The Frame–Budget Approach (FBA)

How time, dynamics, and geometry emerge from budget flows

An operational bridge between quantum mechanics and general relativity

Part I: FBA – Foundations

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January 21, 2026

Contents

What is Time?

Time meets us as something self-evident and yet eludes any simple definition. In everyday life it is, for some, a stream running in the background; for others, an elementary timekeeper. In experience it separates our "now" from a "before" and an "after"; in thought it can sometimes feel like fate. We talk about it as if it were a thing ("time is money"), fantasize about it as a dimension that is—at least in theory—traversable, and yet mostly accept it as something that is simply *there*.

But does time have to belong to the essence of existence at all? Can it be conceived apart from our reality, independent of everything that exists? Or, put differently: would time still "pass" if nothing existed — and conversely, would there be *anything* if there were no time? Is it more like a river behind our world, or the stage on which whatever happens becomes possible in the first place? Are we right to speak of a fourth dimension tied to *space*? And if so: is its direction fundamental, or could it even be reversible in suitable descriptions?

Classical Answers and Open Questions

In physics, three familiar readings stand side by side: (i) Newton's universal external time, (ii) Einstein's spacetime with relative simultaneity and proper time along worldlines, (iii) the block universe, which considers four-dimensionality "all at once".

All of them are fruitful — and yet fundamental questions remain open: Where do the direction we experience and the (seemingly) unassailable irreversibility come from? Why is there a maximum speed of propagation and the light-cone structure? How does relativistic geometry arise from operative resources rather than as a metric postulate? And how can all of this be thought consistently together with quantum mechanics?

Intuition – and how we formalize it

To approach these open questions in a new way, we adopt a change of perspective that begins with a simple observation: **time and change belong together.** Where something changes, we order it into "before" and "after". Conversely: where nothing changes, distinguishability disappears — and with it the word "time" loses its *operative* meaning. Time thus appears in two guises: as an expression of *order* (what follows what?) and as a *measure* (how much has happened?).

Instead of continuing to look for a possible "smallest unit of time", we therefore ask: *What is the smallest change that still counts as a change at all?*

For every change can be further decomposed in thought — down to a boundary: to a point at which a "step" is just barely still able to distinguish a state from its successor. One step smaller, and we would have indistinguishability.[1]

If we think change sufficiently finely, we arrive at *minimally distinguishable* states whose succession is what first separates a "now" from a "before" and an "after". Like the individual frames of a film, such a state has no "duration" in the narrative sense: it *is* — until it passes, through an update that is just barely still distinguishable, into a new state. Not because the world "breaks into pictures", but because our description reaches a natural resolution at the point where "same" and "different" can no longer be meaningfully separated.

[1]Important: for all the philosophical potential of this viewpoint, this boundary is not a metaphysical claim about the "nature of the world"; in reality it is rather an *operative* statement, tied to what a concrete measurement and calibration scheme can actually *register* as a difference.

Framed by their respective predecessor and successor, such states form a sequence of monotonically linked, minimally distinguishable updates — the basic building blocks of what we call "before, now, and after".

We call our view of such a (operatively recognized-as-identical) state — by analogy with the film image — a *frame*: a "snapshot" of the world relative to a previously fixed resolution. We call the transition from one frame to the next a *minimal event* (ME): the smallest change distinguishable within the protocol that turns "same" into "different".

Now the second, decisive step enters: in the world we observe, no transition is completely free of charge. Even the minimal event has a price — not in an economic sense, but as an expenditure of realizability: the capacity to bring about a difference at all is a scarce good, which we call *budget*. Typically, part of this budget flows *internally* (self-change) and part *externally* (change of relations: position, relation, order). Moreover, a share of the internal expenditure can be irreversible — which later leads us to *aging* in the physical sense.

On this reading, time is not a substance, but the *accumulated, budgeted internal work* along a path through the sequence of frames: the **"Frame-Budget Approach" (FBA)**.

And it is precisely here that the idea shows its teeth — before any metric is postulated: from clean calibration and budget balance follow two hard consequences. First, there is a practical bound on how fast changes can propagate from A to B — a *front*, fixed by calibration. Second, the direction of what happens does not arise from a given arrow, but from the fact that certain portions of the expenditure cannot be undone free of charge.

If one wants to hold it in three short sentences: (i) Reality appears *operatively* as a sequence of frames; distinguishability arises through minimal events. (ii) Every transition accounts budget, internal and external — and can be irreversible. (iii) From calibration and budget balance follow a front, proper time, and (in the appropriate limit) relativistic geometry.

What exactly does this buy us?

This change of perspective not only "tells the story differently", it "carries differently in operation":

- **Order without a clock:** The index counts updates, not seconds. "Time" as a measured quantity arises only from integrated *internal* budget flow.
- **Front instead of postulate:** A limiting speed drops out as a calibration ratio from measurement protocols — not as a metaphysical axiom.
- **Arrow from monotonicity:** Irreversibility becomes operationally graspable as budget and information monotonicity (unselective; selection/feedback mark a protocol change).

The point is: the FBA does not merely want to "rename everything". Rather, it aims to *derive* where things have often been *posited* — and to offer a *shared operative grammar* where, so far, *two languages* are spoken (QM vs. geometry).

Preview: What the FBA explains – and how it can fail

And here comes the "spoiler": if we really go through with it — take frames, minimal events, and budget seriously *and* calibrate cleanly — then light cones, proper time, the arrow of time, and admissible dynamics need not be introduced as separate postulates; they emerge as a coherent logic of balance.

This is more than a pretty picture: it produces *overdetermined* consistency relations between geometry, dynamics, and dissipation. That is exactly where the approach can fail — and exactly what makes it physically interesting: either the relations close in realistic protocols, or the FBA is simply wrong.

With that, we take a brief preview of the main implications of the FBA. The corresponding details are laid out in Parts II–X;[2] here, for now, only a few highlights:

- **Calibration first:** "cost per duration/distance" fixes, operatively, what becomes measurable as c and how resolution is bound to protocols. Thus c is no longer merely postulated, but *derivable and explainable* as a calibration ratio.
- **Light cones as balance:** Front + accounting of proper time ⇒ Minkowski structure in the reversible limit; curvature appears as inhomogeneity of flows and calibrations.
- **QM from admissible channels:** Dynamics as budget-constrained processes (discrete CPTP, continuous GKLS); measurement as controlled coarse-graining rather than mysticism.
- **Arrow of time & thermodynamics:** The second law becomes *operatively* graspable as monotonicity (DPI/Spohn) for *unselective* descriptions; "aging" appears as integrated irreversibility — additional to proper time.
- **Effective gravitation:** Geometry and dynamics are not determined only by "mass density" (in the naive sense), but appear as consequences of budget geometry; horizons can be read as "budget cut surfaces" with entropy signatures.
- **Cosmic scales:** time-dilation-induced calibration effects provide null tests beyond the standard picture (expansion, drift, dualities) and allow an early-phase acceleration *without* a postulated inflaton. In the same logic, "singularity pressure" is softened: the origin appears as a regime/calibration boundary rather than a compulsory "point of explanatory distress".
- **Constants as regime calibrations:** $\hbar, c, G$ act as normalizations in the respective regime; "running" means scale dependence of calibration relations, not necessarily a "time variation of nature".
- **Bridge tests:** Maps FBA → QM and FBA → Geo yield commutative/incommutative diagnostics — built for falsification rather than folklore.

If you want a quick map, you can already skim across: [3] [4] [5] [6] [7] [8] [9]. An overview of all parts of the FBA treatise, including download links, can be found in Section I.10 of this document.

[2] An overview of all parts of the FBA treatise, including download links, can be found in Section I.10 of this document.
[3] See FBA Part II: Time, Proper Time & Minkowski Geometry, Minkowski & Calibration.
[4] See FBA Part III: Quantum Kinematics & CPTP Channels, Channels/QM.
[5] See FBA Part V: Spacetime, Light Cones & Local Field Theory, Locality & Field Theory.
[6] See FBA Part VI: Gravity & Geometry from Budget Flows, Gravitation.
[7] See FBA Part VIII: Classical Limit, Thermodynamics & Aging, Thermo & Limits.
[8] See FBA Part IX: Cosmic Dynamics, Time Dilation & Inflation (TDI), Cosmology.
[9] See FBA Part X: Predictions, Falsifiability & Bridge FBA → QM ↔ GR, Tests/Bridge.

What this treatise accomplishes

The present Part I lays the formal foundations of the FBA so that the later parts can build on them without introducing new meta-assumptions. For all the necessary formalism, we try to transition narratively and to motivate *why* the next step is necessary, sensible, or even compelled — and then to state precisely *what* is being claimed.

A brief guide:

- **Section I.1** Context, scope, notation.
- **Section I.2** Primitives: states, minimal events, partial causal order.
- **Section I.3** Budget calculus per step; preparation of the front bound.
- **Section I.4** Proper time from internal budget; aging from irreversibility; quadratic form in the limit.
- **Section I.5** Admissible dynamics as budget-constrained channels; operative arrow of time.
- **Section I.6** Composition, locality, no-signalling.
- **Section I.8** Interfaces to Parts II–X;
- **Section I.9** Pass/fail checklist.

Expectation management

At this point, it may be worth drawing the line once more: the FBA does not claim that time is “only” information. Rather, it offers a model of thought in which *time as a measurable quantity* arises as an *integrated internal budget flow.* In this way, microdynamics (channels, divergences, dissipation) and macro-geometry (proper time, light cones) are coupled within an operative framework. Minkowski structure in the reversible limit is then not a surprise, but a balance identity (under the respective calibration and regularity assumptions stated in each place). Conversely, any violation of budget balance is a red flag: operatively visible as superluminality, as a “free” restoration of distinguishability, or as a non-physical multiplication of resources.

From theory to physics: calibration and predictions

Through *calibration* of budgets to measurement procedures, the theory becomes experimentally addressable. External accounting couples coordinate duration and growth of reach and thereby defines c; internal accounting couples to degrees of freedom and dynamics (Hamiltonian and Lindblad contributions). From this follow testable signatures: time dilation as a budget redistribution between external and internal; aging as integral dissipation; Lieb–Robinson-like propagation bounds as macroscopic manifestations of the front.

Where do we go from here?

Right after this foreword we begin with clear definitions and derivations built upon them. We deliberately speak of *primitives* and *operative basic assumptions*, motivated by simple intuition and then made formally robust. Each formal box is framed: short enough to keep the narrative thread; precise enough that nothing has to be "added later". We start with the stage (states, events, order) and develop step by step accounting, the arrow — and finally the geometry of time.

An overview of all content built on this foundations part, Parts II through X (including download links), can be found in Section I.10 of this document.

Part I

FBA – Foundations

I.1 Positioning, Scope, Notation

I.1.1 Aim & Reading Guide

In the Frame–Budget Approach (FBA) we understand time as a *balance quantity*: the world updates in discrete *update steps*; a step can consist of one or more smallest realized changes (minimal events). Each step consumes a budget, which we split into *internal* (proper progress) and *external* (re-ordering of relations/position). An irreversible part of the internal budget accumulates as *aging*. This opening section provides the terms and the minimal mathematics needed to make this idea precise and interoperable—without anticipating specific models from the subsequent parts.

Before we jump into definitions, we sketch the thread of this section: first we set the stage (global states and jumps), then we introduce actors and measured quantities, and finally we anchor the metrological calibration.[10] [11]

What you should take away from this section

1. **Stage:** Global states U_n form a sequence; each jump $U_n \to U_{n+1}$ is an update step that can bundle one or more minimal changes (ME).
2. **Actors:** Subsystems S carry *worldlines* γ (sequences of step participations).
3. **Measured quantities:** There is a difference function $d(U_{n+1}, U_n)$ and a step budget ΔB_n with a decomposition into internal/external; proper time and aging are sums/integrals over internal and irreversible shares, respectively, along γ.
4. **Calibration:** Measurement procedures set "cost per duration/distance"; from this follow a front bound and, in the limit, the Minkowski quadric—without postulating c.[a]

[a]See FBA Part II: Time, Proper Time & Minkowski Geometry, Secs. II.5–II.7.

The points above lay out the logical track. In the next subsection we provide the minimal concepts in a short, operational form; the actual calibration (and thus the front) comes only later, so that we do not smuggle in additional assumptions unnoticed.

I.1.2 Terms (Brief Overview)

The next three boxes define "stage", "actors", and "measured quantities". We deliberately separate the operational reading (what is *done*?) from the metrological embedding (how do we *measure* it?), which will be calibrated only later.

[10]See FBA Part II: Time, Proper Time & Minkowski Geometry, Secs. II.5–II.7.
[11]See FBA Part V: Spacetime, Light Cones & Local Field Theory, Sec. V.4.

Global states, sequence, minimal event

Global states U_n are complete world configurations (operationally: everything relevant for the next step is encoded in them).

The **sequence** $\{U_n\}$ orders the updating of reality into discrete jumps.

A **minimal event (ME)** is one of the smallest realized changes; a step $U_n \to U_{n+1}$ can contain one or more MEs.

With that, the stage is set; next we name the carriers along which sums/integrals (proper time/aging) are formed, and we mark the observer role without fixing measurement protocols yet.

Subsystems, worldlines, observer perspective

A **subsystem** S is a well-defined part of the world description (e.g., a field region, an atom, a measuring device).

A **worldline** γ of S is the sequence of steps in which S participates.

Observers are special subsystems with access to calibration procedures (cf. Subsection I.1.4).

Once carriers and sequence are fixed, we define the quantities that are balanced. Here we already set up what will later be sharpened as the thermodynamic arrow and an aging measure.[12]

Difference function, budget, proper time, aging

Difference function $d(U_{n+1}, U_n) \geq 0$: measures the realized change per step; it is (at least) subadditive and vanishes only if no realized change is present.

Step budget $\Delta B_n \geq 0$: accounts the step's "resources" and splits into ΔB_n^{int} (proper progress) and ΔB_n^{ext} (relation/position).

Proper time $\tau[\gamma]$: sum/integral of the internal shares along a worldline.

Aging $A[\gamma]$: sum/integral of the *irreversible* part of the internal shares along γ.

Comment. The terms above are deliberately "model-lean". The formal embedding into admissible channels (CPTP), the data-processing arrow (DPI), and GKLS flows comes later.[13] [14]

I.1.3 Notation & Minimal Assumptions

Between the conceptual frame and calibration it is helpful to fix notation; this prevents argumentative weight from slipping into a notational convention.

[12] See FBA Part VIII: Classical Limit, Thermodynamics & Aging, Secs. VIII.6–VIII.8.

[13] See FBA Part III: Quantum Kinematics & CPTP Channels, Secs. III.4–III.5.

[14] See FBA Part IV: Dynamics & Measurement (GKLS), Secs. IV.3–IV.7.

Remark I.1.3.1: Notation at a glance

Indices n count update steps; ΔX_n means the quantity "in step $n \to n+1$".

$\mathbf{x}$ denotes spatial/relational components; $\|\cdot\|$ is the Euclidean norm.

Relations over many steps are written as sums $\sum_{n\in\gamma}$ (discrete) or integrals $\int \mathrm{d}\tau$ (limit).

d denotes differentials in the limit; the difference function $d(\cdot,\cdot)$ is notationally separated from that.

We reserve ε for model/closure/approximation parameters (e.g. $\mathcal{O}(\varepsilon)$); δ_* for empirical tolerances/error bands (windows, bootstrap, residuals, pass/fail).

Statements about robustness under refinement (finer step partitioning) are to be read as well-defined limiting and/or additivity assumptions in each case.

I.1.4 Calibration & Units (Without Anticipation)

Up to this point we have deliberately fixed neither scales nor units: we know *that* there are update steps and *that* changes are to be accounted for, but not yet how these abstract accounts relate to real measurement procedures (clocks, rulers, communication protocols). Exactly this bridge is called *calibration* in the FBA.

The logical purpose of this subsection is therefore narrow: we introduce *only* the calibration theorems that later (i) couple an observer-side coordinate-time assignment Δt to the *external* account and (ii) mark a range change $\|\Delta\mathbf{x}\|$ as *externally costly.* From this coupling a front bound follows immediately. Only *after that* (in the following sections) do we specify how internal accounting leads to proper time and how, in the appropriate limit, the Minkowski quadric is reconstructed from it.[15]

Calibration theorems κ

$\kappa_t > 0$: cost rate (budget per duration) for external duration Δt.

$\kappa_x > 0$: cost rate (budget per range) for external range increase $\|\Delta\mathbf{x}\|$.

$\kappa_\tau > 0$: cost rate (budget per proper time) defining $\Delta\tau_n$ via $\Delta\tau_n = \Delta B_n^{\mathrm{int}}/\kappa_\tau$.

