updates (Section II.3), (2) geometric proper time τ_{geo} as reversible internal flow (calibrated via κ_τ), and (3) aging A as an irreversible contribution with a monotonicity structure. In the next Section we couple these intrinsic quantities to the external reference structure: front calibration (Section II.5) fixes the comparison scale and links it operationally to the maximal propagation rate c.

II.5 External calibration, front, and invariance of c

So far we have (i) constructed a time order as an embedding of the frame sequence (Section II.3) and (ii) defined intrinsic, coordinate-free quantities along a worldline—namely geometric proper time and aging as integrated budget shares (Section II.4). What is still missing is the metrological bridge: How are numerical values of t, τ_{geo}, and lengths coupled so that different systems and laboratories reconstruct the same scale? This is exactly what *front calibration* provides. It introduces a reproducible reference procedure that fixes the coupling between external accounting, coordinate time, and range, and therefore does not postulate c but determines it as the ratio of calibrated cost rates.[16]

The logical payoff is immediate: Only once c is fixed operationally can the later budget quadric be normalized such that its null directions are precisely the signal fronts. In this way, the light cone is not assumed but anchored as a limiting structure of admissibility.

Definition II.5.1: Calibration via a signal front (front costs)

A *(front) calibration* fixes two external cost rates $\kappa_t, \kappa_x > 0$ (budget per duration and budget per range) such that for every admissible step:

1. **Time calibration (external duration):** The observer-side coordinate-time increment Δt is tied to the external budget share per step:
$$\delta b_{\text{ext}} \;=\; \kappa_t \,\Delta t.$$
2. **Length calibration (external range):** A spatial relation change by $\|\Delta\vec{x}\|$ costs at least the following amount of external budget:
$$\kappa_x \,\|\Delta\vec{x}\| \;\leq\; \delta b_{\text{ext}}.$$
3. **Definition of the front constant:** The front constant is the derived ratio
$$c \;:=\; \frac{\kappa_t}{\kappa_x}.$$

A *front step* (signal front as calibration reference) is an admissible step that saturates the range coupling, $\kappa_x\|\Delta\vec{x}\| = \delta b_{\text{ext}}$, *and* in the idealized limit carries no reversible internal share,

$$\delta b_{\text{int}}^{\text{rev}} = 0 \qquad \text{(idealized limit additionally: } \delta b_{\text{irr}} = 0).$$

It then realizes $\|\Delta\vec{x}\|/\Delta t = c$ and, at the same time, $\Delta\tau_{\text{geo}} = 0$. Together with the internal calibration κ_τ (Section II.4), front calibration thus couples the units of t, τ_{geo}, and $\|\cdot\|$ in a reproducible way.

Why is this the right approach? Because it cleanly separates two things: (1) The class of admissible signals/processes is determined by dynamical and budget assumptions, not by a choice of coordinates. (2) Metrology is bound to external accounting via κ_t, κ_x; c emerges as the ratio of these two calibration rates and is therefore derived, not postulated. The extremal picture (front as a saturating limit case) is stable, because additional internal or irreversible

[16] See FBA Part I: FBA – Foundations, Section I.3: calibration, front bound, signal front.

accounting does not increase the external budget available per step for range gain.

Lemma II.5.1: Front bound from calibration and budget positivity

Under Definition II.5.1, for every admissible step,

$$\frac{\|\Delta\vec{x}\|}{\Delta t} \le c \qquad (\Delta t > 0).$$

If $\Delta t = 0$, then $\delta b_{\text{ext}} = \kappa_t \Delta t = 0$ and $\kappa_x \|\Delta\vec{x}\| \le \delta b_{\text{ext}}$ necessarily imply $\Delta\vec{x} = 0$ (no spatial displacement without a time increment).

For the reading path, it suffices to take the inequality as a direct consequence of the calibration couplings; an optional short sketch is given in the next box.

Proof Sketch II.5.1: Front bound from calibration and budget positivity

For $\Delta t > 0$: From $\kappa_x \|\Delta\vec{x}\| \le \delta b_{\text{ext}}$ and $\delta b_{\text{ext}} = \kappa_t \Delta t$ it follows that $\kappa_x \|\Delta\vec{x}\| \le \kappa_t \Delta t$, hence $\|\Delta\vec{x}\| \le (\kappa_t/\kappa_x)\Delta t = c\Delta t$, and therefore $\|\Delta\vec{x}\|/\Delta t \le c$. The case $\Delta t = 0$ is handled separately in Lemma II.5.1.

This bound is more than a "speed limit": it makes fronts an operational boundary at which geometric proper time vanishes. This is exactly the structure that appears, in the Minkowski limit, as the null direction of the quadric.

Corollary II.5.1: Signal fronts and null proper time

Worldlines/steps that saturate the front bound and are realized as front steps in the sense of Definition II.5.1 are *signal fronts.* For them, in the (idealized) limit:

$$\Delta\tau_{\text{geo}} = 0 \quad \text{and} \quad \frac{\|\Delta\vec{x}\|}{\Delta t} = c.$$

Thus, fronts mark the null-like limiting cases of the budget quadric introduced later: they span the light cone and fix the cone structure metrologically.
A more detailed development of the cone structure and its microcausal role can be found in *Part V: spacetime, light cones & local field theory.*[a] The group theory of the invariance follows in Section II.7.

[a] See FBA Part V: Spacetime, Light Cones & Local Field Theory, Sections V.3 - V.5: light cones & microcausality.

Metrological invariance of c

Core statement. c is not a postulated constant of nature, but the *metrological* fixed point of a calibration protocol: $c = \kappa_t/\kappa_x$.

Justification (structured).

1. *Fixing via cost rates.* The front protocol ties external duration and external range to the same external accounting: $\delta b_{\text{ext}} = \kappa_t \Delta t$ and $\kappa_x \|\Delta\vec{x}\| \le \delta b_{\text{ext}}$. Hence the front bound $\|\Delta\vec{x}\|/\Delta t \le \kappa_t/\kappa_x$ and the definition $c := \kappa_t/\kappa_x$ follow (see Definition II.5.1 and Lemma II.5.1).