The ratio $c := \kappa_t/\kappa_x$ is the *front constant*; it arises from calibration, not as a postulate.[a] [b]

[a] See FBA Part II: Time, Proper Time & Minkowski Geometry, Sec. II.5.
[b] See FBA Part VII: Constants, Scales & Renormalization, Secs. VII.3–VII.4.

With this, the operational role split is fixed: Δt is (for now) a *coordinate-time* assignment from *external* accounting; $\Delta\tau$ is a *system-bound* quantity from *internal* accounting. What is still missing is the one formal step that turns this coupling into a bound. That is exactly what the next lemma provides: it uses no dynamics, only (a) budget positivity and (b) the definition of calibration.

[15] See FBA Part II: Time, Proper Time & Minkowski Geometry, Sec. II.5.

Lemma I.1.4.1: Front bound from calibration and budget positivity

Assume that per step the step budget splits as

$$\Delta B_n \;=\; \Delta B_n^{\text{int}} + \Delta B_n^{\text{ext}}, \qquad \Delta B_n^{\text{int}}, \Delta B_n^{\text{ext}} \ge 0,$$

and that calibration couples external duration to the external budget via

$$\Delta B_n^{\text{ext}} = \kappa_t \, \Delta t_n, \qquad \kappa_x \, \|\Delta \mathbf{x}_n\| \le \Delta B_n^{\text{ext}}.$$

Then

$$\kappa_x \, \|\Delta \mathbf{x}_n\| \le \kappa_t \, \Delta t_n \quad \Rightarrow \quad \|\Delta \mathbf{x}_n\| \le c \, \Delta t_n, \qquad c := \kappa_t/\kappa_x.$$

The statement is deliberately *operational*: it does not constrain "reality", but the set of processes that count as *realizable* in one step under a given external accounting. The proof is accordingly one-line bookkeeping; we present it as a sketch to keep the logical status transparent.

Proof Sketch I.1.4.1: Front bound from calibration and budget positivity

(i) By calibration, the externally realized duration per step Δt_n is coupled to the external budget: $\Delta B_n^{\text{ext}} = \kappa_t \Delta t_n$.

(ii) Any external range change $\|\Delta \mathbf{x}_n\|$ must be paid from the same external budget; hence $\kappa_x \|\Delta \mathbf{x}_n\| \le \Delta B_n^{\text{ext}}$.

(iii) Combining (i) and (ii) yields $\kappa_x \|\Delta \mathbf{x}_n\| \le \kappa_t \Delta t_n$ and thus the claim.

The operational statement is: external re-ordering costs budget and competes with external duration for the same budget share.[a]

[a] See FBA Part IV: Dynamics & Measurement (GKLS), Sec. IV.7.

To obtain a geometric structure from the single-step bound, one iterates it along a chain of steps and then considers a well-defined limit. The next corollary states exactly this transition: from a budget-calibrated bound to a cone structure, i.e., a signal front.

Corollary I.1.4.1: No superluminal transport & cone structure

If one iterates the bound over a chain of steps and takes a continuous limit, then for every worldline γ one has $\|\mathrm{d}\mathbf{x}\| \le c \, \mathrm{d}t$. This yields a cone structure (signal front) with opening c. Outside the cone, processes that are realized solely via the externally calibrated budget in finite $\mathrm{d}t$ are not reachable.[a]

[a] See FBA Part V: Spacetime, Light Cones & Local Field Theory, Sec. V.4.

So far this concerned only the *external* account. The next step (as a preview) is conceptual: if, in addition, the *internal* accounting is calibrated to proper time τ and the irreversible share is small or can be accounted for separately, then the front bound couples the three quantities $(t, \mathbf{x}, \tau_{\text{rev}})$ in a quadric. We state this here deliberately as a *preview formula*; the clean derivation, regularity assumptions, and symmetries are given in Part II.[16]

Formula Box I.1.4.1: Budget quadric and Minkowski limit (preview)

In the (nearly) reversible limit we define $\mathrm{d}\tau_{\mathrm{rev}} := \mathrm{d}B^{\mathrm{int}}_{\mathrm{rev}}/\kappa_\tau$.

Under stationary calibration and small irreversible share, the balance together with the front implies

$$(\mathrm{d}\tau_{\mathrm{rev}})^2 \;=\; (\mathrm{d}t)^2 \;-\; \frac{1}{c^2}\,\|\mathrm{d}\mathbf{x}\|^2 \;+\; \mathcal{O}\big((\Delta t)^2 + \|\Delta\mathbf{x}\|^2\big),$$

where $c = \kappa_t/\kappa_x$. Thus the Minkowski quadric emerges without postulating c.[a] [b]

[a]See FBA Part II: Time, Proper Time & Minkowski Geometry, Secs. II.6–II.7.
[b]See FBA Part V: Spacetime, Light Cones & Local Field Theory, Sec. V.4.

The content guideline is thus clear: *calibration* fixes how balance accounts are translated into measured quantities; *budget positivity* turns this into a bound; *iteration/limit* turns that into a cone structure; and only with *internal* calibration does the quadric appear.

Remark I.1.4.1: Why this calibration?

Calibrations tie abstract balance quantities to real measurement procedures (clocks, rulers, communication protocols). In this way the front bound, proper time, and—in the reversible limit—the Minkowski quadric are empirically anchored. Irreversibility contributions later couple to entropy production (DPI/Spohn, unselective) and define *aging* as the integrated irreversible share.[a]

[a]See FBA Part VIII: Classical Limit, Thermodynamics & Aging, Secs. VIII.6–VIII.8.

I.1.5 Example (Reading Bridge)

Before we state the formal *basic assumptions*, the following minimal example illustrates the interplay of internal and external shares and makes the role of the front visible. Immediately afterwards we name which quantities are measured *operationally* and where the bridge to CPTP/GKLS runs.

[16]See FBA Part II: Time, Proper Time & Minkowski Geometry, Secs. II.6–II.7.

Two Qubits and a Detector

A detector D couples briefly to two qubits A, B. One step ($n \to n+1$) consists of:

(i) local interaction A–D (internal share at A and D), (ii) transport of a signal to B (external share, bounded by the front),

(iii) registration in B (internal share at B).

The *proper time* of A grows through (i); *aging* increases if the coupling is dissipative; the external share in (ii) bounds range per duration and thus yields a maximal propagation speed c.

Operational measurement reading. (i) and (iii) can be modeled as CPTP steps with possible dissipation; (ii) is a front-limited transport protocol. For the *unselective* description this yields—via DPI/Spohn for suitable divergences—directed monotonicities and nonnegative entropy production; selective updates/feedback are to be treated as a protocol change (register extension/conditioning).[17] [18]

I.1.6 What Comes Next

Next we sharpen the *primitives* (states, events, order) and the *formal basic assumptions*. The terms introduced here remain unchanged; they will be spelled out quantitatively in the following sections and linked to the balance and DPI statements. An overview of which detailed derivations appear in which subsequent parts is deliberately bundled in an overview box to avoid footnote deserts:

Guide to the subsequent parts (Overview/Import)

- Quadric and Lorentz symmetries: Part II *Time, Proper Time & Minkowski Geometry* (Secs. II.6–II.7).
- Light cones, causality, and local field algebras: Part V *Spacetime, Light Cones & Local Field Theory* (Secs. V.4–V.6).
- Admissible dynamics (GKLS) and protocols (incl. DPI/Spohn): Part IV *Dynamics, Measurement & GKLS* (Secs. IV.3–IV.7) as well as Part III *Quantum Kinematics & CPTP Channels* (Secs. III.4–III.5).
- Aging, thermodynamics, and classical limit: Part VIII *Classical Limit, Thermodynamics & Aging* (Secs. VIII.6–VIII.8).
- Constants/scales and renormalization: Part VII *Constants, Scales & Renormalization* (Secs. VII.3–VII.5).
- Pass/fail criteria and bridge tests: Part X *Predictions, Falsifiability & Bridges* (Secs. X.3–X.7).

[17] See FBA Part IV: Dynamics & Measurement (GKLS), Secs. IV.4–IV.7.
[18] See FBA Part VIII: Classical Limit, Thermodynamics & Aging, Secs. VIII.6–VIII.7.

I.2 Primitives & Basic Assumptions of the FBA

Before we account budgets (Section I.3) or formulate dynamics (Section I.5), we need a stage on which "a step" makes sense at all. This stage consists of three building blocks: (i) *global states* and their *sequence* in update steps, carried by minimal events, (ii) a *causal structure* together with the right to *co-actualize* causally independent changes, and (iii) a *difference function* that quantifies "how much happened?" per step in a minimal, model-independent way. The following basic assumptions are deliberately concise: they do not say *what* the world is, but only *how* we count changes. Everything that follows—budgets, front, proper time—will build on this.

I.2.1 Global states, frames, and minimal events

First we fix what we mean by a "moment of the world", how such moments stand in a *sequence*, and what a *minimal event* accomplishes. The "frame" is the operational lens with which we prepare the next step—this is where budgets will later be accounted and calibrations chosen.

Definition I.2.1.1: Basic assumptions (A1–A4) and terminology: global states, frames, minimal events

Basic assumption (A1) State space. There exists a state space $\mathcal{U}$. An element $U \in \mathcal{U}$ is called a *global state*.

Basic assumption (A2) Update steps and minimal events. Reality updates as a sequence $\{U_n\}_{n\in\mathbb{N}}$ of global states. The transition $U_n \to U_{n+1}$ is called an *update step*. There exists a set of elementary changes ME (minimal events). In a given representation (frame), each step $n \to n+1$ is assigned a finite set $E_n \subset \mathsf{ME}$ such that the step $U_n \to U_{n+1}$ realizes, in this sense, exactly the co-actualized minimal events E_n. The index n counts updates, not seconds.

Basic assumption (A3) Frames $\{F_n\}$. A *frame* F_n is the operational view of U_n, including the chosen accounting/calibration of the upcoming step $n \to n+1$ (cf. Section I.3).

Basic assumption (A4) Subsystems & worldlines. A decomposition into disjoint subsystems $\{S^i\}$ is given. For a subsystem S, its *worldline* γ_S is the (partial) sequence of steps in which S participates (i.e., in which at least one $e \in E_n$ affects S).

Terminology. The elements $e \in \mathsf{ME}$ are called *minimal events*; the sets E_n are called the *(co-actualized) event sets* of a step. These notions are purely operational: they fix *how* changes are counted, not *what* they consist of ontologically.

Remark I.2.1.1: Why no time parameter?

The sequence $\{U_n\}$ is primary and purely ordinal. Metric time arises only through calibration of budgets (proper time) and/or external accounting (Δt); this keeps "order" (what comes before what?) and "measure" (how much?) cleanly separated. A reconstruction of Minkowski geometry happens *after* calibration, not before.[a]

[a]See FBA Part II: Time, Proper Time & Minkowski Geometry, Secs. II.6–II.7.

I.2.2 Sequence, causal order, and co-actualization

We now fix which constraints the sequence $\{U_n\}$ must respect in order to be physically meaningful. The central idea: only the *causal* order is physical; the granularity (how many intermediate steps we insert) is a representational degree of freedom.

Definition I.2.2.1: Causal structure and co-actualization

(A5) Partial order. The set of minimal events carries a partial order $(\mathsf{ME}, \preceq)$ that represents causal dependencies. We write

$$e \parallel e' \;:\Longleftrightarrow\; \neg(e \preceq e') \;\wedge\; \neg(e' \preceq e),$$

and then call e and e' *incomparable* or *causally disconnected* (this is a statement about order, not about statistical (un-)correlation).

(A6) Co-actualization (representational freedom). For $e \parallel e'$, their order in the step sequence $\{U_n\}$ is a *representation artifact*: in a *coarser* representation they may be assigned to the same step (the same set E_n), or in a *finer* representation distributed across different steps, *provided* the quantities under consideration depend only on start/end and (once introduced) on refinement-invariant balance sums. No-signalling will be made precise later.[a]

[a]See FBA Part V: Spacetime, Light Cones & Local Field Theory, Sec. V.4.

In the FBA we treat *refinements* of a step as representational freedom: inserting or removing purely intermediate steps should not generate physics of its own. We capture this idea as an equivalence notion.

Definition I.2.2.2: Refinement equivalence and refinement-invariant quantities

A *(finite) representation* of a transition $U \to U'$ is a finite chain

$$U = V_0 \to V_1 \to \cdots \to V_m = U'.$$

A representation $U = V_0 \to \cdots \to V_m = U'$ is called a *refinement* of the "coarse" representation $U \to U'$ if it has the same endpoints U, U' (i.e., it arises only by inserting intermediate steps). Two representations are called *refinement-equivalent*, written $\sim_{\text{ref}}$, if they can be transformed into one another by finitely inserting/removing intermediate steps.

A quantity Q assigning values to transitions is called *refinement-invariant* if Q is constant on $\sim_{\text{ref}}$-equivalence classes (equivalently: Q does not change under insertion/removal of intermediate steps).

Convention/working rule: In what follows, "operationally relevant" quantities are always understood as refinement-invariant unless explicitly stated otherwise. Once budgets are introduced, total balances are, in particular, refinement-invariant because by construction they depend only on the (additive) total accounting of a transition.

Remark I.2.2.1: Comment

Refinement equivalence formalizes that intermediate steps can be pure bookkeeping: they do not change the endpoints and (after introducing budget accounting) do not change the total balance. It is therefore consistent to tie predictions only to $\sim_{\text{ref}}$-classes.

Later, when steps are modeled as CPTP or GKLS evolution, this quotienting aligns with the fact that admissible dynamics are closed under serial composition and that balance quantities are typically carried additively over compositions.[a] [b]

[a]See FBA Part III: Quantum Kinematics & CPTP Channels, Secs. III.4–III.5.
[b]See FBA Part IV: Dynamics & Measurement (GKLS), Secs. IV.3–IV.7.

Remark I.2.2.2: What would go wrong without co-actualization?

Without co-actualization, the granularity of the representation would be observable. Then pure bookkeeping could seemingly create superluminality or "back-reactions"—an artifact of representation, not of physics.[a]

[a]See FBA Part V: Spacetime, Light Cones & Local Field Theory, Sec. V.4.

I.2.3 Difference function and minimal difference

For "a step" to make quantitative sense, we need a measure of *how much* changed between U_n and U_{n+1}—independently of *what* (internal/external) produced the change. We require only the minimal structures we later need for budget and DPI arguments.

Definition I.2.3.1: Difference function and operational minimal difference

(A7) Difference function. There exists a pseudometric $d : \mathcal{U} \times \mathcal{U} \to \mathbb{R}_{\geq 0}$ with $d(U,U) = 0$, symmetry $d(U,V) = d(V,U)$, and triangle inequality $d(U,W) \leq d(U,V) + d(V,W)$. Operationally: $d(U,V) = 0$ means that no realized (observable) change is present between U and V; states are identified up to this indistinguishability.

(A8) Operational minimal difference. For a given frame there exists a resolution threshold $\delta_{\min,*} > 0$ such that every event $e \in \mathsf{ME}$ resolved as minimal in this frame realizes a nontrivial change: if e is represented in the description by a transition $V \to V'$ between (global) states (typically with local support), then $d(e) := d(V',V) \geq \delta_{\min,*}$. $\delta_{\min,*}$ is a *calibration* and *resolution* quantity, not a universal "time atom"; under refinement, the effective threshold (frame- and protocol-dependent) can tend to 0.

The following pictures help anchor the role of $\delta_{\min,*}$ and of sums over many small steps.

Pragmatic picture: a counter with a sensitivity threshold

Think of an experiment with "clicks": the counter registers only changes above its sensitivity threshold $\delta_{\min,*}$. Whether we describe the evolution in five coarse steps or fifty fine steps does not change the *summed value* of observable functionals—as long as threshold, frame, and calibration are handled consistently.

Remark I.2.3.1: No "time atom"

The minimal difference is operational: it arises from measurement accuracy and the chosen representation. The continuous limit is a *refinement limit*, not a presupposed dense time axis. This stance makes it possible to *extract* geometry (Minkowski) from budgets rather than *postulate* it.[a]

[a] See FBA Part II: Time, Proper Time & Minkowski Geometry, Secs. II.6–II.7.

Interim summary. With (A1)–(A8) we have the order of events (what precedes what), representational freedom (refinement), and a measure of "how much" per step. In the next section (Section I.3) this structure is equipped with a balance: we split the step budget into *internal* and *external*, introduce an irreversibility account, and calibrate external costs such that a front bound and later the Minkowski quadric emerge.[19] [20]

[19] See FBA Part II: Time, Proper Time & Minkowski Geometry, Secs. II.5–II.7.
[20] See FBA Part IV: Dynamics & Measurement (GKLS), Secs. IV.3–IV.7.

I.3 Stepwise Budget Calculus

With Section I.2 we have set the stage: a sequence of global states, a minimal causal structure, and a measure of "how much" per step. What is still missing is the *bookkeeping* that distinguishes between a system's *proper progress* and the *re-ordering of relations/position.* This separation is not decoration; it supports the later constructions of *proper time* and *front* (Section I.4) and makes the Minkowski quadric accessible as a limit—without postulating it.[21] The reading path of this section has three stages: (i) we define the one-step budget and its decomposition, (ii) we fix the balance rules under serial/parallel/refinement as postulates of the budget assignment, (iii) we calibrate the external costs and read off a front bound. What matters throughout is the separation of status: *definition/assignment* (what is counted?) comes before *calibration* (how is it measured?).

I.3.1 One-step budget: internal/external/irreversible

Intuitively, each step $U_n \to U_{n+1}$ records two kinds of expenditure: *internal* (what participating subsystems accomplish "in themselves") and *external* (what it costs to re-order relations/position). The internal share further splits into reversible and *irreversible*—this will later become *aging* (Section I.4).

The following box is the formal core of this intuition: it fixes which accounts we keep per step. No physics is "explained" by this yet; we merely fix *how* a budget assignment must be structured so that proper time/front/aging can be defined at all.

Definition I.3.1.1: One-step budget and decomposition

(B1) Total budget. Each transition $U_n \to U_{n+1}$ is assigned a nonnegative *total budget* $\Delta B_n \geq 0$.

(B2) Decomposition (frame-relative). For each step there is an external share $\Delta B_n^{\text{ext}} \geq 0$ as well as internal shares $\Delta B_{n,S}^{\text{int}} \geq 0$ for the subsystems $S \in \mathcal{S}_n$ participating in this step, such that

$$\Delta B_n \;=\; \Delta B_n^{\text{ext}} \;+\; \sum_{S \in \mathcal{S}_n} \Delta B_{n,S}^{\text{int}}.$$

$\Delta B_{n,S}^{\text{int}}$ accounts the subsystem's *proper progress* (proper-time limit along its worldline), and ΔB_n^{ext} accounts the *relational/positional share* (relational). For a fixed worldline γ we often write, for brevity, $\Delta B_n^{\text{int}} := \Delta B_{n,S}^{\text{int}}$ with S as the subsystem fixed by γ.

(B3) Irreversibility account. For each participating subsystem the internal share further splits as

$$\Delta B_{n,S}^{\text{int}} = \Delta B_{n,S}^{\text{rev}} + \Delta B_{n,S}^{\text{irr}}, \qquad \Delta B_{n,S}^{\text{irr}} \geq 0.$$

Aging along γ is the integral (sum) of the ΔB^{irr}-shares of the subsystem belonging to γ.

The decomposition is frame-relative, but not arbitrary: it is chosen precisely so that it can carry the three later components separately—proper time (internal), front (external), and arrow/aging (irreversible). The next box makes this necessity explicit, to keep clear what role the decomposition plays in the overall construction.

[21] See FBA Part II: Time, Proper Time & Minkowski Geometry, Secs. II.5–II.7.

Remark I.3.1.1: Why this decomposition is necessary

Without the int/ext split we could (i) not define a *proper time* that is reparametrization-invariant and bound to the system, and (ii) not derive a *front* from the external costs. Without rev/irr we could (iii) not formulate an operational arrow of time via dissipation (Spohn/DPI).[a] [b]

[a] See FBA Part IV: Dynamics & Measurement (GKLS), Secs. IV.4–IV.7.
[b] See FBA Part VIII: Classical Limit, Thermodynamics & Aging, Secs. VIII.6–VIII.8.

A short minimal example helps avoid mixing the three accounts: "motion" is (in the FBA sense) external, "reversible dynamics" is internal-reversible, and "dissipation" is internal-irreversible.

Minimal example: relocation vs. local rotation

A two-level system S is (a) rotated locally and unitarily ($\Delta B_S^{\text{int}} = \Delta B_S^{\text{rev}} > 0,\ \Delta B^{\text{ext}} = 0$), (b) displaced by a distance $\|\Delta\mathbf{x}\|$ ($\Delta B^{\text{ext}} > 0$, internal demand minimal), (c) cooled dissipatively ($\Delta B_S^{\text{irr}} > 0$). Only (b) contributes to the external range balance; only (c) accumulates *aging*.

I.3.2 Balance equations: serial, parallel, refinement

Bookkeeping is only as good as its additivity. Serial composition (successive steps) and parallel composition (disjoint sub-processes) must neither "create" nor "destroy" budget in the *chosen budget calculus*. Likewise, a finer description (refinement) must not change the *sums*.[22]

These postulates are deliberately "dry": they ensure that the accounts do not depend on representational artifacts. Precisely thereby later integrals (proper time/aging) become robust, and precisely thereby the front can be cleanly iterated over many steps. The following box fixes the three invariances in a form that leads directly to Lemma I.3.2.1.

[22] Reading: (F1)–(F3) are postulates/definitions of the budget assignment in the FBA. Physical content emerges through the later operationalization/calibration of the accounts; with incomplete accounting one may read these equations as idealized limiting forms or with explicit tolerance/slack terms $\delta_{B,*}$, without giving up the calculus principle.

Formula Box I.3.2.1: Balance postulates & invariances (bookkeeping)

(F1) Serial. For serial composition $P = P_2 \circ P_1$ one has

$$\Delta B_P = \Delta B_{P_1} + \Delta B_{P_2}, \qquad \Delta B_P^{\text{ext}} = \Delta B_{P_1}^{\text{ext}} + \Delta B_{P_2}^{\text{ext}},$$

and analogously componentwise for the internal accounts along a fixed worldline and/or for each subsystem S.

(F2) Parallel (disjoint). For disjoint sub-processes $P_A \parallel P_B$ one has additively

$$\Delta B_{P_A \parallel P_B}^{\text{ext}} = \Delta B_{P_A}^{\text{ext}} + \Delta B_{P_B}^{\text{ext}}, \qquad \Delta B_{P_A \parallel P_B, S}^{\text{int}} = \Delta B_{P_A,S}^{\text{int}} + \Delta B_{P_B,S}^{\text{int}} \text{ (where defined).}$$

(F3) Refinement invariance. For any decomposition of a step $U_n \to U_{n+1}$ into m sub-steps, $\sum_{k=0}^{m-1} \Delta B_{(k)} = \Delta B_n$, and analogously componentwise for ext and for the relevant internal accounts (including rev/irr).

From (F1)–(F3) it follows immediately: coarse and fine segmentations do not differ in what the budget calculus accounts as "what really happened". This is exactly the statement we isolate in the next lemma, because it will later be used repeatedly as a technical step (proper time/aging as sums; front iteration in the limit).

Lemma I.3.2.1: Refinement-invariance of the budget balance

Under Formula Box I.3.2.1, coarse and refined segmentations are *budget-equivalent*. In particular, proper time formed from ΔB^{int} (along a worldline) and aging formed from ΔB^{irr} are invariant under any finite refinement.

The proof sketch is intentionally short: it is pure bookkeeping, not dynamics.

Proof Sketch I.3.2.1: Refinement-invariance of the budget balance

A refinement replaces one step by a finite series of sub-steps. By (F1), budgets add under serial composition, and by (F3) the sum over sub-steps is by construction equal to the coarse-step budget, also componentwise. Hence neither proper-time functionals (sum of internal shares along γ) nor aging functionals (sum of irreversible shares along γ) can be changed by mere re-segmentation.
Later, this bookkeeping is also consistent with representing admissible dynamics as serial composition (CPTP/GKLS).[a]

[a] See FBA Part IV: Dynamics & Measurement (GKLS), Secs. IV.4–IV.7.

It is also important what we *do not* fix: the balance postulates fix sums and invariances, but not the microscopic "assignment policy" within a frame. This leaves room for modeling without losing formal robustness.

Remark I.3.2.1: What this balance does *not* say

The equations fix sums, not the microscopic assignment of individual contributions: internal accounts are unambiguous along a worldline, but different representations may vary the stepwise partition as long as the sums are preserved. Calibrations (Subsection I.3.3) make the relevant external assignments operational.

I.3.3 Front bound and signal front

External re-ordering costs proportionally to range; an observer-side coordinate-time accounting is coupled to the external account by calibration. These cost rates suffice to derive a front bound—and with it an operational conversion quantity c that will later carry the light-cone structure.

Again the logical status is two-stage: first we define the *calibration* (which external accounting corresponds to Δt, which minimal accounting corresponds to $\|\Delta\mathbf{x}\|$?). Afterwards the front bound is an immediate consequence of the same budget positivity on which the entire calculus rests.

Definition I.3.3.1: Calibration and front costs

There exist calibrated cost rates $\kappa_t, \kappa_x > 0$ for external accounting such that

(i) a spatial change of relations by $\|\Delta\mathbf{x}\|$ requires at least $\kappa_x\|\Delta\mathbf{x}\|$ external budget:

$$\kappa_x\|\Delta\mathbf{x}\| \le \Delta B^{\text{ext}}.$$

(ii) the observer-side coordinate-time increment Δt is calibrated by the external accounting in the chosen frame, i.e.

$$\Delta B^{\text{ext}} = \kappa_t\,\Delta t.$$

(In particular, Δt is a *coordinate-time* assignment from external accounting here, not proper time.)

The *front constant* is $c := \kappa_t/\kappa_x$.

From (i) and (ii) the bound follows directly: range increase cannot be "paid for" faster than external duration. The lemma states exactly this consequence, without additional assumptions about dynamics, channels, or geometry.

Lemma I.3.3.1: Front bound from budget accounting

Under Definition I.3.3.1, for each step

$$\|\Delta\mathbf{x}\| \;\le\; c\,\Delta t.$$

That is: range increase and coordinate time are coupled by external budget accounting; c is fixed by calibration as the ratio κ_t/κ_x and is determined experimentally/operationally in applications (it is not needed as an additional dynamical assumption).

The proof is a one-liner; we present it as a sketch to keep the status as pure bookkeeping transparent.

Proof Sketch I.3.3.1: Front bound from budget accounting

From (i) we have $\kappa_x\|\Delta\mathbf{x}\| \le \Delta B^{\mathrm{ext}}$, from (ii) $\Delta B^{\mathrm{ext}} = \kappa_t\,\Delta t$.

Combining yields $\kappa_x\|\Delta\mathbf{x}\| \le \kappa_t\Delta t$, hence $\|\Delta\mathbf{x}\| \le (\kappa_t/\kappa_x)\Delta t = c\,\Delta t$.

Refinement invariance (Formula Box I.3.2.1) ensures that finer segmentation does not loosen the bound.

The bound is initially a *single-step* statement. Its geometric content appears when one iterates it over many steps and interprets it, in the appropriate limit, as a cone structure. The following corollary formulates this operational content: it is about the (budget-bound) possibility of establishing *new* influences/correlations over range.

Corollary I.3.3.1: Signal front (operational content)

For two spatially separated stations A, B the following holds: in a single-step process, without sufficient external budget no operationally detectable *influence/communication* from A to B over distances $\|\Delta\mathbf{x}\| > c\,\Delta t$ can occur. In particular, starting from *initially uncorrelated* preparations (product state and no pre-shared resource between A and B), no *new* correlations between A and B over $\|\Delta\mathbf{x}\| > c\,\Delta t$ can be *operationally generated* in one step.[a]
In the continuous limit, c and Lemma I.3.3.1 define the preferred structure of the later light cones; together with internal calibration this leads to the Minkowski quadric in Section I.4.[b] [c]

[a]Important: pre-shared correlations/entanglement are not excluded by this; they allow correlations without signalling. The front content concerns the creation of *new* correlations and/or the transmission of information/influence.

[b]See FBA Part II: Time, Proper Time & Minkowski Geometry, Secs. II.6–II.7.

[c]See FBA Part V: Spacetime, Light Cones & Local Field Theory, Sec. V.4.

That c is not just a symbol but is experimentally accessible as a calibration ratio is shown by the next example. It is deliberately readable as a “reviewer test” (which measurement curve yields c?).

Operational test of the front

Two lab stations A and B are separated by a variable distance $\|\Delta\mathbf{x}\|$. A protocol permits in each step only a maximal external budget $\Delta B^{\text{ext}}_{\text{max}}$. The associated coordinate-time assignment is (in the chosen frame) defined by calibration (ii) as $\Delta t := \Delta B^{\text{ext}}/\kappa_t$.

If A in each step randomly chooses an input/setting variable and B tests for a dependent *signalling effect* (not mere correlation), then the front bound yields as a necessary condition

$$\Delta t \;\geq\; \kappa_x \|\Delta\mathbf{x}\|/\kappa_t \;=\; \|\Delta\mathbf{x}\|/c.$$

Varying $\|\Delta\mathbf{x}\|$ makes c fall out as the slope of the measurement—a *calibration quantity* (conversion factor between external accounts), not an additional postulate.[a]

[a]See FBA Part VII: Constants, Scales & Renormalization, Secs. VII.3–VII.4.

Interim summary. The budget calculus provides three pillars: (i) a decomposition into internal/external/irreversible, (ii) additive balance equations under composition, (iii) a front bound from external calibration. In the next section (Section I.4) we define *proper time* and *aging* as integrals of the internal and irreversible shares, and couple them to the front to obtain the Minkowski quadric.[23] [24]

[23]See FBA Part II: Time, Proper Time & Minkowski Geometry, Secs. II.5–II.7.
[24]See FBA Part VIII: Classical Limit, Thermodynamics & Aging, Secs. VIII.6–VIII.8.

I.4 Proper Time and Aging

In the budget calculus (Section I.3) we have clarified the *types* of expenditure: internal (proper progress), external (re-ordering/position), and the irreversible share. The next step is to build *invariant* quantities for individual subsystems from these accounts. The guiding idea is simple: *proper time* is the integrated internal accounting along a worldline—independent of how we choose external coordinates. *Aging* is the integrated irreversibility—the part of internal accounting that cannot be "turned back" for free.

Accordingly, the reading path of this section has three stages. First we define τ as a system-bound sum/integral quantity and fix minimal requirements (additivity, refinement invariance, reparametrization freedom). Then we define A as the irreversible part of the same accounting and anchor its status as an operational "arrow counter". Finally we couple τ and A to the external calibration and formulate in what sense, in the (nearly) reversible limit, the Minkowski quadric can be read as an effective structure from the balance.[25]

I.4.1 Proper time as integrated internal budget flow

Proper time should do three things: (i) be bound to a subsystem (*intrinsic*), (ii) be additive over serially concatenated evolution segments, and (iii) remain unaffected by mere reparametrization or by finer segmentation of the representation (refinement invariance). Before we formalize, recall: ΔB^{int} was identified in the budget calculus as the share that accounts the proper progress of a system (Subsection I.3.1). We now close this internal flow into a system-bound "clock": the definition is deliberately minimal and strictly separates (a) the internal account flow and (b) the choice of a calibration scale.

Definition I.4.1.1: Proper time along a worldline

Let γ be a worldline of a subsystem S, represented as a sequence of steps $n \to n+1$ with internal budget shares $\Delta B^{\text{int}}_{n,S} \geq 0$ (Subsection I.3.1). A *proper-time calibration* is the choice of a constant rate $\kappa_\tau > 0$ (budget per proper time) such that the discrete proper time

$$\Delta \tau_{n,S} := \frac{\Delta B^{\text{int}}_{n,S}}{\kappa_\tau}, \qquad \tau[\gamma] := \sum_{n \in \gamma} \Delta \tau_{n,S}$$

is well-defined.[1, 2]

Notation convention (rates). If, in addition, an external parametrization t is calibrated and one considers the refinement limit, we write

$$\mathrm{d}\tau = \frac{b^{\text{int}}_t}{\kappa_\tau}\,\mathrm{d}t, \qquad b^{\text{int}}_t := \frac{\mathrm{d}B^{\text{int}}}{\mathrm{d}t} \geq 0.$$

If later τ itself is used as a parameter, we correspondingly write $b^{(\cdot)}_\tau := \mathrm{d}B^{(\cdot)}/\mathrm{d}\tau$ in order to strictly separate rates per t and per τ.

For the definition to be more than a symbol, it must behave "correctly" under concatenation of evolution segments and must not depend on an external parametrization. These requirements are not extra physics, but consistency requirements on a system-bound balance quantity.

[25] See FBA Part II: Time, Proper Time & Minkowski Geometry, Secs. II.6–II.7.

Formula Box I.4.1.1: Properties of proper time

(E1) Additivity. $\tau[\gamma_1 \cdot \gamma_2] = \tau[\gamma_1] + \tau[\gamma_2]$ (serial composition).