2. *Replication without extra assumptions.* If the same protocol is repeated in arbitrary local inertial situations, it operates on the same class of admissible processes and the same cost relations. The reconstructed c is therefore protocol- and location-independent as long as the underlying admissibility assumptions hold.

3. *Geometric anchoring as the next stage.* In Section II.6 we identify fronts as null directions of the budget quadric. In Section II.7 it then follows that the isometries of the quadric preserve the light cone and thus keep c invariant in all inertial frames.

Consequences.

- c is a *calibration constant*: its value follows from a reproducible measurement protocol (fixing κ_t, κ_x), not from a postulate.
- Every correctly executed calibration must yield the same c; deviations indicate protocol errors or a violation of the underlying admissibility assumptions.

With front calibration, the external units are now fixed, and the front bound provides the operational limiting structure that is geometrized next: In Section II.6 we rewrite the balance so that it can be read as a quadric whose null directions are exactly the signal fronts defined here.

II.6 Budget Quadric and Minkowski Limit

With Section II.5 the metrological bracket is closed: time increments Δt, lengths $\|\Delta\vec{x}\|$, and the front bound $\|\Delta\vec{x}\|/\Delta t \leq c$ are operationally fixed. What is still missing is an *invariant* line element that realizes the geometric proper time from Definition II.4.1 as a line length, and whose null set coincides exactly with the signal fronts. This is precisely where the quadric emerges: under the symmetry assumptions of the kinematic, homogeneous, and (nearly) reversible limiting case, it is (up to normalization) the natural locally quadratic invariance structure that (i) has the fronts as null directions and (ii) is compatible with spatial isotropy and reversal symmetry. The full development of calibration and balance invariances is given in the FBA – Foundations.[17]

The crucial point is not that we "introduce" Minkowski language, but that the balance together with the front calibration already determines *which* combination of Δt and $\Delta\vec{x}$ must be invariant if fronts are to represent the boundary cases of admissible signals. In this way, the light cone becomes an operational boundary, and τ_{geo} becomes the corresponding line length.

Formula Box II.6.1: Budget quadric (discrete) and continuous limit

Standard reference: Minkowski interval and proper time as line length [1, 2].

Let Δt and $\Delta\vec{x}$ be the (calibrated) time and space increments between two consecutive frames. Let $\kappa_\tau > 0$ be the proper-time calibration (cf. Definition II.4.1). Let the corresponding reversible internal budget contribution on the step be $\delta b_{\text{int}}^{\text{rev}}$. Then the geometric proper-time increment on the step is

$$\Delta\tau_{\text{geo}} := \frac{\delta b_{\text{int}}^{\text{rev}}}{\kappa_\tau} \qquad \text{(in the continuum: } d\tau_{\text{geo}} = db_{\text{int}}^{\text{rev}}/\kappa_\tau\text{).}$$

In the reversible limit ($\delta A = 0$, equivalently $\delta b_{\text{irr}} = 0$) a locally quadratic relation of the form

$$-c^2\,(\Delta\tau_{\text{geo}})^2 \;=\; -c^2\,(\Delta t)^2 \;+\; \|\Delta\vec{x}\|^2 \;+\; \mathcal{O}(\|\Delta\|^3),$$

holds for small increments, where $\|\Delta\|$ denotes a common small-scale parameter for the increments and c is the calibration constant from Definition II.5.1.
In the continuous limit $\Delta \to 0$ this yields the Minkowski line element

$$-c^2\,d\tau_{\text{geo}}^2 \;=\; -c^2\,dt^2 \;+\; \|d\vec{x}\|^2,$$

with signature $(-\,+\,+\,+)$.
Fronts (Corollary II.5.1) are exactly the null directions of the quadric, i.e. (for $dt > 0$)

$$d\tau_{\text{geo}} = 0 \quad\Longleftrightarrow\quad \frac{\|d\vec{x}\|}{dt} = c.$$

(For $dt = 0$, $d\vec{x} = 0$ is forced; cf. Lemma II.5.1.)

The statement in Formula Box II.6.1 is deliberately local: it fixes the quadric via (i) the null structure of fronts and (ii) symmetry assumptions in the kinematic, homogeneous limit. The next step is the isometry group of this quadric, i.e. Lorentz symmetries, which preserve

[17] See FBA Part I: FBA – Foundations, Kapitel I.3: Kalibration, Frontschranke, Signalfront.

precisely this null structure (Section II.7).

Proof Sketch II.6.1: Budget quadric (discrete) and continuous limit

Starting point. For a step we have (with the accounting/normalization fixed in Part I) the step balance

$$\delta b_{\text{int}}^{\text{rev}} + \delta b_{\text{ext}} + \delta b_{\text{irr}} = 1,$$

and in the reversible limit $\delta b_{\text{irr}} = 0$. Refinement invariance ensures that the decomposition and the balance remain stable under refinement.[a]

Key idea. We seek a local invariance relation between $\Delta\tau_{\text{geo}}$, Δt, and $\Delta\vec{x}$ that reproduces fronts as boundary cases ($\Delta\tau_{\text{geo}} = 0$, $\|\Delta\vec{x}\|/\Delta t = c$) and, in the reversible limit, prefers no spatial direction.

Steps.

1. *Front calibration as a null condition.* By Definition II.5.1, Lemma II.5.1, and Corollary II.5.1 it is fixed: fronts saturate $\|\Delta\vec{x}\|/\Delta t = c$ and carry no reversible internal contribution, i.e. $\delta b_{\text{int}}^{\text{rev}} = 0$ or $\Delta\tau_{\text{geo}} = 0$. Hence, the desired invariance relation is *null-like* on the front.
2. *Local quadric from symmetries.* In the kinematic, homogeneous limit (no gradients, no distinguished spatial directions), the leading relation between increments depends only on Δt and $\|\Delta\vec{x}\|$. Reversal symmetry in the reversible limit eliminates linear terms, and spatial isotropy eliminates mixed $\Delta t\,\Delta x_i$ terms. Therefore, the leading approximation must have the form
$$Q(\Delta t, \Delta\vec{x}) = -\alpha\,(\Delta t)^2 + \beta\,\|\Delta\vec{x}\|^2 \quad \text{with} \quad \alpha, \beta > 0.$$
3. *Fixing by the null structure.* Since $Q = 0$ is supposed to describe exactly the fronts, we must have $\|\Delta\vec{x}\|/\Delta t = c$ for front steps. Substitution gives $-\alpha + \beta c^2 = 0$, hence $\alpha/\beta = c^2$.
4. *Identification of the line measure.* We define $\Delta\tau_{\text{geo}}$ as the (up to normalization) unique line measure parametrizing Q:
$$-c^2(\Delta\tau_{\text{geo}})^2 := Q(\Delta t, \Delta\vec{x}).$$
The calibration fixes the units so that (via the chosen spatial normalization) one can set $\beta = 1$, and thus $\alpha = c^2$. This yields exactly Formula Box II.6.1 in the limit $\Delta \to 0$.
5. *Remark on higher-order terms.* The $\mathcal{O}(\|\Delta\|^3)$ terms collect non-ideal effects (e.g. deviations from the homogeneous, strictly kinematic limit). In this part, they are suppressed in the flat limit; their role as potentially testable deviations is systematized later in the pass/fail section.[b]

Dissipation. An irreversible contribution $\Delta A > 0$ (with $\Delta A = \delta b_{\text{irr}}/\kappa_\tau$) increases the *total* proper time $d\tau_{\text{tot}} = d\tau_{\text{geo}} + dA$, but does not change the geometric line element; see Definition II.4.2 and Formula Box II.4.2.

[a] See FBA Part I: FBA – Foundations, Kapitel I.3: Refinement-Invarianz der Bilanz.
[b] See FBA Part X: Predictions, Falsifiability & Bridge FBA → QM ↔ GR, Kapitel X.6 - X.8: Tests & Vergleich.

The quadric thus provides the operational causal structure: fronts are the null directions, timelike increments carry positive geometric proper time, and spacelike increments would require an effective slope $> c$ and are therefore not realizable by signal-mediated causality. This is exactly the language that is carried forward to microcausality in *Part V: Spacetime,* *light cones & local field theory.*[18]

Light-cone structure

Definition. With $x^\mu = (ct, x, y, z)$, $dx^\mu = (c\,dt,\, d\vec{x})$, and $\eta = \mathrm{diag}(-1, 1, 1, 1)$, the budget quadric

$$\eta_{\mu\nu}\, dx^\mu dx^\nu \;=\; -c^2 dt^2 + \|d\vec{x}\|^2$$

defines the *light cone* as the null set $\eta_{\mu\nu} dx^\mu dx^\nu = 0$ (cf. Formula Box II.6.1).

Classification.

null $\eta_{\mu\nu} dx^\mu dx^\nu = 0 \;\Leftrightarrow\; d\tau_{\text{geo}} = 0$. The cone mantle is saturated by fronts; $\|d\vec{x}\|/dt = c$ (Corollary II.5.1).

timelike $\eta_{\mu\nu} dx^\mu dx^\nu < 0 \;\Rightarrow\; d\tau_{\text{geo}} > 0$. Points are causally connectable by admissible signals; τ_{geo} measures the invariant line length.

spacelike $\eta_{\mu\nu} dx^\mu dx^\nu > 0$. A connection would require an effective slope $> c$ and is thus excluded by the front bound; local dynamics respects no-signalling.

Past/future. The sign of dt separates future and past cones; the sets $J^\pm$ arise as unions of timelike or null paths with $\pm dt \geq 0$.

Continuation. Lorentz isometries preserve the null set (light cone) and thus the front slope c; see Section II.7.

[18] See FBA Part V: Spacetime, Light Cones & Local Field Theory, Kapitel V.3 - V.5: Lichtkegel & Mikrokausalität.

II.7 Relativity & Lorentz Symmetries from the Quadric

Section II.6 provided the decisive bridge: from balance accounting and front calibration we obtain a line element whose null directions are exactly the signal fronts. This yields not merely a "speed limit", but a complete local causal structure. The next step is then: If $-c^2dt^2 + \|d\vec{x}\|^2$ is the operational line element in the kinematic, homogeneous limit, then precisely those coordinate systems are *equivalent* in which this quadric has the same form. The relativity of simultaneity and Lorentz symmetries are therefore not additional assumptions, but statements about the isometries of this quadric (in the kinematic, homogeneous limit).[19] [1, 2]

We package this in two steps: First, time dilation follows immediately as the ratio of geometric proper time to coordinate time along a worldline. Second, we identify the coordinate transformations that leave the quadric invariant—these are Lorentz transformations.

Lemma II.7.1: Time dilation as a consequence of the quadric

Standard reference: time dilation / proper-time formula [2, 7].

For a system with (locally) constant relative speed

$$v = \left\| \frac{d\vec{x}}{dt} \right\|$$

with respect to a calibrated inertial frame, we have

$$d\tau_{\mathrm{geo}} \;=\; dt\sqrt{1-\frac{v^2}{c^2}} \qquad \Longleftrightarrow \qquad \frac{d\tau_{\mathrm{geo}}}{dt} \;=\; \gamma^{-1}, \quad \gamma \equiv \left(1-\frac{v^2}{c^2}\right)^{-1/2}.$$

In the non-inertial case, the integrated form is

$$\tau_{\mathrm{geo}}[\Gamma] = \int_\Gamma dt\sqrt{1-\frac{\|\dot{\vec{x}}(t)\|^2}{c^2}},$$

where Γ denotes the system's worldline (and $dt > 0$ fixes the time orientation).

For the reading path, the dilation formula can be taken as an immediate consequence of the line element; an optional sketch is given next.

Proof Sketch II.7.1: Time dilation as a consequence of the quadric

From Formula Box II.6.1 we obtain in the continuum

$$-c^2\, d\tau_{\mathrm{geo}}^2 \;=\; -c^2\, dt^2 + \|d\vec{x}\|^2 \;=\; -c^2\, dt^2 + v^2\, dt^2.$$

Dividing by $-c^2$ and taking the (positive) square root for $dt > 0$ yields the claim.