(E2) Refinement invariance. Any finite refinement of the sequence leaves $\tau[\gamma]$ unchanged (Lemma I.3.2.1).

(E3) Reparametrization freedom. Changing external coordinates or using a different segmentation into steps does not change τ; τ is bound to S, not to a reference frame.

The scale κ_τ is the only place where "seconds" in the usual sense enter at all: it couples the intrinsic counter to real clocks. We keep this coupling, for now, as a choice and connect it systematically to the external calibration only in the next subsection.

Remark I.4.1.1: Choice of the scale κ_τ

κ_τ is a unit and calibration choice (budget per proper time) that will later be linked to the external calibration (Subsection I.4.3). In practice, κ_τ is chosen such that τ agrees with standard clocks when dissipation is negligible.[a]

[a] See FBA Part II: Time, Proper Time & Minkowski Geometry, Secs. II.5–II.7.

Proper time typically becomes tangible only when comparing two identical systems that are externally "loaded" differently: then external and internal accounting compete for the same step budget. The following example serves as a reading bridge to time dilation, which we later sharpen as a consequence of the quadric.

Two clocks—one heavy, one light

Two identical oscillator clocks A (at rest) and B (moved back and forth). Both have the same internal calibration κ_τ.

With the same step budget (or the same budget allocation), more external accounting flows into ΔB^{ext} for B (motion), leaving less internal budget per step available for the clock dynamics.

Result (nearly reversible regime): $\tau[B] < \tau[A]$ (time dilation as budget redistribution; cf. Lemma I.4.3.1).

I.4.2 Aging as integrated irreversible share

From τ to *aging* is a small but decisive step: we separate, within the internal account, the irreversible share. This yields a second intrinsic counter that operationally carries the arrow of time. What matters is the clean separation: τ counts total internal flow; A counts only the share that, under the given protocol, cannot be undone for free.

Definition I.4.2.1: Aging

Along a worldline γ of a subsystem S, let the decomposition hold

$$\Delta B^{\rm int}_{n,S} = \Delta B^{\rm rev}_{n,S} + \Delta B^{\rm irr}_{n,S}, \qquad \Delta B^{\rm irr}_{n,S} \geq 0.$$

We define the aging increments and the integrated aging by

$$\Delta A_{n,S} := \frac{\Delta B^{\rm irr}_{n,S}}{\kappa_\tau}, \qquad A[\gamma] := \sum_{n\in\gamma} \Delta A_{n,S}.$$

In the refinement limit (with calibrated t) we write

$$\mathrm{d}A = \frac{b^{\rm irr}_t}{\kappa_\tau}\,\mathrm{d}t, \qquad b^{\rm irr}_t := \frac{\mathrm{d}B^{\rm irr}}{\mathrm{d}t} \geq 0,$$

and analogously $b^{\rm irr}_\tau := \mathrm{d}B^{\rm irr}/\mathrm{d}\tau$ if τ is used as a parameter.

For A to truly measure dissipative *load* (and not merely rename things), two robust, purely balance-type inequalities suffice: nonnegativity and the trivial bound by total internal flow.

Formula Box I.4.2.1: Inequalities for aging

(Ag1) Nonnegativity. $A[\gamma] \geq 0$.

(Ag2) Bound. $A[\gamma] \leq \tau[\gamma]$, because pointwise $\Delta A_{n,S} = \Delta B^{\rm irr}_{n,S}/\kappa_\tau \leq (\Delta B^{\rm rev}_{n,S} + \Delta B^{\rm irr}_{n,S})/\kappa_\tau = \Delta\tau_{n,S}$, and summing over γ yields the bound. Equality holds exactly when $\Delta B^{\rm rev}_{n,S} \equiv 0$ along γ (purely dissipative evolution).

The reading is operational: where protocols (channels) irreversibly dump information into inaccessible degrees of freedom, A grows. Where the evolution is reversible, only τ grows. The connection to monotonicities (Spohn/DPI) becomes sharp only once we have specified admissible dynamics as CPTP/GKLS; here A remains, for the moment, a balance quantity.

Remark I.4.2.1: Physical reading

"Aging" is not a metaphor but a balance quantity: where Spohn's inequality or data-processing monotonicity (DPI) is *strict* (unselective description), irreversible budget flows. Reversible (unitary) segments contribute to τ but not to A.[a] [b]

[a]See FBA Part IV: Dynamics & Measurement (GKLS), Secs. IV.4–IV.7.
[b]See FBA Part VIII: Classical Limit, Thermodynamics & Aging, Secs. VIII.6–VIII.8.

A brief look at two extreme cases anchors the intuition.

Thermostat vs. frictionless dynamics

A qubit in a bath-coupled relaxation channel has $b_t^{\mathrm{irr}} > 0 \Rightarrow \mathrm{d}A > 0$.

An isolated spin under pure H (no Lindblad terms) has $b_t^{\mathrm{irr}} = 0 \Rightarrow \mathrm{d}A = 0$, even though $\mathrm{d}\tau > 0$.

I.4.3 Budget-quadric and Minkowski-limit

With τ and A the internal counters are ready. To obtain geometry as an effective structure, we couple them to the *external* cost rates. The front bound (Subsection I.3.3) provides the cone structure; an appropriate choice of κ_τ puts both accountings into a form from which the quadric can be read off.

What matters for the reading path is that we make two things transparent at once. First: which parts are *definition/convention* (e.g. the choice of κ_τ)? Second: which parts are *consequences* of the front bound and the budget balance already introduced? The full derivation, regularity assumptions, and symmetry analysis are collected in Part II; here the minimal operational reconstruction suffices.[26]

Definition I.4.3.1: External calibration and the choice of κ_τ

Let $\kappa_t, \kappa_x > 0$ be the external cost rates (Definition I.3.3.1) and $c := \kappa_t/\kappa_x$ the front constant. As a unit matching, we choose a proper-time scale κ_τ in the same dimension (budget per time).

Convention. In what follows we set $\kappa_\tau := \kappa_t$. This choice only fixes the τ-unit relative to the external t-unit: any alternative $\kappa_\tau = \alpha\,\kappa_t$ with constant $\alpha > 0$ rescales τ globally and leaves the cone structure and the budget inequalities invariant. With $\kappa_\tau = \kappa_t$, in the rest case ($\Delta\mathbf{x} = 0$) in the minimal-reversible regime one has $\Delta\tau_{\mathrm{rev}}^{\mathrm{min}} = \Delta t$, and the quadric takes the standard form.

With this choice, the minimal *reversible* internal requirement of a step in the (nearly) reversible regime can be brought into a standard form. We therefore formulate the quadric as (i) a definition of a minimal reversible share and (ii) a consistency condition that lower-bounds the actually realized reversible accounting.

[26] See FBA Part II: Time, Proper Time & Minkowski Geometry, Secs. II.6–II.7.

Formula Box I.4.3.1: Budget-quadric (discrete) & Minkowski-limit (continuous)

(Q1) Definition (minimal reversible). For a step with external accounting $(\Delta t, \Delta\mathbf{x})$ we define the *minimal reversible* proper-time increment share $\Delta\tau_{\mathrm{rev}}^{\min}$ by

$$\kappa_\tau\,\Delta\tau_{\mathrm{rev}}^{\min} \;:=\; \sqrt{(\kappa_t\,\Delta t)^2 - (\kappa_x\,\|\Delta\mathbf{x}\|)^2}.$$

The front bound guarantees the nonnegativity of the square root.[1–3]

(Q1') Working principle (minimal-reversible regime). Let $\Delta\tau_{\mathrm{rev}} := \Delta B^{\mathrm{rev}}/\kappa_\tau$. In the idealized minimal-reversible regime (no additional reversible "load" beyond the minimal accounting enforced by $(\Delta t, \Delta\mathbf{x})$) one has $\Delta\tau_{\mathrm{rev}} = \Delta\tau_{\mathrm{rev}}^{\min}$; in general we assume as a consistency condition

$$\Delta\tau_{\mathrm{rev}} \geq \Delta\tau_{\mathrm{rev}}^{\min}.$$

(Q2) Standard form. With $\kappa_\tau = \kappa_t$ it follows that

$$(c\,\Delta\tau_{\mathrm{rev}}^{\min})^2 = (c\,\Delta t)^2 - \|\Delta\mathbf{x}\|^2.$$

(Q3) Continuous limit (refinement). In a refinement limit in which the worldline can be described by a (piecewise) smooth curve $(t, \mathbf{x}(t))$, the minimal form reads

$$c^2\,\mathrm{d}\tau_{\mathrm{rev}}^{\min\;2} = c^2\,\mathrm{d}t^2 - \|\mathrm{d}\mathbf{x}\|^2.$$

For the internal accounts actually realized along γ one has componentwise

$$\mathrm{d}\tau = \frac{\mathrm{d}B^{\mathrm{rev}}}{\kappa_\tau} + \frac{\mathrm{d}B^{\mathrm{irr}}}{\kappa_\tau} = \mathrm{d}\tau_{\mathrm{rev}} + \mathrm{d}A.$$

From (Q2) the usual time-dilation formula drops out immediately—*in the minimal-reversible or idealized regime* and under the assumption $\Delta t > 0$. The lemma serves here as orientation: it shows how the familiar form follows from the balance structure, not vice versa.

Lemma I.4.3.1: Time dilation as budget redistribution

With $v := \|\Delta\mathbf{x}\|/\Delta t$ one has $\Delta\tau_{\mathrm{rev}}^{\min} = \Delta t\,\sqrt{1 - v^2/c^2} \leq \Delta t$ and in the limit $\mathrm{d}\tau_{\mathrm{rev}}^{\min} = \mathrm{d}t\,\sqrt{1 - v^2/c^2}$.

Proof Sketch I.4.3.1: Time dilation as budget redistribution

Insert $\kappa_\tau = \kappa_t$ into (Q2) and solve for $\Delta\tau_{\mathrm{rev}}^{\min}$. The front bound $\|\Delta\mathbf{x}\| \leq c\,\Delta t$ guarantees positivity of the square root.

The geometric interpretation is thus clear: geometry does not *impose* the balance; rather, the balance *induces* an effective geometric structure in the appropriate limit. The quadric is not a claim about ontology, but a compact bookkeeping relation that reproduces the observed time/space relations in the relevant regime.

Remark I.4.3.1: Interpretation: geometry from balance

The quadric is a *balance identity* in the minimal-reversible regime: external accounting "consumes" a share of a step's budget, leaving less minimal-reversible internal share available. In the limit, Minkowski structure appears as an effective description of this accounting relation.[a] [b]

[a]See FBA Part II: Time, Proper Time & Minkowski Geometry, Secs. II.6–II.7.
[b]See FBA Part V: Spacetime, Light Cones & Local Field Theory, Sec. V.4.

To close, a measurement bridge: how to make c and the quadric visible *operationally* in the same data, without presupposing geometry as the starting point.

Operational calibration test for c

Determine the minimal external duration $\Delta t_{\min}$ needed to generate correlations over distance $\|\Delta\mathbf{x}\|$ under a fixed budget limit.

Measuring $\Delta t_{\min}(\|\Delta\mathbf{x}\|)$ yields a linear relation with slope $1/c$.

An independent measurement of proper-time differences (nearly reversible regime) validates (Q2)/time dilation.

Interim summary. τ and A are the two internal integrals we need: τ as an *intrinsic clock*, A as an *irreversible counter*. Coupling them to the external calibration via the front yields Minkowski structure in the (nearly) reversible limit—and prepares the ground for formulating admissible dynamics as budget-constrained channels in the following Section I.5.[27]

[27]See FBA Part IV: Dynamics & Measurement (GKLS), Secs. IV.3–IV.7.

I.5 Admissible Dynamics as Budget-Constrained Channels

With Section I.4 we have built the *clock* (proper time) and the *counter* for irreversibility (aging); with Section I.3 the *bookkeeping* (internal/external/irreversible) and the *front*. What is still missing is the *how* of evolution: which classes of processes are physically "admissible" under budget constraints, and which *arrow of time* follows from that?

The reading path of this section has three stages: (i) discrete update steps as CPTP channels (including measurement as a special case), (ii) the refinement limit as a GKLS effective description in proper time, (iii) the data-processing inequality (DPI) and Spohn's inequality as an explicitly operational arrow. The central dividing line throughout is: *non-selective* (deterministic description) vs. *selective* (instrument branches/conditioning as a protocol change).

I.5.1 Discrete steps: CPTP-channels & measurement

The discrete view fits directly to the sequence $\{U_n\}$: for a subsystem S, each update step $U_n \to U_{n+1}$ in the frame F_n induces a transformational description $\rho_{n,S} \mapsto \rho_{n+1,S}$. We treat this effective description as a CPTP channel $\Phi_{n,S}$. If initial system–environment correlations or memory effects are relevant, S is correspondingly enlarged in the frame (environment/memory as part of the effective system description), so that the step map can again be formulated as CPTP.[28]

The next definition box makes two things explicit at once: (a) *admissibility* as the CPTP property (compatibility with ancillas/reference systems) and (b) *budget binding* via a cost functional. The concrete choice of $\mathcal{C}$ is a modeling decision in the FBA sense: it is part of what is counted as the "cost" of a channel in the frame.

[28]The point is purely operational: one expands the system boundary until the step description in the chosen frame can be formulated as a CPTP channel; the cost of this expansion becomes visible later via $\mathcal{C}$ and the budget allocation.

Definition I.5.1.1: Admissible channels (discrete)

The *admissible discrete dynamics* on a subsystem S is a completely positive, trace-preserving map [4–7]

$$\Phi : \mathcal{D}(\mathcal{H}_S) \longrightarrow \mathcal{D}(\mathcal{H}_S).$$

It satisfies a *budget constraint* via a cost functional $\mathcal{C}$:

$$\mathcal{C}(\Phi) \;\leq\; \Delta B_S^{\text{int}} + \Delta B_{\text{alloc}}^{\text{ext}}.$$

Here ΔB_S^{int} is the internal budget share assigned to subsystem S in the step (cf. Subsection I.3.1). The term $\Delta B_{\text{alloc}}^{\text{ext}}$ is the external share allocated to this operation in the frame, with

$$0 \leq \Delta B_{\text{alloc}}^{\text{ext}} \leq \Delta B_n^{\text{ext}}, \qquad \sum_{\text{parallel considered ops } j} \Delta B_{\text{alloc}}^{\text{ext}}(j) \leq \Delta B_n^{\text{ext}},$$

and in particular $\Delta B_{\text{alloc}}^{\text{ext}} = 0$ for purely internal operations.
The functional $\mathcal{C}$ is (i) additive under disjoint parallel composition and (ii) subadditive under serial composition, compatible with Formula Box I.3.2.1.

Domain & finite normalization. In Part I we work in the finite-dimensional case $d := \dim(\mathcal{H}_S) < \infty$. For $\mathcal{C}$ to be well-defined as a finite budget quantity, we fix in the frame an operational state domain (resolution/cutoff parameter $0 < \varepsilon \leq 1$)

$$\mathcal{D}_\varepsilon(\mathcal{H}_S) := \{\rho \in \mathcal{D}(\mathcal{H}_S) : \rho \geq \varepsilon\, \mathbb{I}/d\},$$

where $\mathbb{I}$ denotes the identity on $\mathcal{H}_S$.

Moreover, for a contractive divergence D we define the (dimensionless) maximal DPI gap

$$\Delta_D^\varepsilon(\Phi) := \sup_{\rho,\sigma \in \mathcal{D}_\varepsilon(\mathcal{H}_S)} \Big(D(\rho\|\sigma) - D(\Phi\rho\|\Phi\sigma) \Big) \in [0, \infty).$$

The cost functional is then normalized by a fixed conversion constant $\kappa_{\mathcal{C}} > 0$ (budget per nat/bit, depending on the choice of D):

$$\mathcal{C}(\Phi) := \kappa_{\mathcal{C}}\, \Delta_D^\varepsilon(\Phi).$$

Without such a domain restriction, $\Delta_D(\Phi)$ (and thus $\mathcal{C}$) can in general be ∞; in that case Φ is, by definition, not admissible at finite budget.

To tie budget constraints to concrete channel forms, we recall two standard representations. They serve here as a *toolbox*: (i) Kraus operators are practical for discrete steps and measurement models; (ii) the Stinespring dilation makes explicit where information/distinguishability can flow into environmental degrees of freedom.

Formula Box I.5.1.1: Kraus- and Stinespring-representations

(K1) Kraus. $\Phi(\rho) = \sum_i K_i \rho K_i^\dagger$ with $\sum_i K_i^\dagger K_i = \mathbb{I}$.