The statement is deliberately phrased here as a direct consequence of the quadric: Once fronts are fixed as null directions and τ_{geo} is the associated line measure, the ratio $d\tau_{\mathrm{geo}}/dt$ can depend only on the local slope v. In the next Section, this coupling is interpreted as a

[19] See FBA Part I: FBA – Foundations, Kapitel I.4: Minkowski-Limes & Quadrik.

budget reallocation.

Corollary II.7.1: Lorentz isometries of the budget quadric

Standard reference: Lorentz group as isometries of the Minkowski form [1, 2, 8].
Write $x^\mu = (ct, x, y, z)$, $dx^\mu = (c\,dt, d\vec{x})$, and $\eta = \mathrm{diag}(-1, 1, 1, 1)$, so that

$$\eta_{\mu\nu}\, dx^\mu dx^\nu \;=\; -c^2 dt^2 + \|d\vec{x}\|^2.$$

Then the linear transformations Λ satisfying

$$\Lambda^\top \eta\, \Lambda = \eta$$

form the group $O(1,3)$, and the physically relevant, orientation- and time-orientation-preserving subgroup is $SO^+(1,3)$. These isometries

1. leave the line element $\eta_{\mu\nu} dx^\mu dx^\nu$ (and hence $-c^2 d\tau_{\mathrm{geo}}^2$) invariant,
2. preserve the null set (light cone) and thus the front slope c (Corollary II.5.1),
3. mix t and $\vec{x}$ via the familiar boost and rotation matrices, e. g. the 1D boost:

$$ct' = \gamma(ct - \beta x), \qquad x' = \gamma(x - \beta ct), \qquad \beta = v/c.$$

For the reading path, the invariance condition $\Lambda^\top \eta \Lambda = \eta$ suffices; an optional local justification is given next.

Proof Sketch II.7.2: Lorentz isometries of the budget quadric

If two local inertial frames are to share the same line element in the form $\eta_{\mu\nu} dx^\mu dx^\nu$, then the coordinate transformation must preserve the associated bilinear form. In the linear (local) approximation, this is exactly the condition $\Lambda^\top \eta \Lambda = \eta$.

Relativity of simultaneity and measurement protocols

The invariance of the light cone is the operational reason for the relativity of simultaneity: If two inertial frames are related by a Lorentz transformation, then the hypersurfaces $t = \mathrm{const}$ are mapped to tilted hypersurfaces $t' = \mathrm{const}$. This does not contradict a shared causal structure; it is its consequence: Only the light cone and the interval are invariant, not the splitting into "pure time" and "pure space".
Calibrated protocols are equivalent because they are tied to the same notion of a front: synchronization and distance fixing proceed via the signal fronts, which have the same null structure in every inertial frame. Coordinate times t and t' therefore differ systematically, while the geometric proper time τ_{geo} remains invariant as a line length (Definition II.4.1 and Formula Box II.6.1).

II.8 Time Dilation as Budget Reallocation

In Section II.4, τ_{geo} was defined as the integrated *reversible internal* budget flow (calibrated via κ_τ). In Section II.5, the external scale was fixed via a signal front, and in Section II.6 the budget quadric emerged as a line element. This already secures time dilation geometrically (Lemma II.7.1). What this section adds is the *operational reading*: Why, in the FBA, is dilation not merely a coordinate phenomenon, but the direct consequence of a per-step budget reallocation?

The logic is simple and strict: Per step, the accounting is bounded, and the external calibration ties reach and coordinate time to external budget. Formally, this rests on the step balance (with the decomposition into external / internal-reversible / irreversible).[20] If a system realizes a translation relative to the calibrated frame, it must account for it externally. What is committed externally cannot, in the same step, appear as reversible internal flow. The quadric makes this reallocation quantitative and invariant.

Definition II.8.1: Joint calibration of c and τ_{geo}

Standard reference: proper-time normalization (at rest) and the SR limit [2, 7].

A c–τ_{geo} *calibration* is a choice of units that simultaneously

1. fixes the front constant c via the external front calibration (Definition II.5.1 and Corollary II.5.1), and
2. normalizes the internal proper-time calibration κ_τ via a (nearly) reversible reference clock at rest such that along its worldline

$$d\tau_{\text{geo}} = dt$$

holds.

This jointly determines t, τ_{geo}, and the length scale. Additional dissipation contributes *additively* to the total proper time (Definition II.4.2 and Formula Box II.4.2).

This joint normalization is the point at which the interpretation as budget reallocation ceases to depend on conventions: Without (2), one could define away any deviation between t and τ_{geo} as a mere rescaling; only fixing a rest reference clock turns the statement "the reversible internal share shrinks under motion" into a physical claim.

[20] See FBA Part I: FBA – Foundations, Secs. I.3: One-step budget, balance equations & invariances.

Formula Box II.8.1: Dilation formula as a budget relation

Standard reference: SR time dilation as a consequence of the interval [2, 7].

Under the c–τ_{geo} calibration, for a system with local speed $v = \|d\vec{x}/dt\|$,

$$d\tau_{\text{geo}} = dt\sqrt{1-\frac{v^2}{c^2}} \qquad \Longleftrightarrow \qquad -c^2 d\tau_{\text{geo}}^2 = -c^2 dt^2 + \|d\vec{x}\|^2.$$

Equivalently, in budget form (definition from Definition II.4.1):

$$db_{\text{int}}^{\text{rev}} \;=\; \kappa_\tau \, d\tau_{\text{geo}} \;=\; \kappa_\tau \, dt\sqrt{1-\frac{v^2}{c^2}}.$$

Thus, the reversible internal budget share per step is reduced by the externally committed share (translation).

For the reading path, Formula Box II.8.1 may be taken as an immediate consequence of the quadric and the rest normalization; an optional sketch is given next.