(K2) Stinespring. There exist an isometry $V : \mathcal{H}_S \to \mathcal{H}_S \otimes \mathcal{H}_E$ and σ_E such that $\Phi(\rho) = \mathrm{Tr}_E\big[V(\rho \otimes \sigma_E)V^\dagger\big]$.[4–7]

With these building blocks, measurement has no special status but is a special case of CPTP. What is critical is the protocol separation: the *non-selective* description is deterministic (CPTP) and falls under DPI/Spohn; the *selective* description conditions on an outcome and must be treated as an instrument with a classical register (protocol change).

Lemma I.5.1.1: Measurement as CPTP / instrument (selective) and coarse-graining

The *non-selective* measurement map (including an optional classical register, without postselection) is CPTP and thus admissible, provided the budget constraint is satisfied. [5–7]

The *selective* branches of an instrument are CP and trace-*non*-increasing (CP-TNI): for Kraus operators K_i, the map $\rho \mapsto K_i \rho K_i^\dagger$ is CP-TNI and the non-selective map is $\rho \mapsto \sum_i K_i \rho K_i^\dagger$ CPTP.

Important (arrow discipline). Statements of the DPI/Spohn type apply to the *non-selective* CPTP map; selective updates (conditioning on i) are a protocol change and must be modeled as such (classical register/feedback).

Proof Sketch I.5.1.1: Measurement as CPTP / instrument (selective) and coarse-graining

POVMs admit Kraus forms. [5–7] Appending a classical register is isometric; the subsequent partial trace is CPTP, hence the non-selective map is CPTP. For individual outcomes i, $\mathrm{Tr}(K_i \rho K_i^\dagger) \le \mathrm{Tr}(\rho)$, hence CP-TNI. Contractive divergences are monotonically non-increasing under CPTP, so $D(\Phi\rho \| \Phi\sigma) \le D(\rho\|\sigma)$.

Why do we insist on CPTP? Because exactly this class guarantees composability with ancillas/reference systems and thus realizes the operational requirement that local steps remain well-defined even in the presence of arbitrary (including entangled) reference systems.

Remark I.5.1.1: Why CPTP is necessary

Violating positivity produces unphysical negative probabilities; failing trace preservation does not describe a deterministic step map. Selective (postselected) steps must be modeled as an instrument (CP-TNI branches) with an explicit classical register; the deterministic description is then the associated non-selective CPTP map. Non-CP is incompatible with the requirement that local operations on a subsystem remain well-defined in the presence of arbitrary (including entangled) reference systems; otherwise locally implemented steps could artifactually emulate “superluminality”. CPTP is precisely the class compatible with *locality* and *no-signalling* (Section I.6).

Before we go to the continuous limit, we anchor the intuition in three elementary discrete steps and explicitly mark *where* the budget load lands in the calculus (internal/external/irreversible).

Three elementary discrete steps

(Simple unitary) $\Phi(\rho) = U\rho U^\dagger$: $\mathcal{C}(\Phi)$ is purely int and $\Delta B^{\mathrm{irr}} = 0$.

(Dephasing) $\Phi(\rho) = \sum_k P_k \rho P_k$: contractive divergences drop; typically $\Delta B^{\mathrm{irr}} > 0$.

(Relocation) Φ as a channel on a position/register component (or swap/transport in a network): $\mathcal{C}(\Phi)$ is charged to $\Delta B^{\mathrm{ext}}_{\mathrm{alloc}}$; the front bound applies (Lemma I.3.3.1).

Interim summary. Discrete CPTP steps are the "atoms" of operational evolution; costs are booked against $\Delta B^{\mathrm{int/ext}}$, irreversibility against ΔB^{irr}. The refinement limit leads to the GKLS effective theory for open systems.

I.5.2 Continuous limit: GKLS-candidates (open systems)

In the limit of many small steps, evolution is described as a semigroup $(\Phi_\tau)_{\tau \geq 0}$ in *proper time* τ (along the worldline of the subsystem under consideration). The generator has GKLS form; budget rates couple dynamics to cost flows. The key point is: we use GKLS here as an *effective description* of refined CPTP composition, not as an independent postulate about microdynamics.

Definition I.5.2.1: GKLS-generator & budget rates

A continuous admissible evolution in proper time τ is a C_0-semigroup [8–10] with

$$\frac{\mathrm{d}}{\mathrm{d}\tau}\rho \;=\; \mathcal{L}(\rho) = -\,i[H,\rho] + \sum_j \Big(L_j \rho L_j^\dagger - \tfrac{1}{2}\{L_j^\dagger L_j, \rho\} \Big).$$

We write budget rates relative to τ (notation as in Definition I.4.1.1) as

$$b^{\mathrm{rev}}_\tau(\tau) := \frac{\mathrm{d}B^{\mathrm{rev}}}{\mathrm{d}\tau}, \quad b^{\mathrm{irr}}_\tau(\tau) := \frac{\mathrm{d}B^{\mathrm{irr}}}{\mathrm{d}\tau}, \quad b^{\mathrm{ext}}_\tau(\tau) := \frac{\mathrm{d}B^{\mathrm{ext}}_{\mathrm{alloc}}}{\mathrm{d}\tau}, \qquad b^{\mathrm{int}}_\tau = b^{\mathrm{rev}}_\tau + b^{\mathrm{irr}}_\tau,$$

where $B^{\mathrm{ext}}_{\mathrm{alloc}}$ denotes the external accounting allocated along the worldline under consideration (or for the operation under consideration) (cf. Definition I.5.1.1).

Cost rate (without smoothness assumptions). Instead of assuming differentiability of $\tau \mapsto \mathcal{C}(\Phi_\tau)$, we require the integrated budget binding: for all $0 \leq \tau_1 < \tau_2$,

$$\mathcal{C}(\Phi_{\tau_2}) - \mathcal{C}(\Phi_{\tau_1}) \;\leq\; \int_{\tau_1}^{\tau_2} \big(b^{\mathrm{int}}_\tau(s) + b^{\mathrm{ext}}_\tau(s)\big)\,\mathrm{d}s.$$

If $\mathcal{C}(\Phi_\tau)$ is (locally) differentiable, this implies pointwise $\frac{\mathrm{d}}{\mathrm{d}\tau}\mathcal{C}(\Phi_\tau) \leq b^{\mathrm{int}}_\tau(\tau) + b^{\mathrm{ext}}_\tau(\tau)$ as shorthand.

Next we make the arrow of time visible in the continuum: Spohn's inequality is a differential form of the DPI for GKLS semigroups. Again, arrow discipline applies: the statement is *unselective* (deterministic), not conditioned on outcomes.

Formula Box I.5.2.1: Spohn-inequality / continuous DPI

Let ω be a faithful stationary state of the semigroup ($\mathcal{L}(\omega) = 0$). [10, 11] Then

$$\frac{\mathrm{d}}{\mathrm{d}\tau} D(\rho_\tau \| \omega) \;\leq\; 0.$$

If $\frac{\mathrm{d}}{\mathrm{d}\tau} D(\rho_\tau \| \omega) = 0$ holds along a trajectory, then the associated (instantaneous) entropy production in this description is zero. Sufficient is, e.g., purely unitary evolution that respects ω (in particular $[H, \omega] = 0$ and no dissipative terms), or that ρ_τ stays in a dissipation-free invariant subalgebra of the dynamics. For additional "if and only if" characterizations, further structural assumptions (e.g. primitivity/detailed balance) are needed. The dissipative flux is nonnegative and is compatible with $\frac{\mathrm{d}A}{\mathrm{d}\tau} \geq 0$.

The semigroup structure is the formal version of "time intervals can be composed": it guarantees clean serial composition and thus matches exactly the (sub)additivity of budgets from the calculus.

Lemma I.5.2.1: Semigroup property & budget subadditivity

For $\tau_1, \tau_2 \geq 0$ one has $\Phi_{\tau_1+\tau_2} = \Phi_{\tau_2} \circ \Phi_{\tau_1}$ and $\mathcal{C}(\Phi_{\tau_1+\tau_2}) \leq \mathcal{C}(\Phi_{\tau_1}) + \mathcal{C}(\Phi_{\tau_2})$.

Proof Sketch I.5.2.1: Semigroup property & budget subadditivity

Semigroup property of GKLS evolution and subadditivity of the cost functional under serial composition. The identity at $\tau = 0$ corresponds to a zero step/zero cost and is compatible with $\Delta B = 0$.

The link "dissipation $\Rightarrow$ arrow $\Rightarrow$ aging" becomes more concrete in the single-qubit example of amplitude damping, which makes $\frac{\mathrm{d}A}{\mathrm{d}\tau} \geq 0$ visible in a standard dynamics.

Amplitude-damping as budget flow

For $\dot{\rho} = \gamma\Big(\sigma_-\rho\sigma_+ - \frac{1}{2}\{\sigma_+\sigma_-, \rho\}\Big)$ with rate $\gamma > 0$, for suitable ω (e.g. a thermal fixed point) one has $\frac{\mathrm{d}}{\mathrm{d}\tau} D(\rho_\tau \| \omega) \leq 0$, hence consistently $\frac{\mathrm{d}A}{\mathrm{d}\tau} \geq 0$.

The reversible contribution resides in the Hamiltonian part; external costs arise if the damping is coupled to relocation/communication (front).

Interim summary. GKLS generators are the continuous effective description of budgeted CPTP steps. Spohn provides a quantitative version of the arrow of time; b_τ^{irr} measures the aging rate.

I.5.3 Data-processing inequality (DPI) as an operational arrow of time

The DPI says: *distinguishability does not increase under admissible (non-selective) channels.* Without additional budget, no admissible process can "for free" enlarge information distances.

This operationally fixes the direction of evolution: as the one in which suitable divergences do not increase along serial composition. We state the principle, derive an immediate consequence, and finally clarify the role of the front.

Definition I.5.3.1: DPI-admissibility & arrow of time

Let $D(\cdot\|\cdot)$ be a divergence contractive under CPTP maps (e.g. quantum relative entropy). [12, 13]

For every admissible (non-selective) Φ,

$$D\big(\Phi(\rho)\,\|\,\Phi(\sigma)\big) \;\leq\; D\big(\rho\,\|\,\sigma\big).$$

The *direction* of physical evolution is thus the one in which (suitable) divergences do not increase along serial composition $\Phi_{\tau_2} \circ \Phi_{\tau_1}$. The (proper-)time measure τ parametrizes this canonical composition direction.

Coupling to irreversibility. If strict contraction occurs for some pair (ρ, σ), then the evolution is not purely reversible in this sense. In the GKLS limit this corresponds (under the usual regularity/stationarity assumptions) to positive entropy production and thus $\frac{\mathrm{d}A}{\mathrm{d}\tau} > 0$ (cf. Formula Box I.5.2.1); purely unitary segments realize equality and do not contribute to A.

An immediate consequence is the prohibition of “free” unscrambling as long as one operates only on the system (without access to environment/register). The statement is deliberately phrased to use only DPI (no additional structural assumptions); it does not exclude *approximate* recovery, but says that perfect restoration of the original distinguishability is impossible once strict contraction occurs.

Corollary I.5.3.1: No free unscrambling (recovery only on S)

If there exist states ρ, σ with $D(\Phi\rho\|\Phi\sigma) < D(\rho\|\sigma)$, then no admissible CPTP *recovery* $\mathcal{R}$ that acts *only* on the system S (possibly with a local ancilla) and has no access to environmental/memory degrees of freedom or to a classical outcome register can perfectly restore this distinguishability:

$$D(\mathcal{R}\Phi\rho\|\mathcal{R}\Phi\sigma) \;\leq\; D(\Phi\rho\|\Phi\sigma) \;<\; D(\rho\|\sigma).$$

Operationally: perfect restoration requires additional degrees of freedom (e.g. access to the Stinespring environment or to measurement protocols) and/or additional budget allocation; infinitesimally, in the GKLS case this is reflected in the (nonnegative) entropy production rate (Formula Box I.5.2.1).

The DPI is channel-based; only through the front does it become a geometric range statement. This clarifies the role of locality in the FBA: the “arrow” is first a statement about information distances under admissible channels; “geometry” emerges when external accounting limits range.

Remark I.5.3.1: Role of the front and locality

The DPI is *channel-based*; the front (Lemma I.3.3.1) makes it *geometrically* effective: even if internal resources were ample, $\Delta B^{\text{ext}}_{\text{alloc}}$ limits the range per step of operations that build up information/distinguishability between distant registers. This yields the budget cone (no-signalling, Section I.6).

Closing this section. We have put the "engine" of evolution into a precise form: *admissible dynamics* are exactly CPTP/GKLS processes *under budget constraint*; the *arrow of time* is not postulated in addition but is the monotonicity of contractive divergences (DPI/Spohn, unselective description); and *geometry* appears as the range structure of the front. In short: dynamics = physics *within* a cost framework, direction = "distinguishability does not increase for free", range = "external costs are finite".

In the next section (Section I.6) we stress-test these components for composability and locality-compatibility: we show that *serial* and *parallel* composition do not leave the class of admissible channels (closure), that budgets remain additive/subadditive (no "cost arbitrage"), and that *locality/no-signalling* is coherent with the front. Practically: budget cones compose, DPI is preserved under "wiring" of channels, measurement/feedback fits into the CPTP framework, and range limits propagate through complex networks just as reliably as through single steps.

I.6 Composition, Locality & No-Signalling

So far, we have kept accounts *within* a step (Section I.3) and defined proper time/aging *along* a worldline (Section I.4); moreover, we know the class of *admissible* dynamics (Section I.5).

However, the theory becomes physically robust only if two additional stabilities are satisfied: First, "admissible" must be stable under the operations that are unavoidable in any protocol: steps in sequence (serial) and independent subprocesses side by side (parallel). If this were not the case, one could construct an inadmissible overall operation from admissible building blocks (or vice versa) by mere re-segmentation or wiring—and "admissibility" would not be a protocol-robust predicate. Second, local interventions must remain local: without such a locality requirement, the budget front $\|\Delta\mathbf{x}\| \le c\,\Delta t$ (Lemma I.3.3.1) would lose its operational meaning, because distant marginals could then (apparently) be influenced without externally budgeted coupling. This section therefore anchors (i) composition as closure of the process class and (ii) locality as no-signalling of the *unconditional* dynamics, so that neither "bookkeeping" nor "wiring" can create an artificial action-at-a-distance.

I.6.1 Symmetric-monoidal structure (serial/parallel/swap)

If processes are to be the building blocks of physics, they must compose the way experiments are actually built: sequentially (substeps) and in parallel (disjoint sub-setups). That disjoint factors can be swapped is not a dynamical assumption but a representational freedom: the same circuit must not depend on whether we write subsystems as $A \otimes B$ or $B \otimes A$.

Definition I.6.1.1: Structure of admissible processes

The class of systems (objects) and admissible channels (morphisms) carries a *symmetric monoidal structure*:

1. **Serial:** composition $\circ$ is associative with identity id.
2. **Parallel:** monoidal product $\otimes$ for disjoint systems, with unit object $\mathbf{1}$.
3. **Symmetry:** swaps $\sigma_{A,B} : A\otimes B \to B\otimes A$ with $\sigma_{B,A} \circ \sigma_{A,B} = \mathrm{id}$.

We work in a (monoidally equivalent) strictified presentation, so that bracketings/unit isomorphisms need not be tracked separately.[a] Admissible channels are CPTP-/GKLS-induced (Section I.5) and respect the budget constraints.

[a] Strictification is a coherence-theoretic simplification: one replaces the monoidal structure by an equivalent strict presentation; the physical statements are invariant under this equivalence.

This structure is physically controlled only if it is *budget-stable*: once circuits can be composed, the budget balance must scale with them; otherwise "budget" would be manipulable by merely choosing a circuit description (cost arbitrage). This is precisely why the balance postulates from Section I.3 must be compatible with $\circ$ and $\otimes$.

Formula Box I.6.1.1: Budget additivity and cost rules under $\circ$ and $\otimes$

Notation. For a process Φ (possibly composed of several steps), ΔB_Φ denotes the *total budget booked* to this process in the chosen frame (sum of the associated step/subprocess budgets in the sense of Formula Box I.3.2.1).

For admissible processes Φ, Ψ one has (serial composition)

$$\Delta B_{\Psi\circ\Phi} = \Delta B_\Phi + \Delta B_\Psi, \qquad \mathcal{C}(\Psi \circ \Phi) \le \mathcal{C}(\Phi) + \mathcal{C}(\Psi),$$

and for disjoint parallel composition

$$\Delta B_{\Phi\otimes\Psi} = \Delta B_\Phi + \Delta B_\Psi, \qquad \mathcal{C}(\Phi \otimes \Psi) = \mathcal{C}(\Phi) + \mathcal{C}(\Psi).$$

This holds componentwise, compatible with Formula Box I.3.2.1: external shares add over disjoint subprocesses, and internal/irreversible accounts add along the respective worldlines and/or over disjoint subsystems. The same holds in the GKLS limit for the rates $b_\tau^{(\cdot)}$ (cf. Definition I.5.2.1).

With these rules, "admissible" becomes protocol-stable: if an experimental realization is built from admissible substeps, then the composed realization must not fall outside the admissible class, provided the total budget is allocated accordingly.

Lemma I.6.1.1: Closure of admissible channels under wiring

Let Φ, Ψ be admissible and satisfy the budget constraints $\mathcal{C}(\Phi) \le \Delta B_\Phi$ and $\mathcal{C}(\Psi) \le \Delta B_\Psi$ (with $\Delta B_\Phi, \Delta B_\Psi$ the respectively allocated total budgets). Then $\Psi \circ \Phi$ is also admissible under the budget $\Delta B_{\Psi\circ\Phi} = \Delta B_\Phi + \Delta B_\Psi$. For disjoint systems, analogously: $\Phi \otimes \Psi$ is admissible under $\Delta B_{\Phi\otimes\Psi} = \Delta B_\Phi + \Delta B_\Psi$.

Proof Sketch I.6.1.1: Closure of admissible channels under wiring

For serial: $\mathcal{C}(\Psi \circ \Phi) \le \mathcal{C}(\Phi) + \mathcal{C}(\Psi) \le \Delta B_\Phi + \Delta B_\Psi = \Delta B_{\Psi\circ\Phi}$ using Formula Box I.6.1.1. For parallel (disjoint): $\mathcal{C}(\Phi \otimes \Psi) = \mathcal{C}(\Phi) + \mathcal{C}(\Psi)$ and analogously budget additivity.

A particularly critical special case are purely formal wiring steps: if identities or swaps could *somehow* carry budget, then statements about budget balance, DPI, and the front would depend on the chosen diagrammatic representation. That must not happen.

Lemma I.6.1.2: No budget inflation through rewiring

Neither renumbering nor inserting/removing *formal* wire-like identities and swaps changes the budget-accounted process-cost balance. In particular, parallelizing disjoint processes cannot "create" budget.

Proof Sketch I.6.1.2: No budget inflation through rewiring

In the process calculus, id and $\sigma_{A,B}$ are pure relabeling/wiring isomorphisms; by convention they carry neither internal nor external accounting, hence $\Delta B = 0$ and $\mathcal{C} = 0$.

The claim then follows directly from additivity/subadditivity under $\circ, \otimes$ (Formula Box I.6.1.1) and closure of admissible channels under wiring (cf. Lemma I.6.1.1).

Note: a *physical* swap of two spatially separated systems is a transport process and is modeled in the FBA as its own channel with external accounting; it must not be confused with the formal swap $\sigma_{A,B}$.

I.6.2 Tensor structure, partial trace, and no-signalling

The budget front (Lemma I.3.3.1) makes statements about range per step physically meaningful only if one cannot influence distant systems by purely *internal* local operations. This is the role of no-signalling: it ensures that *without* communication and *without* outcome transmission, a local operation does not change the distant marginal.

Lemma I.6.2.1: Local operations, global independence (No-Signalling)

For disjoint systems A, B and a local *CPTP* channel Φ_A, for all global states ρ_{AB},

$$\mathrm{Tr}_A\big[(\Phi_A \otimes \mathrm{id}_B)(\rho_{AB})\big] \;=\; \mathrm{Tr}_A(\rho_{AB}).$$

That is, the B-marginal remains unchanged (no-signalling) *for the unconditional dynamics*, i.e. without postselection and without transmitting a measurement outcome. (Conditioned states after selective instrument branches can change; that is steering and does not violate no-signalling.)[6]

Proof Sketch I.6.2.1: Local operations, global independence (No-Signalling)

Let $\rho'_B := \mathrm{Tr}_A\left[(\Phi_A \otimes \mathrm{id}_B)(\rho_{AB})\right]$. For every observable M_B,

$$\mathrm{Tr}\big[M_B\, \rho'_B\big] = \mathrm{Tr}\big[(\mathbb{I}_A \otimes M_B)\,(\Phi_A \otimes \mathrm{id}_B)(\rho_{AB})\big] = \mathrm{Tr}\big[(\Phi_A^\dagger(\mathbb{I}_A) \otimes M_B)\,\rho_{AB}\big] = \mathrm{Tr}\big[(\mathbb{I}_A \otimes M_B)\,\rho_{AB}\big],$$

since Φ_A is trace-preserving, hence $\Phi_A^\dagger(\mathbb{I}_A) = \mathbb{I}_A$. Complete positivity ensures that this remains well-defined even under arbitrary entanglement with B.

This secures *channel-based* locality. The *geometric* range statement then arises if one additionally requires that building *new* influences/correlations between separated regions is modeled as an *external* coupling and correspondingly charged to the external budget. Here the front enters: it ties the externally available accounting per step to Δt, thereby coupling range to a calibrated time assignment.

Corollary I.6.2.1: Causality cone from the budget front

Since external accounting guarantees the front bound $\|\Delta\mathbf{x}\| \le c\,\Delta t$ (Lemma I.3.3.1), a single step cannot establish an operationally detectable *signal effect* over distances outside the budget cone (here Δt is the coordinate time calibrated in the frame via ΔB^{ext}, cf. Definition I.3.3.1).

In particular: from *initially uncorrelated* preparations (product state and no pre-shared resource between the regions), no *new* correlations between separated regions can be built up in one step over $\|\Delta\mathbf{x}\| > c\,\Delta t$. Pre-existing correlations (e.g. entanglement) are compatible with this and do not violate no-signalling.

Bell correlations without signalling

Two distant stations A, B evaluate entangled pairs. Measurements are local CPTP (non-selective) and wired in parallel. Strong (Bell) correlations are compatible with Lemma I.6.2.1 because the B-marginal is independent of the choice of measurement basis in A. Conditioned states (steering) do not change this as long as no outcome/setting is communicated from A to B. Any *active* information transfer would require additional external accounting and is limited by $\|\Delta\mathbf{x}\| \le c\,\Delta t$.

I.6.3 Local GKLS generators and propagation bounds

If GKLS is read as the refinement limit of admissible, budgeted steps (Subsection I.5.2), then the discrete range structure must reappear in the continuum: an effective dynamics that systematically allowed faster propagation in the limit than the underlying front calibration would be inconsistent with its discrete origin. Locality at the generator level is the standard way to enforce this consistency dynamically.

Definition I.6.3.1: Local GKLS generators

A GKLS generator $\mathcal{L} = \sum_X \mathcal{L}_X$ is called *local* if each term $\mathcal{L}_X$ acts only on a finite region X and the terms are short-ranged (e.g. finite range or sufficiently fast decay of $\|\mathcal{L}_X\|$ with the diameter of X).

Remark I.6.3.1: Lieb–Robinson-type bound

Under standard assumptions on local (Lindblad) generators, Lieb–Robinson-type propagation bounds exist with an effective velocity v_{LR}, such that influences outside a cone $\|\Delta\mathbf{x}\| > v_{\mathrm{LR}}\,\Delta t$ are strongly suppressed.[14]

Here Δt is a (calibrated) coordinate time; a comparison with the front constant c is therefore to be understood in the same units (if needed using the calibration between t and τ along the worldline under consideration, cf. Definition I.4.1.1).

In the FBA, the front is read as a consistency condition: admissible local open dynamics must be compatible with the budget front (operationally: effective propagation must not outrun the limit set by external accounting, i.e. $v_{\mathrm{LR}} \lesssim c$ in calibrated units).

I.6.4 Coherence of the arrows: DPI, front, and composition

To conclude, we bundle the consistency requirements: the information arrow (DPI), the geometry arrow (front), and the composition arrow (wiring) must not undermine one another. This is not cosmetic, but the condition that allows global consequences to be drawn from local statements: DPI must be preserved under wiring; the front must remain stable under iteration/refinement; and no-signalling must be compatible with both.

Formula Box I.6.4.1: Coherence criteria

(K1) DPI is stable under $\circ, \otimes$ (for a contractive divergence D and CPTP channels).

(K2) Front bounds are stable under serial composition and refinement (Formula Box I.3.2.1) and invariant under mere relabeling/wiring.

(K3) No-signalling (Lemma I.6.2.1) holds for unconditional local operations and is compatible with local GKLS generators.

Lemma I.6.4.1: No contradiction of the arrows

If the channels involved are CPTP (or GKLS-induced) and budgets are allocated consistently according to Section I.3, with external allocations respecting the front, then (K1)–(K3) hold. In particular, no admissible composition can violate DPI or the front without leaving the respective premises (CPTP/GKLS or budget/calibration rules).

Proof Sketch I.6.4.1: No contradiction of the arrows

CPTP $\Rightarrow$ DPI/monotonicity, and CPTP is closed under $\circ, \otimes$.

Budget additivity/refinement invariance (Section I.3) and calibration of external accounting $\Rightarrow$ stability of the front bound under composition.

Partial trace + parallel structure $\Rightarrow$ no-signalling for unconditional local operations.

Taken together, these three arrows exclude circumventions via mere wiring/resegmentation.

Interim summary. Composition makes the theory *computable*, locality makes it *physical*. Together—carried by budget balance and the front—they ensure that admissible channels cannot, by representational tricks (wiring/refinement), produce apparent action-at-a-distance or budget arbitrage. This structure is made more concrete in the subsequent parts in the context of field/causal dynamics and geometry.[29] [30] [31]

[29]See FBA Part IV: Dynamics & Measurement (GKLS), Secs. IV.3–IV.7.
[30]See FBA Part V: Spacetime, Light Cones & Local Field Theory, Secs. V.4–V.6.
[31]See FBA Part VI: Gravity & Geometry from Budget Flows, Secs. VI.2–VI.6.

I.7 Positioning Relative to Standard Views

The FBA does not replace the assumption of a prior, universal time parametrization with a "new" metaphysical picture, but with an operational bookkeeping: a sequence of minimal events, budget accounts, and their calibration. This shifts the priority: not "Which metric holds?" comes first, but "Which accounts are operationally accessible, and which bounds follow solely from positivity, additivity, and calibration?"

To situate this change of perspective cleanly, we contrast three familiar views: (i) Newton's "external time parameter", (ii) the block universe of four-dimensional geometry, and (iii) standard quantum mechanics with an external time t. The goal is not polemics, but *translation*: In which regimes does the FBA reduce to the standard language? Where is the standard language an effective shorthand for a balance statement (front/proper time)? And where does it become visible which extra assumptions one has implicitly made when time is postulated as a parameter from the outset?

I.7.1 Newtonian time vs. FBA sequence

Newton's time is a parameter that is the same everywhere and progresses "uniformly". In the FBA, instead, there is an ordinally given sequence $\{U_n\}$ and an external calibration Δt from external accounting (Subsection I.3.3). The decisive question is therefore not whether t "exists", but whether t in a given regime is suitable as an *effective* time parameter, without thereby distorting a system's intrinsic clock (proper time).

This is exactly the Newton limit in the FBA: if the front is practically non-binding and dissipation is negligible, then external and internal accounting are so decoupled that the system-bound proper time practically coincides with the externally calibrated coordinate time. The following statement makes this regime characterization transparent.

Lemma I.7.1.1: Newton limit as a weakly relativistic, weakly dissipative regime

If $v := \|\Delta\mathbf{x}\|/\Delta t \ll c$, then Lemma I.4.3.1 yields

$$\Delta\tau_{\text{rev}}^{\min} = \Delta t\,\sqrt{1 - v^2/c^2} \approx \Delta t.$$

Thus, in the *minimal-reversible* regime (i.e. $\Delta\tau_{\text{rev}} \approx \Delta\tau_{\text{rev}}^{\min}$) and with negligible dissipation ($\Delta B^{\text{irr}} \approx 0$, hence $A \approx 0$), from $\tau = \tau_{\text{rev}} + A$ it follows that $\tau \approx t$ (for $\kappa_\tau = \kappa_t$, Definition I.4.3.1).

In this approximation, FBA dynamics can be described by an external time parametrization t, compatible with the ordinal sequence.

This is the precise sense in which "Newtonian time" appears in the FBA: not as a basic structure, but as a *regime concept*. As soon as either external relocation becomes relevant (near the front) or irreversibility is not negligible, the identification $\tau \approx t$ breaks down in a measurable way.

Remark I.7.1.1: Reading

The FBA does not require a presupposed "flow of time". The sequence counts updates; the quantity t is a *calibration* that becomes an effective time parameter in suitable regimes. This preserves the separation of *order* (before/after) and *measure* (how much).

A laboratory timekeeper

A laboratory clock at rest experiences $\Delta B^{\text{ext}} \approx 0$ and (with good isolation) $\Delta B^{\text{irr}} \approx 0$; its proper-time tick is then an excellent realization of Δt. If the clock is accelerated or dissipatively loaded, t and the ideal-reversible proper-time share diverge measurably—standard time dilation and/or aging.

I.7.2 Block universe vs. ME propagation

The block universe sees "everything at once": a four-dimensional structure (typically: a manifold with a metric) in which "past" and "future" appear as different slices. The FBA, by contrast, is *procedural*: it starts from sequence, budget, and monotonicities and only then asks which geometric effective structure this balance supports in the appropriate limit. This produces no contradiction, but a shift in logical order: geometry is the result of a reconstruction, not a starting postulate.[3, 15]

Remark I.7.2.1: Compatibility via reconstruction

The Minkowski quadric obtained from Subsection I.4.3 yields, in the reversible limit, an effective Lorentz-signature-like quadric. Combined with the causal partial order (Subsection I.2.2), this gives a *causal-metric effective structure* that *reconstructs* the block view in suitable regimes.

A smooth 4D manifold with a metric in the strict geometric sense requires additional regularity/limit assumptions (made explicit in the later parts); in Part I, the operational reconstruction from balance+order suffices.

It is important here what "procedural" does *not* mean: the sequence is not a license for retrocausal modeling. Causal precedence is fixed at the ME level; *only* the bundling of causally separated MEs into common steps is representational freedom (refinement), i.e. a descriptive "gauge".

Corollary I.7.2.1: No retrocausality; co-actualization as a gauge

If $\preceq$ encodes the causal precedence of minimal events (A5), then causal cycles on the ME level are excluded (in particular, no closed precedence loops $e_0 \prec \cdots \prec e_0$). Co-actualizing causally separated MEs is a *representational freedom* (refinement) with no observable consequence—not a physical "intervention into the past".

Two remotely coupled measurements

Two space-like separated measurements can, in the FBA, be co-actualized in one step or represented in two steps. Both representations are equivalent for all observables that depend only on start/end and the balance (Definition I.2.2.2); there is no signalling path that "pops up" by the choice of representation.

I.7.3 Standard QM: external time parameter vs. proper time/sequence

In standard QM, time appears as an external parameter t in the Schrödinger equation.[16, 17] The FBA classifies this as a regime statement: t is an *external calibration* (κ_t), whereas *proper time* τ emerges from internal budgets. As long as one stays in the (nearly) reversible, weakly relativistic regime, this distinction is barely noticeable ($\tau \approx t$); once the front or dissipation becomes relevant, it becomes operationally decisive. In the FBA, the *natural* continuous description of open dynamics is therefore a GKLS evolution in proper time τ; a t-parametrized form is then a derived description via the chain rule: for $\rho(t) := \rho(\tau(t))$,

$$\frac{d}{dt}\rho(t) = \frac{d\tau}{dt}\,\mathcal{L}(\rho) \quad \text{(with } \mathcal{L} \text{ from } Definition\ I.5.2.1\text{)},$$

where $d\tau/dt$ is determined by the budget calibration along the worldline under consideration (Definition I.4.1.1).

Definition I.7.3.1: Unitary limit and effective Schrödinger dynamics

If the effective evolution in the regime under consideration can be modeled as *reversible* (e.g. $b_\tau^{\mathrm{irr}} \equiv 0$ and no dissipative terms, hence effectively $L_j \equiv 0$), then Definition I.5.2.1 reduces to

$$\frac{d}{d\tau}\rho(\tau) = -\,i[H(\tau), \rho(\tau)].$$

In a regime where also $\tau \approx t$ (e.g. the Newton limit) and $\kappa_\tau = \kappa_t$ (Definition I.4.3.1), one equivalently obtains the usual t-parametrized Schrödinger form. Dissipative contributions (the L_j terms) describe open dynamics in proper time τ; with $d\tau/dt$ one obtains the corresponding t-parametrized master equation.

This also sharpens the role of "measurement" compared to the standard presentation: the relevant difference is not philosophical, but protocol-level. The non-selective description is CPTP and hence unconditional dynamics; selection/feedback is a protocol change that must be made explicit as an instrument plus a classical register. Exactly here the information arrow becomes operational.