Proof Sketch II.8.1: Dilation formula as a budget relation

Rest normalization $v = 0 \Rightarrow d\tau_{\text{geo}} = dt$ (Definition II.8.1) and inserting $v = \|d\vec{x}/dt\|$ into the quadric (Formula Box II.6.1) yields the claim.

The equation in Formula Box II.8.1 is formally identical to SR dilation, but it is anchored differently in interpretation: here v is not only a kinematic slope, but a marker of how strongly the step must be accounted for externally in order to realize the observed translation. That the square-root form appears is a direct consequence of the quadric, whose null directions are fixed by the front calibration.

Corollary II.8.1: With dissipation: total vs. geometric proper time

Decompose the internal flow into a reversible and an irreversible share:

$$d\tau_{\text{tot}} = d\tau_{\text{geo}} + dA, \qquad dA \geq 0$$

(Definition II.4.2 and Formula Box II.4.2). Then

$$d\tau_{\text{tot}} \;=\; dt\sqrt{1-\frac{v^2}{c^2}} + dA \;\geq\; dt\sqrt{1-\frac{v^2}{c^2}}.$$

Minkowski geometry is tied to τ_{geo} (the ideal, reversible clock); dA models additional irreversible budget work of the system.

This also clarifies why the FBA keeps two layers separate: the quadric governs reversible geometry, while irreversible aging carries an additional monotone structure. Together, both are experimentally relevant: real clocks are never exactly reversible, yet these deviations remain modelable as systematic corrections.

Operational consequences

Transport clocks and the twin setup. Comparison experiments separate τ_{geo} (geometric) from A (irreversible): the kinematic dilation follows from Formula Box II.8.1; additional deviations in the overall balance are booked as dA and are therefore no longer interpretable as a mere "coordinate effect".

Ideal clocks as a limit case. Measurement protocols that minimize dA realize ideal clocks and directly test the relation in Formula Box II.8.1. Conversely, protocols with controllably increased dissipation provide a way to disentangle geometric and dissipative contributions experimentally.[a]

Test logic. In the FBA, the standard dilation is thus not only a consequence of inertial symmetry, but a budget statement: if $\|\dot{\vec{x}}\|$ increases, the reversible internal share (and hence $d\tau_{\text{geo}}/dt$) must decrease. A pass/fail framework that systematizes exactly such separations is formulated explicitly later.[b]

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[a] See FBA Part VIII: Classical Limit, Thermodynamics & Aging, Secs. VIII.6–VIII.8: DPI/Spohn & an aging measure.

[b] See FBA Part X: Predictions, Falsifiability & Bridge FBA $\rightarrow$ QM $\leftrightarrow$ GR, Secs. X.6–X.8: Tests & comparison.

II.9 Causality, Light Cones & Local Field Structure (Outlook)

The budget quadric (Formula Box II.6.1) yields not only the dilation formula, but the entire light-cone structure: fronts (Corollary II.5.1) form the null mantle and thus operationally separate causally reachable from causally unreachable regions.

This section is an outlook, because it already becomes visible here why the later locality and field assumptions are not "additional": once a cone structure is fixed, "local" becomes a statement about which operations can influence which regions at all. The required compositional and locality structure is introduced in the FBA – Foundations.[21]

We first formulate the usual causal sets $J^{\pm}$ directly from the quadric. After that we give an operational no-signalling statement: spacelike separated regions remain decoupled at the level of expectation values, provided the dynamics is implemented locally. The actual elaboration as a local field structure (local GKLS densities, microcausality, continuity equations) follows in *Part V: Spacetime, Light Cones & Local Field Theory*.[22]

Definition II.9.1: Causal regions from the budget quadric

Standard reference: causal structure in Minkowski space [2, 8].

Fix a time orientation via the coordinate time t (local inertial frame). For two events p, q, define the interval

$$s^2(p,q) \;\equiv\; -c^2(t_q - t_p)^2 + \|\vec{x}_q - \vec{x}_p\|^2,$$

corresponding to the Minkowski line element

$$-c^2 d\tau_{\text{geo}}^2 = -c^2 dt^2 + \|d\vec{x}\|^2$$

from Formula Box II.6.1.
The causal sets are

$$J^{\pm}(p) = \Big\{ q \;\Big|\; \exists \text{ timelike or lightlike path } \Gamma : p \to q,\; d\tau_{\text{geo}} \geq 0,\; \pm dt \geq 0 \text{ along } \Gamma \Big\},$$

i.e. the set of all events reachable by a non-spacelike, time-oriented path. Its boundary $C^{\pm}(p) = \partial J^{\pm}(p)$ is formed by null paths (fronts).
Points with $s^2(p,q) > 0$ are *spacelike* separated (outside the cones).

Why these definitions here? Because they mark exactly the interface between geometry and dynamics: the quadric says which connections are kinematically admissible at all (front bound), and the composition and locality axioms say how processes may be composed without undermining this admissibility.

[21] See FBA Part I: FBA – Foundations, Sec. I.6: Symmetric-monoidal structure, local operations & causal cones.

[22] See FBA Part V: Spacetime, Light Cones & Local Field Theory, Secs. V.3 - V.6: Light cones, microcausality & local generators.

Lemma II.9.1: No-signalling & causal order (operational)

Standard reference: no-signalling/CPTP [4] and GKLS generators [6, 9, 10].

Let two regions A, B be spacelike separated, i.e. every connection between their points is spacelike in the sense of Formula Box II.6.1, or equivalently $s^2 > 0$ from Definition II.9.1. Under the principles introduced in the FBA – Foundations (symmetric-monoidal composition, budget additivity, local operations, and the causal-cone corollary), the following holds:[a]

1. **No-signalling (marginal).** For any CPTP operation $\mathcal{E}_A$ implemented locally in A, the reduced statistics in B remains unchanged, provided the global implementation satisfies the locality assumptions from Part I (no direct coupling $A \leftrightarrow B$ outside the cone):

$$\rho'_B = \mathrm{Tr}_A[(\mathcal{E}_A \otimes \mathrm{id}_B)(\rho_{AB})] = \mathrm{Tr}_A(\rho_{AB}) = \rho_B.$$

2. **Causal support of local generators (outlook).** Any admissible GKLS evolution whose generator is supported locally in A is compatible with the cone order: under local implementation there is, at the level of expectation values, no influence outside the causal regions defined by $J^{\pm}$.

[a] See FBA Part I: FBA – Foundations, Sec. I.6: Local operations, global independence & causal cones.

Proof Sketch II.9.1: No-signalling & causal order (operational)

(i) Spacelike separation means: a direct coupling $A \to B$ would require an effective slope $> c$ and is therefore incompatible with the front bound.

(ii) The composition and locality principles required in Part I (including budget additivity) exclude implementations that would, by chaining local steps, generate an effective signal transfer outside the cone.

(iii) Hence: CPTP maps implemented locally in A cannot change the marginals in B; analogously, locally supported GKLS generators are compatible with the cone order (outlook toward Part V).

Corollary II.9.1: Preview: local field generators

Note (outlook/heuristic): the concrete field construction (support, densities, continuity) is carried out axiomatically/technically only in Part V.[a]

From the cone structure and the locality axioms follows the natural form of a field description: a *local* GKLS generator can be written as a density $\updownarrow(x)$ whose action is locally integrable over a spacelike hypersurface Σ (e.g. $t =$ const in the local inertial frame),

$$\mathcal{L} = \int_{\Sigma} \updownarrow(x)\, d^3x.$$

Locality then means: contributions from spacelike separated regions are causally decoupled in the sense of Lemma II.9.1; in the unitary (Hamiltonian) limit this corresponds to the usual microcausality (commutator structure) in field theory.

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[a] See FBA Part V: Spacetime, Light Cones & Local Field Theory, Secs. V.4 - V.6: Local GKLS densities, microcausality & continuity equations.

Path map to the subsequent Parts

The cone geometry and front invariance established here are the shared foundation for the next steps:

- **Part IV: dynamics, measurement, GKLS:** Admissible channels, measurement protocols, and semigroup dynamics as a CPTP/GKLS structure, compatible with budget and locality.[a]
- **Part V: spacetime, light cones & local field theory:** Expansion of the causal order sketched here into a local field structure (local generators, microcausality, preservation of the cone structure at the level of expectation values).[b]
- **Part VI: gravitation & geometry from budget flows:** Deviations from the flat limit due to budget gradients ($\nabla b \neq 0$) as effective curvature and gravitational time dilation.[c]

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[a] See FBA Part IV: Dynamics & Measurement (GKLS), Secs. IV.1 - IV.5: CPTP/GKLS & measurement protocols.
[b] See FBA Part V: Spacetime, Light Cones & Local Field Theory, Secs. V.3 - V.6: Light cones & local field structure.
[c] See FBA Part VI: Gravity & Geometry from Budget Flows, Secs. VI.3 - VI.5: Budget geometry, curvature & redshift.

II.10 Comparison & Positioning relative to Special Relativity

Up to this point, we have built a continuous chain in the flat, kinematic limit: sequence → budget → front calibration → quadric → Lorentz isometries → dilation. The comparison with Special Relativity (SR) therefore serves not as a new justification, but as a consistency check: in the idealized limit, the FBA construction must reproduce SR kinematics, because both rest on the same Minkowski quadric.[1, 2, 7, 8] The distinction then lies in the interpretation and in the additional structure that becomes available in the FBA via budget accounting, metrology, and dissipation.

Mapping FBA ↔ SR (flat, kinematic limit)

Standard reference: SR kinematics via the Minkowski interval and proper time [1, 2, 7].

- **Proper time:** FBA *geometric* proper time τ_{geo} (Definition II.4.1) ≡ SR: proper time along timelike worldlines.
- **Metric/quadric:** Budget quadric $-c^2 d\tau_{\text{geo}}^2 = -c^2 dt^2 + \|d\vec{x}\|^2$ (Formula Box II.6.1) ≡ SR: Minkowski interval $ds^2 = \eta_{\mu\nu} dx^\mu dx^\nu$ with $x^0 = ct$ and $\eta = \text{diag}(-1, 1, 1, 1)$, i.e. $ds^2 = -c^2 dt^2 + \|d\vec{x}\|^2$ and for timelike worldlines $ds^2 = -c^2 d\tau^2$.
- **Light cone/front:** Null directions of the quadric ⇔ signal fronts (Corollary II.5.1) ≡ SR: the light cone.
- **Symmetries:** Isometries of the quadric form $SO^+(1, 3)$ (Corollary II.7.1) ≡ SR: the Lorentz group.
- **Dilation:** $d\tau_{\text{geo}} = dt\sqrt{1 - v^2/c^2}$ (Formula Box II.8.1) ≡ SR: time dilation.
- **Clock hypothesis (operational):** Ideal clocks have $dA = 0$ (Definition II.4.2 and Formula Box II.4.2) and measure τ_{geo}; in the kinematic limit, their rate thus depends only on the worldline (in particular on the local slope $v(t)$), not on additional dissipative effects. Real clocks add $dA \geq 0$ (rate deviation without changing the geometry).
- **Causality/no-signalling:** Spacelike separation ⇒ no influence at the level of expectation values (composition/locality; cf. Lemma II.9.1).

The core of the mapping is: once (i) fronts are operationally fixed as null directions and (ii) τ_{geo} is identified as the line measure of the quadric, SR kinematics in the flat limit is already contained. What SR states as postulates (invariance of the speed of light, Lorentz symmetry) appears in the FBA as a consequence of calibration and the quadric.

Formula Box II.10.1: Identification in the Minkowski limit

Standard reference: SR as the theory of the Minkowski quadric [1, 2, 8].

Under the assumptions *flat, kinematic* (no backreaction, homogeneous budgets), c-τ_{geo} calibration (Definition II.8.1) and idealized reversible clock dynamics ($dA = 0$) one has:

The kinematic measurement relations for timelike and null processes derived in Part II agree with SR in the Minkowski limit.

Precisely:

- the invariant line measure is τ_{geo} (Definition II.4.1),
- the isometries are Lorentz transformations (Corollary II.7.1),
- the null set consists of fronts (Corollary II.5.1),
- measurement relations (e.g. dilation) follow from Formula Boxes II.6.1 and II.8.1.