Remark I.7.3.1: Measurement, CPTP, and DPI as a bridge

Measurements are CPTP in the *non-selective* description (Subsection I.5.1); selective measurement branches are CP-TNI (instrument) and are operationalized via a classical register. Coarse-graining reduces contractive divergences (DPI, Subsection I.5.3).

Thus the "arrow of time" of standard QM is *operationally* grounded in the FBA: not the postulation of a directed t, but monotonicity of distinguishabilities fixes the direction.

When do t and τ diverge?

Rapid relocation (near the front, Lemma I.3.3.1) leads, in the ideal-reversible regime, to $\Delta\tau_{\mathrm{rev}}^{\mathrm{min}} < \Delta t$ (time dilation). Strong dissipation ($\Delta B^{\mathrm{irr}} > 0$) leads to measurable aging $A > 0$ at fixed t-protocol and decouples the real clock dynamics from the ideal-reversible Minkowski tick.

Conclusion of this section. The FBA does not *replace* the familiar views, but *underpins* them: Newtonian time appears as the regime $\tau \approx t$, the block universe as a reconstruction from sequence+calibration, and Schrödinger dynamics as a reversible special case of budgeted GKLS evolution. The added value is a single, shared operational grounding of *front*, *proper time*, and *arrow of time* from one and the same balance logic.[32] [33] [34]

[32] See FBA Part II: Time, Proper Time & Minkowski Geometry, Secs. II.6–II.7.
[33] See FBA Part V: Spacetime, Light Cones & Local Field Theory, Secs. V.4–V.6.
[34] See FBA Part IV: Dynamics & Measurement (GKLS), Secs. IV.3–IV.7.

I.8 Interfaces to the Specialist Treatises (Parts II–X)

Up to this point, we have built a self-contained base layer: a sequence of global states (Section I.2), the budget calculus (Section I.3), proper time/aging and the Minkowski limit (Section I.4), as well as admissible dynamics, arrows (DPI, front), and composition (Sections I.5 and I.6). The following specialist treatises (Parts II through Part X) refine individual aspects.

Precisely because the series is modular, a typical reviewer risk point arises: over long chains of text, meaning often shifts *gradually* (symbols get relabeled, invariants are used only implicitly, or a later geometric assumption is "accidentally" already used in an earlier proof). This Section is therefore not a didactic repetition, but a *stability measure*: we make explicit what is fixed as an import, in which direction arguments are allowed to flow, and which invariants must never be silently abandoned.

For orientation, we deliberately begin with a compact separation *Import vs. Extension*. This separation is the core of modularity: only if it is clear what remains unchanged can a later part introduce additional structure without retroactively reinterpreting the foundation.

I.8.1 Reference anchors per treatise

The specialist treatises Parts II through Part X take over the following building blocks *without* modification and build on them. For each part we name the *goal*, the *imported structures* from the present Part I, and the *extending statements*. The form is deliberately "contractual": *Import* means "do not redefine"; *Extension* means "make precise, but remain compatible".

Imports & extensions at a glance

1. **Part II.** *Goal:* Operationalization of the quadric and geodesics. *Import:* Subsections I.4.1 and I.4.3. *Extension:* smooth limit, worldline variation, geodesics as extremals of the (minimal-)reversible internal accounting $\tau_{\mathrm{rev}}^{\mathrm{min}}$, experimental calibration $\kappa_\tau = \kappa_t$.

2. **Part III.** *Goal:* State spaces, channels, composition. *Import:* Sections I.5 and I.6. *Extension:* concrete divergences/cost functionals $\mathcal{C}$ (incl. domains/cutoffs), structure of measurements/instruments and classical registers.

3. **Part IV.** *Goal:* Generators, Spohn, open systems. *Import:* Subsections I.5.2 and I.5.3. *Extension:* stationary/NESS references, flows/rates $b_\tau^{\mathrm{rev}}, b_\tau^{\mathrm{irr}}, b_\tau^{\mathrm{ext}}$, operational bounds.

4. **Part V.** *Goal:* Local field equations under front/locality. *Import:* Subsection I.3.3 and Section I.6. *Extension:* local GKLS generators, Lieb–Robinson-type bounds, effective light cones.

5. **Part VI.** *Goal:* Geometrization of the flows. *Import:* Subsection I.4.3. *Extension:* effective metrics from calibrations (κ_t, κ_x) and internal stresses; coupling of budget/divergences to curvature.

6. **Part VII.** *Goal:* Scale management of the κ relations. *Import:* Definitions I.3.3.1 and I.4.3.1. *Extension:* flow equations for $\kappa_t, \kappa_x, \kappa_\tau$; stability regimes of $c = \kappa_t/\kappa_x$.

7. **Part VIII.** *Goal:* Macroscopic behavior from $A[\gamma]$. *Import:* Subsection I.4.2 and Formula Box I.5.2.1. *Extension:* entropy production, Euler–Lagrange forms for irreversible flows, effective transport equations.

8. **Part IX.** *Goal:* Global sequence, scale flow of cosmic calibrations. *Import:* Sections I.3 and I.4. *Extension:* budget equations on large-scale slices, time-dilation inflation as calibration dynamics.

9. **Part X.** *Goal:* Testable differences, mapping FBA $\leftrightarrow$ QM/GR. *Import:* all of the above. *Extension:* protocols, limiting-case tests, consistency checks (pass/fail).

The overview is deliberately to be read *strictly*: if a later part wants to formulate something “more conveniently” at some point (e. g. geometrically instead of in balance terms), then this is only permissible if the corresponding effective structure is derived *from* the imports or declared as an explicit additional limit assumption. This is exactly how one avoids “proof by vocabulary shift”.

I.8.2 Dependencies & data flow

Besides “what is imported” one also needs “in which direction may one argue”. The point is not didactic, but logical: without directed data flow one gets circular reasoning (e. g. geometry is used to justify the front while the front is supposed to justify geometry). We therefore fix a simple arrow structure: foundations $\to$ kinematics/dynamics $\to$ locality/geometry/scales $\to$ macroscopic behavior/cosmos $\to$ tests.

Formula Box I.8.2.1: Dependency graph (informal)

$$(\text{I) Foundations} \to \{(\text{II) Time, (III) Kinematics, (IV) Dynamics}\}$$
$$\to \{(\text{V) LFT, (VI) Grav, (VII) Scales}\}$$
$$\to \{(\text{VIII) Thermo, (IX) Cosmos}\} \to (\text{X) Tests}$$

From this arrow structure follow practical reading paths: if one reads a block in isolation, one can move forward along the arrows without importing prerequisites "from the back" that are meant to be derived only in the treatise in question.

Remark I.8.2.1: Reading guide

- **Geometry:** (II)→(V)→(VI) – quadric ⇒ light cones ⇒ geometry from budget flows.
- **Open systems:** (III)→(IV)→(VIII) – kinematics ⇒ GKLS/Spohn ⇒ thermodynamics & aging.
- **Cosmos:** (VII)→(IX) – scale management of the κ's / stability of c ⇒ TDI & cosmic dynamics.

I.8.3 Interface contracts (invariants)

The dependency arrows alone are not enough: a treatise can go "forward" and still break invariants (e. g. refinement leaks, hidden postselection, superluminal auxiliary constructions). Therefore Parts II–X commit to a small set of explicit invariants that secure exactly the typical breakpoints: (i) concept drift (primitives), (ii) representational artifacts (refinement), (iii) arrow/causality violations (DPI/front/locality).

Definition I.8.3.1: I0: Imported primitives

The treatises (Parts II–X) import unchanged: Definitions I.2.1.1, I.2.3.1, I.3.1.1 and I.5.3.1, the balance equations Formula Box I.3.2.1, and the front bound Lemma I.3.3.1. No treatise introduces a new "time variable" as a postulate; metric quantities are *calibrations*.

Definition I.8.3.2: I1: Refinement invariance

All observable functionals (proper time, aging, cost/divergence drops) are invariant under finite refinements of the sequence (Lemma I.3.2.1 and Definition I.2.2.2). Proofs must use or preserve this invariance explicitly.

Definition I.8.3.3: I2: DPI/Spohn as the arrow

Every dynamical statement (discrete or GKLS) respects contractive divergences (Subsection I.5.3) and, *where applicable* (appropriate reference states such as stationary/NESS), in the limit the Spohn monotonicity (Formula Box I.5.2.1). Time direction is argued *only* via monotonicity, not via an external time postulate.

Definition I.8.3.4: I3: Front/locality

The front bound limits, per step, *causal influence/signalling* in the externally calibrated geometry (Lemma I.3.3.1 and Corollary I.3.3.1). In particular, the *build-up* of new correlations from locally uncorrelated initial situations can occur per step only within the budget cone; here "uncorrelated" is to be understood operationally (product preparation and no pre-shared resource / no outcome transfer between the regions). Pre-existing (e. g. entangled) correlations are compatible with this. Local statements rest on CPTP locality and partial trace (no-signalling, Subsection I.6.2). No result may bypass these bounds.

Definition I.8.3.5: I4: Calibrations κ & metric

$\kappa_t, \kappa_x, \kappa_\tau$ are measurement/calibration relations (Definitions I.3.3.1 and I.4.3.1). Effects on "constants" are treated as *scale flow* (Part VII), *not* as postulates of new constants.

I.8.4 Examples of using the contracts

The contracts are only helpful if one actually carries them along as side conditions in typical proof steps. The following examples therefore mark the critical point in each case: where exactly a contract prevents artifacts (and does not merely "sound nice").

Part II (Time & Minkowski) uses I0–I4

From Formula Box I.4.3.1 geodesics are derived as an extremal problem of the (minimal-)reversible internal accounting. I1 ensures that the choice of geodesics does not depend on the granularity of the sequence. I2/I3 prevent "shortcuts" across the cone boundary; I4 couples to clocks/rulers.

Part V (LFT) uses I1–I3

Local GKLS generators define field dynamics; in combination with I3 (front/locality) and standard locality assumptions, Lieb–Robinson-type propagation bounds follow. I2 sets the arrow of time; I1 guarantees that lattice refinement (discretization) does not generate artifacts.

Part IX (TDI) uses I0, I4

Cosmic budget equations handle κ-flows on large-scale slices. TDI appears as variation of κ_t/κ_τ along the global sequence; the interpretation remains calibrational (I4), *without* new postulates about "cosmic time" (I0/I2).

I.8.5 Typical failure modes (and how to avoid them)

The most common problems in modular series are rarely calculation errors, but silent protocol changes: one uses geometry before it is calibrated; one argues "forward in t" where DPI/Spohn should actually supply the direction; or one implicitly builds in communication via outcome forwarding and then calls it "local". The following list is therefore meant as an anti-drift check.

Antipatterns

(F1) Implicit postulate. c or Minkowski as an axiom instead of as a calibration/limit. *Symptom:* Direct use of ds^2 or a light-cone structure in hypotheses without prior calibration. *Countermeasure:* Derive strictly via the budget quadric and front calibration (Subsection I.4.3).

(F2) Refinement leak. Results depend on step granularity. *Symptom:* Values change under rebinning/refinement. *Countermeasure:* Prove refinement invariance explicitly (contract I1) via Lemma I.3.2.1 and Definition I.2.2.2.

(F3) Arrow of time "from outside". Direction justified by a t-postulate instead of monotonicity. *Symptom:* "Forward in t" used as justification for entropy increase. *Countermeasure:* Derive the arrow exclusively from DPI/Spohn (Subsection I.5.3 and Formula Box I.5.2.1); t remains a calibration.

(F4) Superluminal auxiliary constructions. Signalling or build-up of new correlations outside the budget cone via a modelling trick. *Symptom:* In a single step, causal influence arises with $\|\Delta\mathbf{x}\| > c\,\Delta t$. *Countermeasure:* Enforce front bound & no-signalling (Lemmas I.3.3.1 and I.6.2.1 and Corollary I.3.3.1); use local CPTP structure.

(F5) Constants confusion. Introducing new "constants" instead of a scale flow of the κ relations. *Symptom:* Fixed parameters without empirical calibration. *Countermeasure:* Treat as calibrations (Definitions I.3.3.1 and I.4.3.1) and handle under scale flow (Part VII).

In practice, a short preflight list is often more useful than long prose: it forces one to carry the three critical invariants (refinement, arrow, front) visibly along with every major result.

Remark I.8.5.1: Checklist for each treatise

1. **Name the imports.** Explicitly list the concretely *imported* definitions/boxes from Part I (e. g. states/MEs, budget calculus, DPI/front) before introducing new structure.
2. **Prove compatibility.** For the main theorems, explicitly check: *(i)* DPI/Spohn monotonicity, *(ii)* front bounds, *(iii)* refinement invariance. Refer to the corresponding foundational results.
3. **Disclose calibrations.** Make all used κ relations ($\kappa_t, \kappa_x, \kappa_\tau$) and their measurement procedures explicit; note whether/where scale flow (Part VII) is used and how empirical anchoring is implemented.
4. **Reduce to standard forms.** Identify a clear regime/limit that reproduces familiar descriptions (e. g. unitary/t-parametrized, Newton limit, effective 4D geometry); locate this in the positioning discussion (Section I.7).

Conclusion of this section. The interface contracts ensure that the treatises Parts II–X share the same foundation: *sequence* instead of presupposed time, *budget* instead of free resources, *calibration* instead of unexplained constants, *DPI* instead of metaphysical arrows, and *front* instead of ad hoc postulated light cones. This keeps the theory modular and testable, and the chain of argument coherent from the qubit to the cosmos.

I.9 Summary & Checklist (Pass/Fail)

The preceding Sections have built the stage (Section I.2), the bookkeeping (Section I.3), clocks/counters (Section I.4), dynamics & arrows (Section I.5), and composition & locality (Section I.6). This final Section condenses all of that into a *working checklist*: How do we tell whether a concrete model, a proof, or an experiment is *FBA-compatible*? And, most importantly: What must one *necessarily* check so that later specializations (Parts II to X) do not silently shift the foundation?[35]

The checklist is deliberately set up as Pass/Fail: it is not meant to "interpret nicely", but to show *early* where an argument fails due to refinement leaks, uncalibrated constants, or selection-induced protocol switches. In empirical applications, the tests always hold *up to tolerances* $\delta_{*,\cdot}$ (windows, residuals, bootstrap, pass/fail bands); these $\delta_{*,\cdot}$ are data-/protocol-level quantities and must not be confused with model/approximation parameters ε.

I.9.1 Core statements in ten lines

Before we test anything, we make explicit the minimal structure that must *not* be changed later without saying so. This is less a "summary" than an *inventory of invariants*: if you deviate here, you no longer have "the FBA" but a different theory.

[35] We mean Parts II to X; an overview of the interfaces is given in Section I.8.

FBA in ten lines

1. Reality updates in an ordinal sequence $\{U_n\}$; an update step $U_n \to U_{n+1}$ can bundle one or more minimal events (no presupposed time parameter, Section I.2).
2. Per step there is a budget $\Delta B = \Delta B^{\text{int}} + \Delta B^{\text{ext}}$ with $\Delta B^{\text{int}} = \Delta B^{\text{rev}} + \Delta B^{\text{irr}}$ (Subsection I.3.1).
3. Additivity under series/parallel composition and refinement invariance are non-negotiable (Formula Box I.3.2.1 and Lemma I.3.2.1).
4. External calibrations define a front bound $\|\Delta\mathbf{x}\| \leq c\,\Delta t$ with $c = \kappa_t/\kappa_x$ (Lemma I.3.3.1).
5. Proper time is the integrated *internal* flux; aging is the integrated *irreversible* flux (Section I.4).
6. The Minkowski quadric is a balance relation in the (minimal-)reversible limit, not a postulate (Subsection I.4.3).
7. Admissible processes are CPTP (discrete) or GKLS (continuous)[a] *with* a budget restriction (Section I.5).
8. The arrow of time is operational: contractive divergences are monotone (DPI; in the GKLS limit, where applicable, Spohn)[b] (Subsection I.5.3 and Formula Box I.5.2.1).
9. Composition is (symmetric-)monoidal; no-signalling follows from CPTP + partial trace, and the front makes reachability geometrically effective (Section I.6).
10. Newton, Schrödinger, and block pictures appear as regimes or reconstructions, not as axioms (Section I.7).

[a] CPTP = *completely positive, trace-preserving.* GKLS = *Gorini–Kossakowski–Lindblad–Sudarshan* form of generators for open-system dynamics.

[b] DPI = data processing inequality (monotonicity of contractive divergences under CPTP); "Spohn" denotes a differential form of this monotonicity under suitable assumptions (e.g. stationary/faithful reference state). See Subsection I.5.3.

From these ten lines it follows: anyone who wants to argue in an FBA-compatible way must *always* show (or cleanly cite as an import from Part I) that refinement produces no artefacts, that c is calibrated (not postulated), and that arrow/locality are not "smuggled in" via selective descriptions.