Scope, tests & additional structure

Scope. Locally inertial, without gravity and without backreaction; accelerated worldlines are allowed (integration of Lemma II.7.1 or Formula Box II.8.1).

Standard tests (SR).

- time dilation of resting vs. moving clocks,
- front invariance (two-way light travel time),
- synchronization protocols via fronts.

All three points are reproduced here by Formula Box II.6.1, Corollary II.7.1, and Definition II.5.1.

Additional (FBA-specific) structure.

1. *Metrology of c:* c is the calibration constant of a reproducible front protocol (formally $c = \kappa_t/\kappa_x$; Definition II.5.1), not a postulate.
2. *Separation τ_{geo} vs. A:* Real clocks measure $\tau_{\text{tot}} = \tau_{\text{geo}} + A$ with $A \geq 0$ (Definition II.4.2 and Formula Box II.4.2); SR corresponds to the idealized limit $A = 0$.
3. *Data-processing arrow:* Unselected CPTP/GKLS evolution implies monotonicity of A (Spohn monotonicity; DPI).[3–5] This yields an operational arrow of time without changing the geometry.[a]

Falsifiability in the flat limit.

(i) Evidence for $\|\Delta\vec{x}\|/\Delta t > c$ despite correct calibration,

(ii) systematic anti-aging $A < 0$ under unselected local CPTP processes (DPI arrow/no-recovery),[b]

(iii) violation of budget additivity under spacelike separation.[c]

Such findings would refute the FBA in the flat, kinematic limit. A structured pass/fail catalog is formulated in *Part X.*[d]

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[a] See FBA Part I: FBA – Foundations, Kapitel I.5: CPTP/GKLS, DPI/Spohn.
[b] See FBA Part I: FBA – Foundations, Kapitel I.5: DPI-Pfeil & No-Recovery.
[c] See FBA Part I: FBA – Foundations, Kapitel I.6: Budget-Additivität & Lokalität.
[d] See FBA Part X: Predictions, Falsifiability & Bridge FBA → QM ↔ GR, Kapitel X.6 - X.8: Tests & Vergleich.

II.11 Summary & Checklist (Pass/Fail)

This closing Section serves two purposes: First, we compress the deductive line of this treatise into a compact mapping onto the SR limiting case. Second, we distill from it a Pass/Fail checklist.

The checklist is intentionally aligned with the logical chain: each item is either a necessary reproduction condition of the flat Minkowski limit, or a direct consequence of the additional structure (budget, dissipation, locality) that the FBA makes explicit relative to a pure postulate-based formulation.

Brief conclusion of the treatise

Standard reference: SR/Minkowski limit and proper time [1, 2, 7].

- **Time as an ordering:** Time is a strictly increasing embedding of the frame sequence; a scale arises only through calibration (cf. Definition II.3.1).
- **Proper time & aging:** τ_{geo} is the reversible internal budget flow; A is irreversible aging; total proper time $\tau_{\text{tot}} = \tau_{\text{geo}} + A$ (Definitions II.4.1 and II.4.2 and Formula Box II.4.2).
- **Calibration & front:** c is a metrologically defined calibration constant of the fastest fronts (Definition II.5.1, Lemma II.5.1, and Corollary II.5.1).
- **Budget quadric ⇒ Minkowski:** $-c^2 d\tau_{\text{geo}}^2 = -c^2 dt^2 + \|d\vec{x}\|^2$ as the continuous limit (Formula Box II.6.1); the light-cone structure follows.
- **Lorentz symmetries & dilation:** Isometries form $SO^+(1,3)$ (Corollary II.7.1); $d\tau_{\text{geo}} = dt\sqrt{1 - v^2/c^2}$ (Formula Box II.8.1).

This brief conclusion also states what is *not* achieved here: curvature, backreaction, and scale running lie outside the flat, kinematic limit and are treated only in later parts as deviations from precisely this reference structure.

Checklist (Pass/Fail) for the flat, kinematic limit

Pass – necessary requirements

1. **Front invariance:** A replicable calibration protocol yields the same c in all local inertial scenarios (Definition II.5.1 and Corollary II.5.1).

2. **SR equivalence in the ideal case:** In flat, local-inertial situations, the measurement relations $\{-c^2 d\tau_{\text{geo}}^2 = -c^2 dt^2 + \|d\vec{x}\|^2,\ d\tau_{\text{geo}} = dt\sqrt{1 - v^2/c^2}\}$ agree with the standard SR tests (Formula Boxes II.6.1 and II.8.1).[1, 2, 7]

3. **Lorentz isometries:** Transformation laws preserve the light cone and τ_{geo} (local isometries of the quadric, Corollary II.7.1).[1, 8]

4. **Separation of τ_{geo} and A:** Ideal clocks are realizable as a limit (near $dA = 0$), and real clocks exhibit an additive irreversible contribution $A \geq 0$, without changing the geometric dilation formula (Definition II.4.2 and Formula Boxes II.4.2 and II.8.1).[3, 5]

5. **Causal structure:** No observable influence across spacelike separation; propagation remains within the cones (Lemma II.9.1; cf. Section II.9).[4]

Fail – falsifying observations (each under correct protocol control)

1. $\|\Delta\vec{x}\|/\Delta t > c$ or systematic anisotropies of c despite identical calibration (Definition II.5.1 and Lemma II.5.1).

2. Observed violation of the monotonicity/irreversibility inequality, e.g. $\mathbb{E}[\delta A] < 0$ for unselected local CPTP/GKLS processes (violation of Formula Box II.4.2).[3–5]

3. Deviations from $-c^2 d\tau_{\text{geo}}^2 = -c^2 dt^2 + \|d\vec{x}\|^2$ for ideal clocks in flat, local-inertial situations (Formula Box II.6.1).[1, 2]

4. Violation of no-signalling and/or causal decoupling at the level of expectation values under spacelike separation (Lemma II.9.1; cf. Section II.9).[4]

The Pass items are not “nice to have”, but the minimal consistency conditions ensuring that, in the flat limit, the FBA really carries the Minkowski reference structure as a deductive result. Accordingly, the Fail items are chosen so that they directly target either the front calibration, the quadric, or the monotonicity of the irreversible contributions.