I.9.2 Pass/Fail criteria (mandatory fields)

The following mandatory fields are deliberately short and technical: they are the points where failure modes typically occur. In purely model/proof settings the criteria are "hard". In empirical settings they hold up to explicit, protocol-dependent tolerances $\delta_{*,\cdot}$ (e.g. $\delta_{B,*}$ for balance residuals, $\delta_{D,*}$ for DPI tests, $\delta_{F,*}$ for front tests).

Formula Box I.9.2.1: Mandatory criteria for FBA compatibility

(C1) Balance & refinement *Requirement:* ΔB is additive under series ($\circ$) and parallel ($\otimes$); all functionals built from $\Delta B^{\mathrm{int/ext/irr}}$ are invariant under finite refinement.
Minimal test: For every decomposition, $\sum_k \Delta B_{(k)} = \Delta B$ and likewise componentwise; τ, A do not change under finer segmentation (up to an explicit residual $\delta_{B,*}$ in empirical accountings).
Reference: Formula Box I.3.2.1 and Lemma I.3.2.1.

(C2) Front *Requirement:* External calibrations κ_t, κ_x are stated explicitly and imply the bound $\|\Delta\mathbf{x}\| \le c\,\Delta t$ with $c = \kappa_t/\kappa_x$.
Minimal test: State the used κ-rules (e.g. $\Delta B^{\mathrm{ext}} = \kappa_t\,\Delta t$ and $\kappa_x\|\Delta\mathbf{x}\| \le \Delta B^{\mathrm{ext}}$) and prove the front bound per step (up to a measurement band $\delta_{F,*}$ if $\Delta t, \Delta\mathbf{x}$ are estimated experimentally).
Reference: Definition I.3.3.1 and Lemma I.3.3.1.

(C3) Proper time & aging *Requirement:* $\tau[\gamma] = \sum \Delta B^{\mathrm{int}}/\kappa_\tau$, $A[\gamma] = \sum \Delta B^{\mathrm{irr}}/\kappa_\tau$ with $0 \le A[\gamma] \le \tau[\gamma]$.
Minimal test: Disclose the choice of κ_τ and check the bound $A \le \tau$; in data form: violations must be reported as estimation/window errors $\delta_{A,*}$, not as "new physics".
Reference: Section I.4.

(C4) Dynamics *Requirement:* Discrete CPTP (Kraus/Stinespring), continuous GKLS; cost functor $\mathcal{C}$ satisfies $\mathcal{C}(\Psi\circ\Phi) \le \mathcal{C}(\Psi) + \mathcal{C}(\Phi)$ and, in the proper-time limit, an integrated budget constraint

$$\mathcal{C}(\Phi_{\tau_2}) - \mathcal{C}(\Phi_{\tau_1}) \;\le\; \int_{\tau_1}^{\tau_2} \big(b_\tau^{\mathrm{int}}(\tau) + b_\tau^{\mathrm{ext}}(\tau)\big)\,d\tau,$$

(if $\mathcal{C}(\Phi_\tau)$ is differentiable, this implies pointwise $\frac{d}{d\tau}\mathcal{C}(\Phi_\tau) \le b_\tau^{\mathrm{int}} + b_\tau^{\mathrm{ext}}$ as a shorthand).
Minimal test: Provide Kraus/Stinespring or a GKLS generator and prove cost subadditivity; disclose domain/cutoff and normalization of $\mathcal{C}$ (e.g. $\mathcal{D}_\varepsilon$, $\kappa_{\mathcal{C}}$); if initial correlations matter, choose system boundary/frame so that the effective step description is CPTP.
Reference: Section I.5.

(C5) Arrows (DPI/Spohn) *Requirement:* For a contractive divergence D, $D(\Phi\rho\|\Phi\sigma) \le D(\rho\|\sigma)$; in the continuum (where applicable, e.g. stationary/faithful reference ω) $\frac{d}{d\tau}D(\rho_\tau\|\omega) \le 0$.
Minimal test: Document the choice of D and show DPI (discrete) or Spohn monotonicity (GKLS limit, under the assumptions actually used) for the Φ or $\mathcal{L}$ employed; mark selective/feedback-based protocols explicitly as protocol switches (otherwise DPI is applied incorrectly).
Reference: Subsection I.5.3 and Formula Box I.5.2.1.

(C6) Locality *Requirement:* No-signalling: $\mathrm{Tr}_A[(\Phi_A \otimes \mathrm{id}_B)\rho_{AB}] = \mathrm{Tr}_A\,\rho_{AB}$ (unconditional local operations); the front bounds, per step, *signalling/causal influence* and in particular the *creation* of *new* correlations from operationally uncorrelated initial conditions (product preparation and no pre-shared resource/no outcome transfer between the regions).
Minimal test: Partial-trace test for local channels and an operational demonstration that causal influence is only possible within $\|\Delta\mathbf{x}\| \le c\,\Delta t$ (up to declared measurement windows $\delta_{NS,*}$, $\delta_{F,*}$).
Reference: Subsection I.6.2 and Lemma I.3.3.1.

The point of the mandatory fields is not to "prove everything", but to control the *places* where otherwise seemingly spectacular effects arise from bookkeeping alone: refinement then produces "physics", selection is sold as "dynamics", or reachability is gained via implicit communication.

Corollary I.9.2.1: Consequence of the mandatory fields

If (C1)–(C6) are satisfied, then: (i) no superluminal communication (in the sense of operational signalling), (ii) no "free" increase of distinguishability under admissible channels, (iii) Minkowski structure as a consistent limit in the (minimal-)reversible regime under a calibrated front, (iv) Newton/Schrödinger as special cases in the respective declared approximation regimes.

I.9.3 Audit protocol (practical checks)

In practice, projects rarely fail because a single statement is wrong, but because budgets/-calibrations/protocols are not fully inventoried. The following audit therefore forces you to clarify *objects and interfaces* first and only then move on to derivations.

Definition I.9.3.1: Model/proof audit in five steps

(S1) Inventory *Goal:* Clarify scope and interfaces.
Output: List of all systems, decomposition into disjoint subsystems, set of used channels $\{\Phi_i\}$ or generators $\{\mathcal{L}_j\}$.

(S2) Balance *Goal:* Consistent bookkeeping.
Check: Per step, report $\Delta B^{\mathrm{int}}, \Delta B^{\mathrm{ext}}, \Delta B^{\mathrm{irr}}$.
Proof: Additivity under series ($\circ$) and parallel ($\otimes$) and refinement invariance of the budget-derived functionals (empirically: log residuals as $\delta_{B,*}$).

(S3) Calibrate *Goal:* Fix external and internal scales.
Data: State $\kappa_t, \kappa_x, \kappa_\tau$ explicitly.
Test: Determine $c = \kappa_t/\kappa_x$ and check $\|\Delta\mathbf{x}\| \le c\,\Delta t$ per step (with measurement band $\delta_{F,*}$ if needed).

(S4) Check arrow *Goal:* Verify the direction of evolution.
Choice: Fix a contractive divergence D.
Proof: Show DPI for all deployed Φ and, in the continuous regime (where applicable), Spohn monotonicity for $\mathcal{L}$. Treat selective/feedback-based protocols as protocol switches (otherwise DPI is misread).

(S5) Test locality *Goal:* Secure no-signalling and reachability bounds.
Tests: Partial-trace test $\mathrm{Tr}_A[(\Phi_A \otimes \mathrm{id}_B)\rho_{AB}] = \mathrm{Tr}_A\,\rho_{AB}$. Operational reachability test: causal influence or creation of *new* correlations from operationally uncorrelated initial conditions (product preparation and no pre-shared resource/no outcome transfer) only within the budget cone $\|\Delta\mathbf{x}\| \le c\,\Delta t$.

A useful side effect of the audit: if something "does not fit", it is clear *which contract* is broken (balance, calibration, arrow, or locality) — instead of retreating into interpretations.

Quick paper-and-pencil sanity check

Check 1 – c/metric without calibration? Is there an implicitly postulated c or a metric rather than κ-calibration anywhere? $\Rightarrow$ **Fail**.

Check 2 – refinement leak? Do statements depend on the number of intermediate steps? $\Rightarrow$ **Fail**.

Check 3 – DPI violated? Is distinguishability increased after CPTP without a budget source? $\Rightarrow$ **Fail**.

Check 4 – front violated? Does causal influence or correlation build-up (from operationally uncorrelated initial conditions without pre-shared resource/outcome transfer) occur with $\|\Delta\mathbf{x}\| > c\,\Delta t$? $\Rightarrow$ **Fail**.

I.9.4 Proof roadmap (minimal required scope)

New results in the series are typically formulated as lemma/theorem/proposition. For these results to be not only "formally correct" but also *FBA-compatible*, they must cover a minimal required scope: well-definedness (refinement), arrow (DPI/Spohn), front (calibration), and clean composition.

Formula Box I.9.4.1: Mandatory proof obligations for new results

(P1) Well-definedness *Goal:* Used functionals are independent of step granularity.
Proof: Show refinement invariance for all relevant quantities, in particular $\tau, A, \mathcal{C}$.
Reference: Lemma I.3.2.1.

(P2) Monotonicity *Goal:* Time direction is fixed operationally.
Proof: Choose a contractive divergence D and establish $D(\Phi\rho\|\Phi\sigma) \leq D(\rho\|\sigma)$; in the continuous regime (under the assumptions actually used, e.g. stationary/faithful reference ω) show $\frac{d}{d\tau}D(\rho_\tau\|\omega) \leq 0$ (Spohn).
Reference: Formula Box I.5.2.1 and Definition I.5.3.1.

(P3) Front compatibility *Goal:* Reachability and time estimates are justified calibratorily.
Proof: Reduce all bounds explicitly to κ-calibration and $\|\Delta\mathbf{x}\| \leq c\,\Delta t$ with $c = \kappa_t/\kappa_x$.
Reference: Lemma I.3.3.1 and Definition I.3.3.1.

(P4) Limit *Goal:* Clear return path to standard regimes.
Proof: State the conditions under which Newton/Schrödinger/Minkowski is obtained (e.g. $v \ll c$, $\Delta B^{\mathrm{irr}} = 0$, $\kappa_\tau = \kappa_t$).
Reference: Lemma I.4.3.1, Definition I.7.3.1, and Subsection I.4.3.

(P5) Composition *Goal:* Closure and clean bookkeeping under series/parallel composition.
Proof: Prove additivity of budgets and subadditivity of costs under $\circ, \otimes$ – no budget inflation by wiring.
Reference: Formula Box I.6.1.1 and Lemma I.6.1.2.

I.9.5 Red flags (typical failure modes)

The red flags are deliberately redundant with the mandatory criteria: they are the fastest indicators that an argument is drifting toward "implicit postulate" or "protocol switch without marking".

Red flags

(R1) Time as an independent variable *Detection:* "Time" is introduced as a dynamical quantity, decoupled from ΔB^{int}.
Immediate action: Treat time exclusively as calibration and tie it to the proper-time definition; use internal fluxes as the primary quantity (Section I.4).

(R2) Uncalibrated c *Detection:* Use of c without establishing $c = \kappa_t/\kappa_x$.
Immediate action: Disclose κ_t, κ_x, calibrate c, and verify the front bound (Definitions I.3.3.1 and I.4.3.1 and Lemma I.3.3.1).

(R3) Refinement leak *Detection:* Statements change under finer segmentation or co-actualization.
Immediate action: Prove refinement invariance explicitly, in particular for $\tau, A, \mathcal{C}$ (Lemma I.3.2.1 and Definition I.2.2.2).

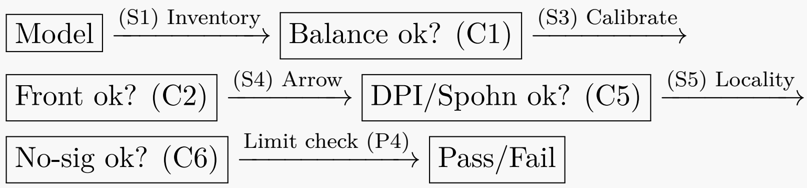
(R4) DPI circumvention *Detection:* Non-CP maps, postselected "miracles" without budget, increasing divergence after CPTP, or applying DPI/Spohn to selective branches without marking the protocol switch.
Immediate action: Use only CPTP or GKLS; treat selective steps as instrument/protocol switch; declare the budget source and prove DPI/Spohn for the *unconditional* description (Definition I.5.3.1, Formula Box I.5.2.1, and Section I.5).

(R5) Front/no-signalling violated *Detection:* Causal influence (or creation of *new* correlations from operationally uncorrelated initial conditions without pre-shared resource/outcome transfer) with $\|\Delta\mathbf{x}\| > c\,\Delta t$, or changing the B-marginal by a local intervention on A.
Immediate action: Run the front bound and partial-trace test; use local generators (Lemmas I.3.3.1 and I.6.2.1 and Subsections I.6.2 and I.6.3).

I.9.6 Decision tree (quickstart)

Finally, a compact flow that makes Pass/Fail operational: it is meant to help decide *in which order* to test, so you do not compute symptoms (e.g. apparent superluminality) while the cause is a missing calibration proof or a refinement leak.

Formula Box I.9.6.1: Quickstart flow



Closing remark. The FBA asks for little, but it is *strict* about that little: balance, front, arrow, locality. If you satisfy these four pillars (with explicit calibrations and stated tolerances $\delta_{*,\cdot}$ in empirical situations), you get geometry (Minkowski limit), dynamics (GKL-S/Schrödinger regime), and causality (budget cone/no-signalling) not as postulates but as testable consequences of clean bookkeeping.

I.10 Appendix: Overview of the FBA Series (Parts I–X)

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1. ***Part I*: FBA-Foundations: Ordering, Budget, Proper Time & Arrows.** *Goal:* Provide the base layer: ordering, budget, proper time/aging, front and the operational arrow of time (DPI); Minkowski limit from the budget quadric; admissible dynamics and locality/no-signalling. *Import:* – (reference for all subsequent parts). *Extension:* interface contracts, pass/fail checklists, reading guide.
2. ***Part II*: Time, Proper Time & Minkowski Geometry.** *Goal:* Capture proper time/quadric operationally and derive geodesics. *Import:* foundations (ordering, budget, proper time, front/DPI). *Extension:* smooth limit, variational principle on worldlines, calibration κ_τ.
3. ***Part III*: Quantum Kinematics & CPTP Channels.** *Goal:* State spaces and channels (CPTP) including composition. *Import:* foundations (budget, channel viewpoint, composition). *Extension:* concrete divergences/cost functionals $\mathcal{C}$, measurements, and classical registers.
4. ***Part IV*: Dynamics, Measurement & GKLS (Open Systems).** *Goal:* Continuous open dynamics (GKLS) and the operational arrow of time. *Import:* channels/DPI. *Extension:* Spohn monotonicity, stationary/NESS references, flows $b^{\mathrm{rev}}, b^{\mathrm{irr}}, b^{\mathrm{ext}}$.
5. ***Part V*: Spacetime, Light Cones & Local Field Theory.** *Goal:* Local field equations under front/locality. *Import:* front, composition, no-signalling. *Extension:* local GKLS generators, Lieb–Robinson-type bounds, effective light cones.
6. ***Part VI*: Gravity & Geometry from Budget Flows.** *Goal:* Geometrization of budget flows. *Import:* budget quadric/proper time. *Extension:* effective metrics from calibrations (κ_t, κ_x) and internal stresses; coupling to curvature.
7. ***Part VII*: Constants, Scales & Renormalization.** *Goal:* Scale running of the calibration theorems. *Import:* $c = \kappa_t/\kappa_x$, κ_τ. *Extension:* flow equations for $\kappa_t, \kappa_x, \kappa_\tau$; stability of c.
8. ***Part VIII*: Classical Limit, Thermodynamics & Aging.** *Goal:* Macroscopic behavior from $A[\gamma]$ (aging) and DPI. *Import:* proper time/aging, Spohn. *Extension:* entropy production, Euler–Lagrange forms for irreversible flows, effective transport equations.
9. ***Part IX*: Cosmic Dynamics, Time Dilation & Inflation (TDI).** *Goal:* Cosmic ordering & calibration flow. *Import:* budget, proper time/front. *Extension:* budget equations on large-scale slices; time-dilation inflation as calibration dynamics.
10. ***Part X*: Predictions, Falsifiability & Bridge FBA → QM ↔ GR.** *Goal:* Testable differences and bridges FBA ↔ QM/GR. *Import:* all foundational building blocks. *Extension:* protocols, limiting-case tests, overdetermined consistency relations (pass/fail).

All parts of the FBA series are available in both English and German at

https://www.frame-budget-approach.eu

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