Entry criteria for follow-up treatises

For Part III – Quantum kinematics & CPTP channels:

- Use τ_{geo} as an invariant line measure and c as a fixed scale anchor (Definition II.5.1 and Formula Box II.6.1).
- Represent admissible processes as a CPTP structure and prepare the dynamical side via the budget and composition principles. [a]

For Part IV – Dynamics, measurement & GKLS:

- Admissible generators are CPTP/GKLS-conform; monotonicity of A as the DPI/Spohn arrow provides an operational irreversibility structure (Formula Box II.4.2).[5, 6, 9, 10] [b]

For Part V – Spacetime, light cones & local field theory:

- Cone geometry as an operational causal order; fronts as null directions (Corollary II.5.1 and Formula Box II.6.1).
- Local generators respect budget additivity and no-signalling (cf. Section II.9). [c]

For Part VI – Gravitation & geometry from budget flows:

- The flat Minkowski limit is the reference; curvature appears as a deviation due to budget gradients and backreaction. [d]

For Part VII – Constants, scales & renormalization, and Part VIII – Classical limit, thermodynamics & aging:

- The separation τ_{geo} vs. A remains central; scale and renormalization questions concern normalizations and effective running, not the quadric signature in the reference limit. [e] [f]

For Part X – Predictions, falsifiability & bridge FBA → QM ↔ GR:

- The Pass/Fail logic formulated here is operationalized there as a protocol and proxy catalogue and mirrored against laboratory, astrophysics, and cosmology. [g]

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[a] See FBA Part III: Quantum Kinematics & CPTP Channels, Secs. III.3–III.5: Zustände, CPTP-Kanäle & Hilbertraum-Formalismus.

[b] See FBA Part IV: Dynamics & Measurement (GKLS), Secs. IV.3–IV.7: Zulässige Prozesse, GKLS & DPI/Spohn.

[c] See FBA Part V: Spacetime, Light Cones & Local Field Theory, Secs. V.3–V.6: Lichtkegel, Mikrokausalität & lokale Generatoren.

[d] See FBA Part VI: Gravity & Geometry from Budget Flows, Secs. VI.3–VI.5: Budgetflüsse, Krümmung & Redshift.

[e] See FBA Part VII: Constants, Scales & Renormalization, Secs. VII.1–VII.4: Kalibration, Normierungen & Skalenfluss.

[f] See FBA Part VIII: Classical Limit, Thermodynamics & Aging, Secs. VIII.6–VIII.8: DPI/Spohn, Thermodynamik & Altersmaß.

[g] See FBA Part X: Predictions, Falsifiability & Bridge FBA → QM ↔ GR, Secs. X.6–X.8: Vorhersagen, Experimente & Vergleich.

II.12 Appendix: Overview of the FBA Series (Parts I–X)

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1. ***Part I*: FBA-Foundations: Ordering, Budget, Proper Time & Arrows.** *Goal:* Provide the base layer: ordering, budget, proper time/aging, front and the operational arrow of time (DPI); Minkowski limit from the budget quadric; admissible dynamics and locality/no-signalling. *Import:* – (reference for all subsequent parts). *Extension:* interface contracts, pass/fail checklists, reading guide.
2. ***Part II*: Time, Proper Time & Minkowski Geometry.** *Goal:* Capture proper time/quadric operationally and derive geodesics. *Import:* foundations (ordering, budget, proper time, front/DPI). *Extension:* smooth limit, variational principle on worldlines, calibration κ_τ.
3. ***Part III*: Quantum Kinematics & CPTP Channels.** *Goal:* State spaces and channels (CPTP) including composition. *Import:* foundations (budget, channel viewpoint, composition). *Extension:* concrete divergences/cost functionals $\mathcal{C}$, measurements, and classical registers.
4. ***Part IV*: Dynamics, Measurement & GKLS (Open Systems).** *Goal:* Continuous open dynamics (GKLS) and the operational arrow of time. *Import:* channels/DPI. *Extension:* Spohn monotonicity, stationary/NESS references, flows $b^{\text{rev}}, b^{\text{irr}}, b^{\text{ext}}$.
5. ***Part V*: Spacetime, Light Cones & Local Field Theory.** *Goal:* Local field equations under front/locality. *Import:* front, composition, no-signalling. *Extension:* local GKLS generators, Lieb–Robinson-type bounds, effective light cones.
6. ***Part VI*: Gravity & Geometry from Budget Flows.** *Goal:* Geometrization of budget flows. *Import:* budget quadric/proper time. *Extension:* effective metrics from calibrations (κ_t, κ_x) and internal stresses; coupling to curvature.
7. ***Part VII*: Constants, Scales & Renormalization.** *Goal:* Scale running of the calibration theorems. *Import:* $c = \kappa_t/\kappa_x$, κ_τ. *Extension:* flow equations for $\kappa_t, \kappa_x, \kappa_\tau$; stability of c.
8. ***Part VIII*: Classical Limit, Thermodynamics & Aging.** *Goal:* Macroscopic behavior from $A[\gamma]$ (aging) and DPI. *Import:* proper time/aging, Spohn. *Extension:* entropy production, Euler–Lagrange forms for irreversible flows, effective transport equations.
9. ***Part IX*: Cosmic Dynamics, Time Dilation & Inflation (TDI).** *Goal:* Cosmic ordering & calibration flow. *Import:* budget, proper time/front. *Extension:* budget equations on large-scale slices; time-dilation inflation as calibration dynamics.
10. ***Part X*: Predictions, Falsifiability & Bridge FBA → QM ↔ GR.** *Goal:* Testable differences and bridges FBA ↔ QM/GR. *Import:* all foundational building blocks. *Extension:* protocols, limiting-case tests, overdetermined consistency relations (pass/fail).

All parts of the FBA series are available in both English and German at

https://www.frame-budget-approach.eu

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